MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
Department of Aeronautics and Astronautics

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Problem Set No.3 Due: Sep 30, 2008

Problem 1: text problem 3.5

Problem 2: text problem 3.6

Problem 3: text problem 3.9

Problem 4: text problem 3.22

Problem 5: text problem 3.61

Problem 6: Derivation of Poisson Process from Random Variables
Let $X_1, ..., X_n$ be mutually independent random variables with the exponential distribution:

$$\text{pdf: } f(x) = \alpha e^{-\alpha x} \qquad \text{cdf: } F(x) = 1 - e^{-\alpha x}, \text{ for } x \geq 0 \tag{1}$$

a) Show that the sum $S_n = \sum_{i=1}^{n} X_i$ has a density g_n and distribution function G_n given by

$$g_n(x) = \alpha \frac{(\alpha x)^{n-1}}{(n-1)!} e^{-\alpha x}$$
$$Pr[S_n \leq x] = G_n(x) = 1 - e^{-\alpha x}\left(1 + \frac{\alpha x}{1!} + \cdots + \frac{(\alpha x)^{n-1}}{(n-1)!}\right)$$

(Hint: Prove by induction using the relationship $g_{n+1}(t) = \int_0^t g_n(t-x) g_0(x) dx$.)
b) Consider a sequence of packet arrivals whose inter-arrival times are i.i.d. with exponential distribution. Assume that the common exponential distribution is given by (1). Denote by X_n the random variable of inter-arrival time between $(n-1)$-th and n-th packets. Let $T_n = \sum_{i=1}^{n} X_i$. Using the result in a), show that the packet arrival process satisfying these properties follows Poisson process.