Wavelength Requirements for Virtual Topology Reconfiguration in WDM Ring Networks

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Abstract—**Through the use of configurable WDM technology including tunable optical transceivers and frequency selective switches, next generation WDM networks will allow multiple virtual topologies to be dynamically established on a given physical topology. We determine the number of wavelengths required to support all possible virtual topologies on a bidirectional ring physical topology. We first determine wavelength requirements for networks using shortest path routing. We then reduce network wavelength requirements by presenting novel adaptive lightpath routing and wavelength assignment strategies. We also show that this reduced wavelength requirement is optimal. These results are first derived for the single port per node case and then extended to networks with multiple ports per node.**

I. Introduction

In Wavelength Division Multiplex (WDM) systems, multiple signals, separated by wavelength, are carried concurrently on an optical fiber. Each wavelength (channel) operates at peak electronic speeds of 1 to 10 Gbps per channel. We consider both single-hop [1] and reconfigurable multihop [2] networks in which each node is typically equipped with a small number of tunable transmitters and receivers.

Configurable optical Add/Drop Multiplexers (ADM) and cross-connects may be used to allow individual wavelength signals to be either *dropped* to the electronic routers at each node or to pass through the node optically. A lightpath between two nodes is formed by tuning the transmitter of one node and the receiver of another node to the same wavelength. Thus a lightpath is unidirectional. The *physical topology* of the network consists of optical nodes and their fiber connections. The *logical topology* describes the lightpaths between the nodes and is determined by the configuration of the transmitters and receivers on each node.

In single-hop networks extremely rapidly tunable transceivers are required to efficiently time share the network transceiver ports. Multihop networks may not need to be reconfigured as rapidly since in a connected logical topology, each node can transmit packets to every other node via store and forward or similar mechanisms. Reconfiguration in multihop networks has been proposed to reduce network delay and electronic processing loads [3], [4]. An important characteristic of both single-hop and multihop WDM networks is the independence between the logical and physical topologies. Any logical topology may be implemented on a given connected physical topology in the absence of wavelength constraints. A network with N nodes and P transceiver ports per node can have up to PN lightpaths. If each lightpath is routed on a different wavelength, PN wavelengths are required. When PN wavelengths are available, tunability is required only on either the transmitters or the receivers. Building networks with PN wavelengths, however, may be an expensive and inefficient use of resources. In a ring physical topology, for example, it may be possible to route multiple lightpaths on the same wavelength by the judicious use of optical ADMs and cross-connects.

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In this work, we determine the minimum number of wavelengths needed to support *all possible* logical topologies on a ring physical topology. If the physical topology is a unidirectional ring, there exist worst case logical topologies that require PN wavelengths. Consider, for example, a logical topology consisting of P rings, where the nodes in each ring are ordered in direction opposite to the physical topology. In this case each lightpath requires a separate wavelength and a total of PN wavelengths are needed. We therefore focus on bidirectional ring physical topologies. We first determine wavelength requirements for networks using deterministic shortest path routing to route lightpaths. We then develop adaptive routing and wavelength assignment strategies that minimize network wavelength requirements. We assume that for each logical topology, all lightpath requests are received simultaneously or that existing lightpaths may be rearranged. Initially, we assume that all logical topologies are connected which ensures that for multihop networks, traffic between every source and destination pair can be continually supported. We then generalize our results to include both connected and unconnected logical topologies.

We consider a bidirectional ring physical topology, shown in Fig. 1, consisting of a minimum of two fibers where half the fibers have wavelengths propagating in the clockwise direction and half the fibers propagate wavelengths in the counter-clockwise direction. We assume throughout that the nodes are labeled in increasing order in the clockwise direction. In determining wavelength provisioning requirements, we assume a set of lightpaths requires one wavelength if the set of lightpaths can be routed on a single wavelength on the same fiber. If a set of lightpaths uses the red wavelength on both the clockwise and counter-clockwise fibers, we say that the set of lightpaths utilizes *two* wavelengths.

We consider two types of networks, protected and unprotected. For the protected network case we assume loop-back protection [5] so that half of the total capacity is reserved for protection. If a lightpath is routed on a wavelength in the clockwise direction fiber, a wavelength on the counter-clockwise direction fiber is reserved for protection and vice versa. On each fiber, the number of wavelengths used for working traffic changes with different logical topologies. However, the total number of working traffic wavelengths is always equal to the

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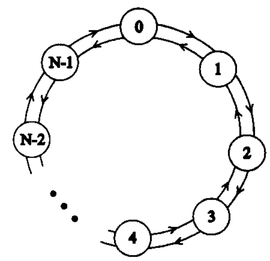

Fig. 1. A bidirectional ring physical topology with a single fiber propagating in each direction.

total number of protection wavelengths. When assessing wavelength requirements, we determine the number of wavelengths needed for working traffic. In the protected network, the working traffic wavelength requirements are not restricted by direction, since one can always allocate an opposite direction wavelength for protection. In an unprotected network, wavelengths on the bidirectional ring network should be allocated in pairs since there is no benefit in reducing the wavelength requirements in only one direction. If a logical topology requires only x wavelengths in the clockwise direction but $x+\Delta$ wavelengths in the counter-clockwise direction, then there also exists a logical topology that requires x counter-clockwise wavelengths and $x+\Delta$ clockwise wavelengths. To see this, simply reverse the lightpath directions of the first logical topology. Thus, in order to accommodate both topologies, the network must provide $x+\Delta$ wavelengths in both directions. These differences in the protected and unprotected network cases lead to differing routing and wavelength assignment strategies as well as different wavelength requirements for the two types of networks.

II. Wavelength Provisioning using Deterministic Shortest Path Routing

Deterministic shortest path routing schemes are often used because they are simple and because they minimize the resources required to route each lightpath. However, in networks without wavelength converters, the number of wavelengths required to implement a logical topology can be substantially larger than optimal. In this section we determine wavelength requirements for a network utilizing deterministic shortest path routing. We present two deterministic shortest path routing schemes and then calculate lower and upper bounds on corresponding network wavelength requirements. These bounds will be used to compare the benefits of our adaptive routing and wavelength assignment algorithms to shortest path routing.

When the number of nodes, N, is even, there are two shortest paths from node i to node $i+\frac{N}{2}$. A deterministic shortest path scheme fixes or pre-determines the directions of all lightpaths. In the Deterministic Odd Even Shortest Path (DOES) [6] routing scheme, a shortest path between nodes i and $i+\frac{N}{2}$ is routed clockwise if i is odd and counter-clockwise if i is even. DOES routing was shown to require fewer wavelengths than routing all length $\frac{N}{2}$ paths in the same direction. An alternative routing scheme for N even which is preferable to DOES in some cases, routes lightpaths from node i to node $i+\frac{N}{2}$ and from node $i+\frac{N}{2}$ to node i in the clockwise (counter-clockwise) direction if i is odd (even) for $0 \le i < \frac{N}{2}$. We call this Deterministic Continuous Ring Shortest Path (DCRS) routing. Note, that DOES and DCRS only differ when N is even and $\frac{N}{2}$ is odd.

TABLE I
Wavelength Requirements under Shortest Path Routing for Single Port per Node Networks

		N odd	N even		
			$\frac{N}{2}$ even	$\frac{N}{2}$ odd (DOES)	$\frac{N}{2}$ odd (DCRS)
Connected	W_{LB}	$N-2$	$N-2$	$N-3$	$N-2$
	W_{UB}	$N-2$	$N-1$	$N-1$	$N-1$
General	W_{LB}	$N-1$	$N-2$	N	$N-2$
	W_{UB}	$N-1$	$N-1$	N	$N-1$

Table I shows upper and lower bounds on the number of wavelengths required to implement all possible logical topologies (connected and unconnected) on a network with N nodes and one port per node. A lower bound of W_{LB} indicates that there exists a logical topology that requires at least W_{LB} wavelengths. An upper bound of W_{UB} implies that no logical topology requires more than W_{UB} wavelengths. An upper bound of N is trivial since N lightpaths can require at most N wavelengths. Since, the lower and upper bounds in Table I are similar and near N, approximately N wavelengths are required to ensure *all possible* logical topologies can be established using deterministic shortest path routing. In subsequent sections we show adaptive routing schemes that significantly reduce network wavelength requirements.

In calculating the bounds in Table I, no restriction was placed on the directions of the wavelength channels, thus these results correspond to the working traffic wavelength requirements in protected networks. Restricting the wavelength directions can only increase wavelength requirements. Trivial upper bounds for P port per node networks can be obtained by multiplying the upper bounds in Table I by P. Proofs for the lower and upper bounds in Table I are omitted in the interest of brevity.

III. Wavelength Provisioning using Adaptive Routing and Wavelength Assignment

In this section, we determine wavelength requirements for networks that must support all possible logical topologies. These wavelength requirements, W_{req}, are determined by showing adaptive routing schemes that can route any set of lightpaths with less than or equal to W_{req} wavelengths. Furthermore, we show that our routing schemes are optimal by finding logical topologies that cannot be supported (under any routing strategy) if fewer than W_{req} wavelengths are available. Adaptive routing strategies and wavelength requirements are determined for both protected and unprotected networks. Although these lightpath routing algorithms do not minimize the wavelength requirements for *each* logical topology, they do minimize the number of wavelengths required to implement *all possible* logical topologies on the bidirectional ring physical topology.

We begin in Section III-A by assuming all logical topologies are connected. In a multihop network, connectivity en-

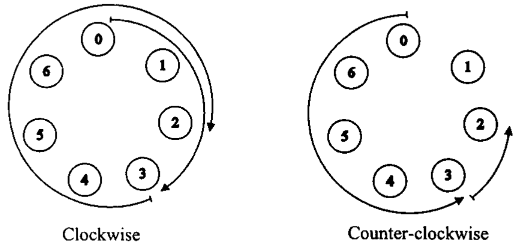

Fig. 2. Two adjacent lightpaths (example $(0, 3)$ and $(3, 2)$) can share a wavelength in either the clockwise or counter-clockwise direction.

sures communications between all source and destination pairs. There may, however, be scenarios where the logical topology can be unconnected, for example in rapidly tunable single-hop networks. We therefore relax the connectivity constraint and consider wavelength requirements for general (connected or unconnected) logical topologies. With minor modifications, the adaptive routing strategies for connected topologies can be used to minimize wavelength requirements for general topologies.

A. *Single port per node networks*

Connectivity in a single port per node network implies a ring logical topology. In protected networks, half the wavelengths are used for working traffic and half are used for protection. Each working traffic wavelength can be assigned in either direction since each clockwise (counter-clockwise) working traffic wavelength is protected by a counter-clockwise (clockwise) protection wavelength. In unprotected networks, wavelengths should be assigned in clockwise/counter-clockwise pairs to minimize wavelength requirements.

A.1 Protected networks

The following theorems show that $\lceil \frac{N}{2} \rceil$ working traffic wavelengths are necessary and sufficient to implement any connected logical topology. The first theorem relies on the following lemma which outlines an efficient routing and wavelength assignment strategy.

Lemma 1: In a bidirectional ring physical topology, every pair of adjacent[1] lightpaths can share a wavelength in one of the two directions.

Proof: Let (i_j, i_k) denote a lightpath from source node i_j to destination node i_k. Consider two adjacent lightpaths (i_1, i_2) and (i_2, i_3). If (i_1, i_2) and (i_2, i_3) cannot share a wavelength in the clockwise direction, then i_2 must lie between i_1 and i_3 on the counter-clockwise direction fiber, hence the two lightpaths can share a wavelength in the counter-clockwise direction as shown in Fig. 2. ■

Theorem 1: The maximum number of wavelengths needed to implement any connected logical topology is equal to $\lceil \frac{N}{2} \rceil$.

Proof: By Lemma 1, each pair of adjacent lightpaths can share a wavelength. Since the logical topology is a ring, the set of lightpaths can be divided into $\lfloor \frac{N}{2} \rfloor$ adjacent pairs plus one lightpath if N is odd. Therefore the maximum number of wavelengths required to route all N lightpaths is $\lceil \frac{N}{2} \rceil$. ■

Theorem 2: For $N > 3$, there exists a connected logical topology that requires $\lceil \frac{N}{2} \rceil$ wavelengths (regardless of the routing strategy).

Proof: We can construct logical topologies for N odd and N even that require $\lceil \frac{N}{2} \rceil$ wavelengths. Example topologies are shown in Fig. 3.

N odd: Consider a logical topology connecting node i to node $(i + \lfloor \frac{N}{2} \rfloor) \bmod N$. Since each lightpath traverses at least $\lfloor \frac{N}{2} \rfloor$ links, at most two lightpaths can share a wavelength.

N even: For N even, the preceding construction does not produce a connected logical topology, therefore we utilize the following alternative construction technique. Any logical ring topology can be defined by the order of its connected nodes $\mathcal{R} = (i_0, i_1, \ldots, i_{N-1})$. Construct a connected logical topology by connecting node i_0 to node $i_1 = i_0 + \frac{N}{2}$. Next connect node i_1 to node $i_2 = i_1 + \frac{N}{2} - 1$. Continue creating lightpaths sequentially in this manner, alternating between adding $\frac{N}{2}$ and $\frac{N}{2} - 1$, until node i_{N-1}. Node i_{N-1} is connected to node i_0. In this logical topology, $\frac{N}{2}$ of the lightpaths traverse $\frac{N}{2}$ links each. Since each of these $\frac{N}{2}$ lightpaths overlap, as shown in Fig. 3, each requires a separate wavelength. Therefore at least $\frac{N}{2}$ wavelengths are needed to support this logical topology. ■

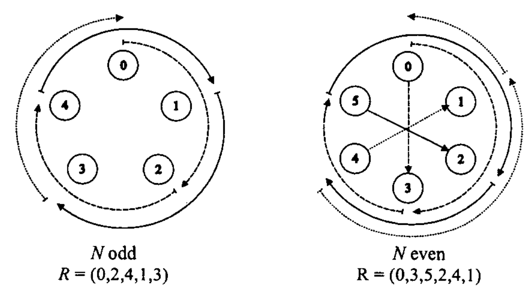

Fig. 3. Example light path topologies that require $\lceil \frac{N}{2} \rceil$ wavelengths for N odd and N even. For N even, the $\frac{N}{2}$ lightpaths $(0, 3)$, $(5, 2)$ and $(4, 1)$ overlap, thus each overlapping lightpath requires a separate wavelength.

These results indicate that by routing pairs of adjacent lightpaths on a single wavelength, any connected logical topology can be supported on a network provisioned with $\lceil \frac{N}{2} \rceil$ wavelengths. Furthermore, since $\lceil \frac{N}{2} \rceil$ is the minimum number of wavelengths required to support all connected logical topologies, this adaptive routing strategy is optimal.

A connected logical topology consists of a single directed circuit. In a single port per node network in which all ports are utilized, a disconnected logical topology is the union of multiple edge-disjoint directed circuits[2] ([7] Theorem 5.6). A general logical topology will consist of K circuits of size M_i, for $i = 1 \ldots K$, where $\sum_i M_i = N$. If adjacent lightpaths in each circuit are routed on a single wavelength, then applying Theorem 1 to each circuit shows that at most $\sum_i \lceil \frac{M_i}{2} \rceil$ wavelengths

[1] Two lightpaths are adjacent if the destination node of one lightpath is equal to the source node of the other lightpath.

[2] A directed circuit in a directed graph G is defined as a finite sequence of vertices $v_0, v_1, \ldots, v_k$, such that (v_{i-1}, v_i) is an edge in G, $v_0 = v_k$, and all other vertices are unique.

will be required. This can be substantially larger than $\lceil \frac{N}{2} \rceil$ if the logical topology consists of many odd size circuits. However, the following lemma can be used to show that at most $\lceil \frac{N}{2} \rceil + 1$ wavelengths are required to support all (connected or unconnected) logical topologies. Furthermore, there exist unconnected logical topologies (e.g., with $M_1, M_2 = 5$) that require $\lceil \frac{N}{2} \rceil + 1$ wavelengths.

Lemma 2: Given three circuits of odd sizes M_1, M_2, and M_3 that, when routed individually, require $\lceil \frac{M_i}{2} \rceil$ wavelengths each, there exists a lightpath from one of the three odd size circuits that can share a wavelength with a lightpath from one of the other two odd size circuits.

Assume without loss of generality that circuits 1 and 2 contain the pair of lightpaths that can share a wavelength. Then the three odd size circuits require $\frac{M_1+M_2}{2} + \lceil \frac{M_3}{2} \rceil = \lceil \frac{M_1+M_2+M_3}{2} \rceil$ wavelengths. Furthermore, Lemma 2 may be applied iteratively to logical topologies with larger numbers of odd size circuits. For example consider a logical topology with 5 odd size circuits, M_1 to M_5. Take any three of these circuits, e.g., circuits 1, 2, and 3, and apply Lemma 2. Two of the three circuits, e.g., circuits 1 and 2, will contain lightpaths that can share a wavelength. These two circuits use $\frac{M_1+M_2}{2}$ wavelengths. There remain three odd size circuits 3, 4, and 5. Applying Lemma 2 to these three circuits shows that two of the three circuits, e.g., 3 and 4, contain a pair of lightpaths that can share a wavelength. Therefore at most $\frac{M_3+M_4}{2} + \lceil \frac{M_5}{2} \rceil$ wavelengths are needed to establish the three circuits. Consequently, a total of $\lceil \frac{M_1+M_2+M_3+M_4+M_5}{2} \rceil$ wavelengths are needed to establish all five circuits. In general, any logical topology contains at most two odd size circuits that require $\lceil \frac{M_i}{2} \rceil$ wavelengths each and that do not allow any further sharing of wavelengths between them. In a connected logical topology, these two circuits would require $\frac{M_1+M_2}{2}$ wavelengths rather than $\lceil \frac{M_1}{2} \rceil + \lceil \frac{M_2}{2} \rceil$. Therefore, at most one extra wavelength, for a total of $\lceil \frac{N}{2} \rceil + 1$ wavelengths, is required to support unconnected as well as connected logical topologies. Due to space considerations, we do not prove Lemma 2 here.

A.2 Unprotected networks

In this section we assume all wavelengths are used for working traffic. Section I showed that in unprotected networks, wavelengths should be allocated in pairs. Simply applying the routing algorithms from the protected case yields requirements of $\lceil \frac{N}{2} \rceil$ wavelengths in each direction. We can do much better. The following theorems show that $\lceil \frac{N}{3} \rceil$ wavelengths in each direction are necessary and sufficient to implement any connected logical topology.

Theorem 3: Any connected logical topology can be implemented with $W = \lceil \frac{N}{3} \rceil$ wavelengths in each direction.

Proof: For any given logical topology, use the following routing and wavelength assignment following algorithm:

1. Divide the lightpaths into sets of three adjacent lightpaths. If N is not perfectly divisible by three, then there will be one set of lightpaths that has either one or two lightpaths in it.
2. Using Lemma 1, route the first two lightpaths in each set on a single wavelength. Route the third lightpath in each set on a wavelength in the opposite direction.

Since there are $\lceil \frac{N}{3} \rceil$ sets, at most $\lceil \frac{N}{3} \rceil$ wavelengths are required in each direction. ∎

The proof indicates a method of routing lightpaths to ensure no more than $\lceil \frac{N}{3} \rceil$ wavelengths are needed in each direction. Fig. 4 illustrates the lightpath routing strategy. The next theorem illustrates that at least $\lceil \frac{N}{3} \rceil$ wavelengths must be provisioned in each direction and hence the optimality of the above routing and wavelength assignment algorithm.

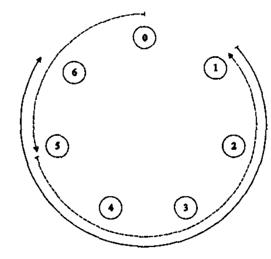

Fig. 4. Any three adjacent lightpaths (example: (0,5) (5,1) and (1,6)) can be routed using one wavelength in each direction.

Theorem 4: For networks with $N > 4$ nodes, if less than $\lceil \frac{N}{3} \rceil$ wavelengths are available in each direction, then there exists a logical topology that can not be supported.

Proof: Suppose $\lceil \frac{N}{3} \rceil - 1$ wavelengths are available in each direction. We can construct logical topologies that *can not* be supported for the cases of N odd and N even.

N odd: Construct a logical topology by connecting node i to node $(i + \lfloor \frac{N}{2} \rfloor) \bmod N$. Since each lightpath traverses a minimum of $\lfloor \frac{N}{2} \rfloor$ links, at most two lightpaths can share a wavelength. Furthermore, since each lightpath traverses $\lceil \frac{N}{2} \rceil$ links in the counter-clockwise direction, adjacent lightpaths can only share a wavelenth in the clockwise direction. Using the $\lceil \frac{N}{3} \rceil - 1$ clockwise wavelengths we can support $2\lceil \frac{N}{3} \rceil - 2$ lightpaths. Each of the $\lceil \frac{N}{3} \rceil - 1$ counter-clockwise wavelengths supports only one lightpath. Thus the total number of lightpaths supported is $3(\lceil \frac{N}{3} \rceil - 1)$ which is always less than N.

N even: Construct a connected logical topology as follows. Sequentially, starting with node i_0 and ending at node i_{N-1}, establish a lightpath between node i_j and node $i_{j+1} = (i_j + \frac{N}{2} - 1) \bmod N$ if node $(i_j + \frac{N}{2} - 1) \bmod N$ is not yet connected in the logical ring topology. If it is already connected, connect node i_j to node $i_{j+1} = (i_j + \frac{N}{2}) \bmod N$. Finally, connect node i_{N-1} to node i_0. When $\frac{N}{2}$ is even, each lightpath traverses at least $\frac{N}{2} - 1$ physical links. When $\frac{N}{2}$ is odd, the connection from i_{N-1} to i_0 traverses $\frac{N}{2} - 2$ links in the clockwise direction and $\frac{N}{2} + 2$ links in the counter-clockwise direction. All other lightpaths traverse at least $\frac{N}{2} - 1$ physical links. Therefore, for $N \geq 8$, at most two lightpaths can share each wavelength. Furthermore, the lightpaths can only share wavelengths in the clockwise direction since each lightpath traverses more than $\frac{N}{2}$ links in the counter-clockwise direction. Using the $\lceil \frac{N}{3} \rceil - 1$ clockwise wavelengths we can support $2\lceil \frac{N}{3} \rceil - 2$ lightpaths. The $\lceil \frac{N}{3} \rceil - 1$ counter-clockwise wavelengths can

each support at most one lightpath. Thus the total number of lightpaths supported is $3(\lceil \frac{N}{3} \rceil - 1)$ which is always less than N. When $N = 6$, three of the lightpaths can share a wavelength in the clockwise direction. However, only one lightpath can be established on each counter-clockwise wavelength. Therefore one clockwise and one counter-clockwise wavelength can carry at most four of the six lightpaths. ■

We have shown that by routing sets of three adjacent lightpaths on a single pair of wavelengths, all connected logical topologies can be supported on a network with $\lceil \frac{N}{3} \rceil$ wavelengths in each direction. It can also be shown that all unconnected logical topologies can also be supported with $\lceil \frac{N}{3} \rceil$ wavelengths in each direction. The proof, which has been omitted for brevity, uses arguments similar to those used for generalizing the protected network results to unconnected logical topologies described in Section III-A.1.

B. Multiple Ports Per Node

A logical topology with P ports per node is a directed graph with nodes of in-degree and out-degree equal to P. If the logical topology is connected, then the directed graph contains a directed Euler trail ([7] Theorem 5.6), where an Euler trail is a closed directed trail[3] which contains all the edges of the graph. Therefore, the PN lightpath logical topology (PN edges of the graph) can be divided into $\lfloor \frac{PN}{2} \rfloor$ pairs of adjacent lightpaths plus one lightpath if PN is odd. For the unprotected network case, the PN lightpath topology can be divided into $\lfloor \frac{PN}{3} \rfloor$ sets of three adjacent lightpaths plus 1 set of (PN) mod 3 lightpaths. The routing strategies described in Section III-A for single port per node networks can thus be directly applied to multiple port per node networks. In protected networks, at most $\lceil \frac{PN}{2} \rceil$ working traffic wavelengths are required to implement all possible logical topologies. In unprotected networks, all logical topologies can be established with $\lceil \frac{PN}{3} \rceil$ wavelengths in each direction.

A disconnected logical topology G can be divided into a set of K connected components where each component, G_i, consists of N_i nodes and M_i lightpaths, where $0 \leq i \leq K - 1$ and $\sum_i M_i = PN$. The set of lightpaths in the ith connected component form an Euler trail on G_i. Thus each set of M_i lightpaths can be routed on $\lceil \frac{M_i}{2} \rceil$ wavelengths in the protected case and $\lceil \frac{M_i}{3} \rceil$ wavelengths in each direction in the unprotected case. Similar to the case of $P = 1$, it can also be shown that all general (connected or unconnected) logical topologies can be established with $\lceil \frac{PN}{2} \rceil + 1$ wavelengths in the protected case and $\lceil \frac{PN}{3} \rceil$ wavelengths in each direction for the unprotected case.

IV. Limited Logical Topology Networks

Thus far we have considered networks that support all virtual topologies. However, by limiting the number of topologies that can be established, it may be possible to reduce network wavelength requirements.

To investigate the tradeoff between the fraction of topologies supported and the number of wavelengths required, we compute a lower bound on the wavelength requirements for each logical topology. For each logical topology, let m be the maximum number of lightpaths cut by any bisection of the logical topology graph. Then a minimum of $\frac{m}{2}$ wavelengths are required to implement this logical topology. We use this lower bound to calculate the minimum wavelength requirements for all connected logical single port per node topologies of size N. These results show that most of the logical topologies require near the maximum number of $\lceil \frac{N}{2} \rceil$ wavelengths. For N even, it can be shown that the number of logical topologies that require $\lceil \frac{N}{2} \rceil$ wavelengths is $(\frac{N}{2}!)^2$.

A lower bound on wavelength requirements for a logical topology using shortest path routing is simply the load on each link. Using this lower bound we can compute the number of logical topologies that use a minimum of 1 to $N - 2$ wavelengths in an N node network. These results indicate that it may be possible to implement a majority of the logical topologies with $\lceil \frac{N}{2} \rceil$ wavelengths if shortest path routing is used in conjunction with wavelength converters.

V. Conclusions

We have determined the minimum number of wavelengths required, W_{req}, to implement all virtual topologies on an N node P port network. For connected logical topologies, $W_{\text{req}} = \lceil \frac{PN}{2} \rceil$ working traffic wavelengths are required on a protected network and $W_{\text{req}} = \lceil \frac{PN}{3} \rceil$ wavelengths in each direction are required on an unprotected network. We have also presented adaptive lightpath routing strategies that ensure that all logical topologies can be established with the minimum W_{req} wavelengths. Furthermore, these adaptive routing schemes significantly reduce the network wavelength requirements in comparison to shortest path routing.

In this work we have focused on the bidirectional ring physical topology. Examining wavelength requirements for more general physical topologies is an area for future work.

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[3] A closed directed trail in a directed graph G is a finite sequence of vertices $v_0, v_1, \ldots, v_k$, such that (v_{i-1}, v_i) is an edge in G, all edges are distinct, and $v_0 = v_k$. Note that a trail can repeatedly visit the same node.