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6.263/16.37	Issued: Oct 14, 2008
Problem Set No.6	Due: Oct 21, 2008

Problem 1: text problem 4.15

Problem 2: text problem 4.17

Problem 3: text problem 4.26

Problem 4
Consider an M/G/1 queue with packet arrival rate λ. The job size Z is distributed according to a heavy tailed distribution, i.e.,

$$P(Z > x) \sim x^{-\alpha},$$

where $\alpha > 1$ is called the *degree* of the distribution. (In the above notation, $f(x) \sim g(x)$ means that

$$\lim_{x \to \infty} \frac{f(x)}{g(x)}$$

is a non-zero real number.)

a) Show that the mean job size $E(Z)$ is finite.

b) For this part and the next, assume that the service discipline is FCFS. Show that there exist values for α such that the expected queue occupancy is *infinite*, even though the loading of the queue is less that one ($\rho = \lambda E(Z) < 1$). Also, find the condition on α for this to happen.

c) How do you reconcile the fact that the expected queue occupancy is infinite in part (b), but the server utilization ρ is less than one? Does this mean that the steady state queue occupancy can be infinite with some positive probability? Discuss.

Problem 5
Let $X(t),\ t \in \mathbb{Z}$ be a discrete time, exactly second order self-similar stochastic process with Hurst parameter H.

a) Write down the auto-correlation function $r(k)$ for the process $X(t)$.

b) Show that if $H \neq 0.5$, $r(k) \sim k^{2H-2}$ for large k. (Hint: Consider the ratio of $r(k)$ and k^{2H-2}).

c) Prove that $X(t)$ exhibits long-range dependence for $\frac{1}{2} < H < 1$.