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Problem Set No.7 Due: Oct 28, 2008

Problem 1: Read Section 4.6 and Solve Text Problem 4.31

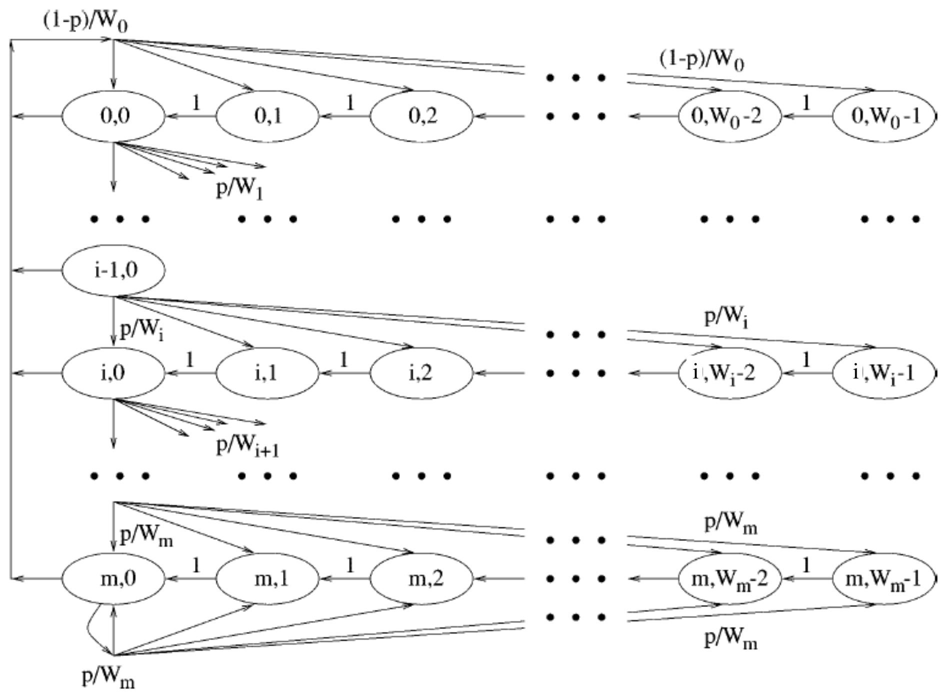
Problem 2: Throughput Analysis of Simplified 802.11
Consider the following random access mechanism:

- There are n users in the system which is time slotted.
- Packet transmission: Each user maintains contention window size W. For current W, it uniformly randomly selects one integer value b from $[0, W-1]$, and counts back from b to 0 after every time slot. When the counter becomes 0, it attempts to transmit a packet. Let $B(t)$ be the stochastic process representing the backoff time counter at time t.
- Update of W: Let $s \in \{0, ..., m\}$ be the backoff stage and W_s be the window size corresponding to stage s. The window size W_s is given by $W_s = 2^s W_0$ where W_0 is a base window size. The backoff stage s is increased by 1 whenever a collision occurs, up to a maximum value of m. On the other hand, upon successful transmission, s is reset to 0. Let $S(t)$ be the stochastic process representing the backoff stage at time t.
- Packet collision: For simplicity, assume that at each transmission attempt, a collision occurs with constant and independent probability p.

a) The bidimensional process $\{S(t), B(t)\}$ can be modeled as a discrete-time Markov chain, as shown in the figure. Prove the following transition probabilities for the Markov chain:

$$
\begin{array}{lll}
P[i,k|i,k+1] = 1, & k \in [0, W_i - 2], & i \in [0, m] \\
P[0,k|i,0] = (1-p)/W_0, & k \in [0, W_0 - 1], & i \in [0, m] \\
P[i,k|i-1,0] = p/W_i, & k \in [0, W_i - 1], & i \in [1, m] \\
P[m,k|m,0] = p/W_m, & k \in [0, W_m - 1] & ,
\end{array}
$$

where $P[i,j|k,l] = \Pr[S(t+1) = i, B(t+1) = j | S(t) = k, B(t) = l]$ and $[k,i]$ is the set of the integers from k to i.



b) Let $q_{i,k} = \lim_{t\to\infty} P[s(t) = i, b(t) = k], i \in [0, m], k \in [0, W_i - 1]$ be the stationary distribution of the chain. Prove the following relations:

$$q_{i,0} = p^i q_{0,0}, 0 < i < m$$

$$q_{m,0} = \frac{p^m}{1-p} q_{0,0}$$

$$\text{for } k \in [1, W_i - 1], q_{i,k} = \frac{W_i - k}{W_i} \cdot \begin{cases} (1-p)\sum_{j=0}^{m} q_{j,0}, & i = 0 \\ p q_{i-1,0}, & 0 < i < m \\ p(q_{m-1,0} + q_{m,0}), & i = m \end{cases}$$

(Hint: Find the balance equation between $q_{i-1,0}$ and $q_{i,0}$ for the first two. For the last, find the expression of $q_{i,k}$ from the right to the left in the Markov chain.)

c) Use the result in b) and the fact $\sum_{i=0}^{m} q_{i,0} = q_{0,0}/(1-p)$, to show

$$q_{i,k} = \frac{W_i - k}{W_i} q_{i,0}, k \in [0, W_i - 1], i \in [0, m].$$

d) From the results in b) and c), it is obvious that $q_{i,k}$ can be expressed as functions of $q_{0,0}$. Using the axiom of probability (adding to 1), obtain the following expression for $q_{0,0}$:

$$q_{0,0} = \frac{2(1-2p)(1-p)}{(1-2p)(W_0+1) + pW_0(1-(2p)^m)}.$$

(Hint: First, using the results in b) and c), show $\sum_{i=0}^{m} \sum_{k=0}^{W_i-1} q_{i,k} = \frac{q_{0,0}}{2} \left[W_0 \left(\sum_{i=0}^{m-1} (2p)^i + \frac{(2p)^m}{1-p} \right) + \frac{1}{1-p} \right]$, then the result easily follows.)

e) Let τ be the probability that a user transmits in a randomly chosen time slot. Show that it is given by

$$\tau = \frac{2(1-2p)}{(1-2p)(W_0+1) + pW_0(1-(2p)^m)}.$$

(Hint: Note that the transmission only occurs when the backoff counter is zero.)

f) Show that the throughput is maximized when the base window size is

$$W_0 = \frac{(2n-1)(1-2p)}{1-2p+p(1-(2p)^m)}$$

(Hint: A successful transmission occurs when there is only one user transmitting. Compute this probability in terms of τ which is the probability that a user attempts to transmit and find an optimal τ maximizing the success probability.)