Paper-review: Aeroelastic analysis of F-16 and F-18/A configurations using adapted CFD-based reduced-order models (D. Amsallem, C. Farhat and T. Lieu)

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Outline

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Fluid ROMs

CFD-based ROMs

- often projection of the equations on a set of basis vectors
- correspond well with full-order
- are sensitive to variation of M_∞ and α

$\Rightarrow$ must be created for every M_∞ and α of interest

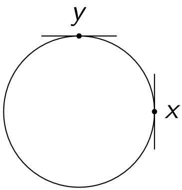
Decrease computational cost

Idea

- Calculate basis vectors at a couple of $M_{\infty,i}$ and α_i
- If other M_∞ and α of interest $\rightarrow$ interpolate basis vectors

$\Rightarrow$ avoids creation of a basis in every M_∞ and α

Manifold

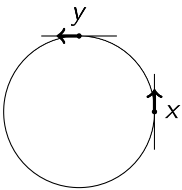


m-dimensional manifold

A space that locally looks like $\mathbb{R}^m$.

- Circle=1-dimensional manifold
- Only valid in neighborhood of $x \in M$

Differential manifold

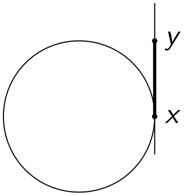


Differential manifold

A manifold for which the derivative of smooth path $\gamma(t)$ on M at $x \in M$ is defined.

- the vector $\gamma'(t)$ lying in $\mathcal{T}_x(M)$
- every $x \in M$ has a tangent space $\mathcal{T}_x(M)$ of the same dimension as M
- $\mathcal{T}_x(M)$ consists of derivatives $\gamma_i'(t)$ of smooth paths $\gamma_i(t)$ through x

Riemannian manifold

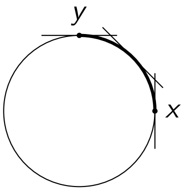


Riemannian manifold

A differential manifold with an inner product $< \, , \, >_x$ uniquely defined on $\mathcal{T}_x(M)$

- distances and angles in $\mathcal{T}_x(M)$

Geodesic



Geodesic

Shortest path on the manifold between two points of the manifold

Distance

The distance $d(x, y)$ between two points on the manifold is the length of the geodesic

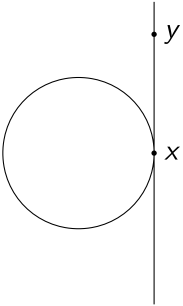
Exp en Log mapping

Exp_x maps from $\mathcal{T}_x(M)$ to M

For every point $x \in M$ there is a unique path starting from x in every direction with derivative $\forall\, y \in \mathcal{T}_x(M) : d(x, Exp_x(y)) = ||y||_x$

Log_x maps from M to $\mathcal{T}_x(M)$

Inverse of Exp_x



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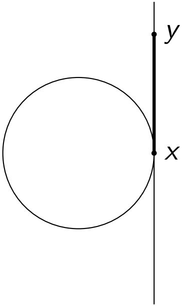

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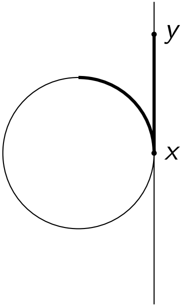

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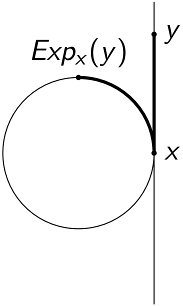
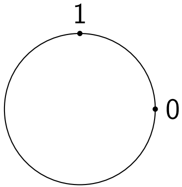


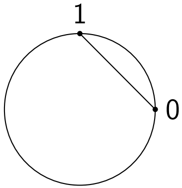
Example



Direct interpolation $\Rightarrow \notin M$

$Log_x \longrightarrow$ interpolate $\longrightarrow Exp_x \Rightarrow \in M$

Example



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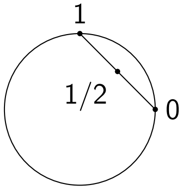

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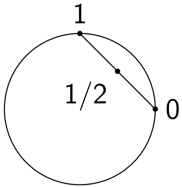

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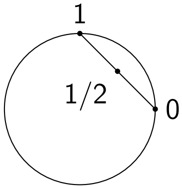

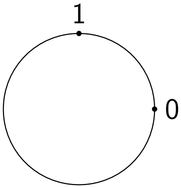

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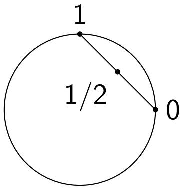

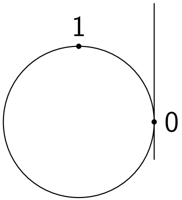

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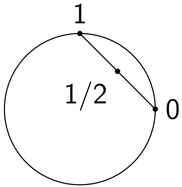
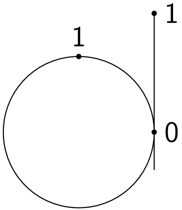


Direct interpolation $\Rightarrow \notin M$



$Log_x \longrightarrow$ interpolate $\longrightarrow Exp_x \Rightarrow \in M$

Example

Direct interpolation $\Rightarrow \notin M$ $\quad$ $Log_x \longrightarrow$ interpolate $\longrightarrow Exp_x \Rightarrow \in M$

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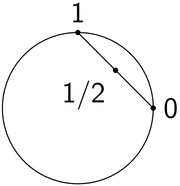

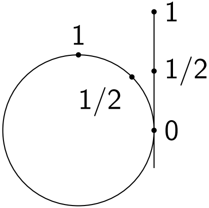

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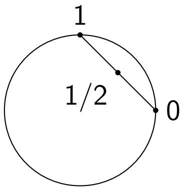

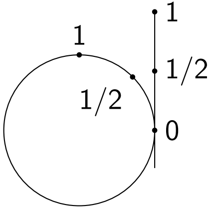

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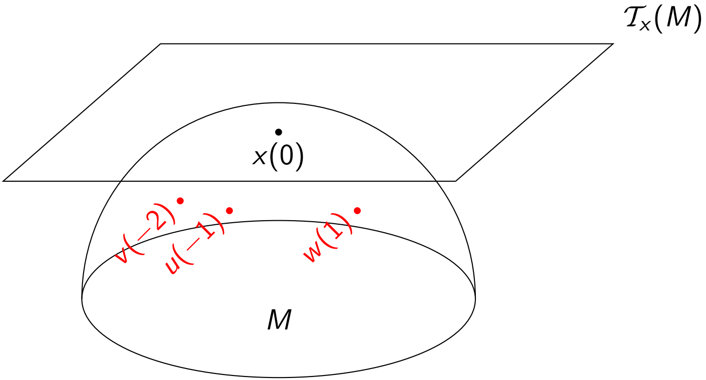
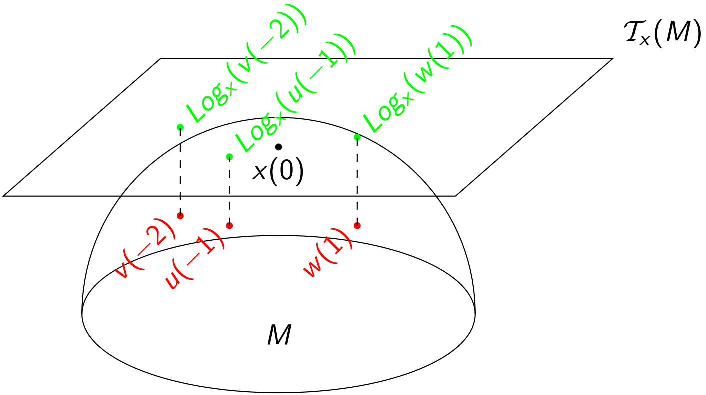
3D Example



Interpolate coordinates independently in $\mathcal{T}_x(M)$

3D Example



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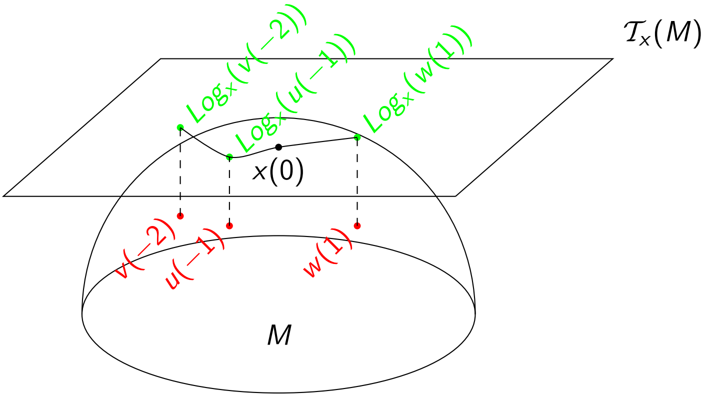

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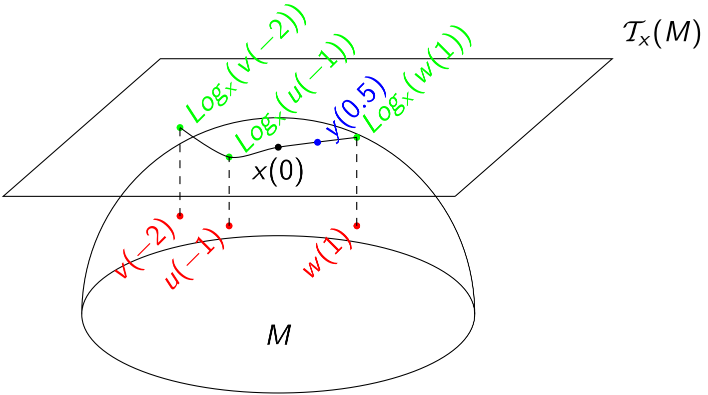

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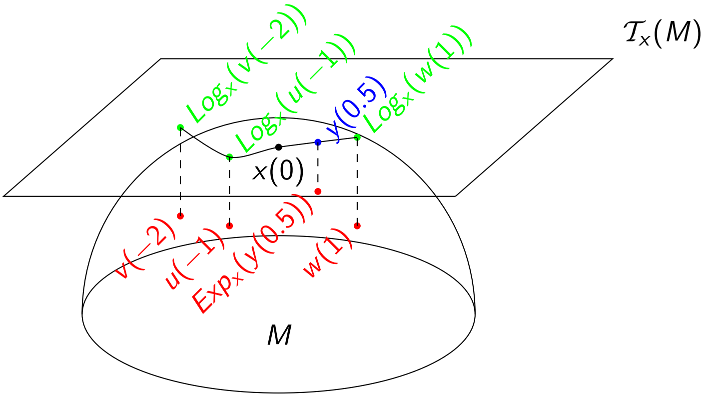

Interpolate coordinates independently in $\mathcal{T}_x(M)$

Grassmann manifold

Grassmann manifold

The Grassmann manifold $\mathcal{G}(k, n)$ is a Riemannian manifold of all k-dimensional linear subspaces of $\mathbb{R}^n$

- A k-dimensional subspace $\mathcal{S}$ of $\mathbb{R}^n$ is the span of k basis vectors, represented by a $n \times k$ orthogonal matrix Φ
- $\Phi_i(M_{\infty,i})$ represents an element $\mathcal{S}_i$ of $\mathcal{G}(k, n)$

Adaptation procedure

Calculate $\tilde{\Phi}(\tilde{M}_\infty)$ from the orthogonal $n \times k$ matrices $\Phi_i(M_{\infty,i})$ $(i = 0, \ldots, N_R)$

1. Choose $M_{\infty,0} \rightarrow \mathcal{T}_{\mathcal{S}_0}(\mathcal{G})$
2. Map Φ_i with $Log_{\mathcal{S}_0}$ to Θ_i
3. Interpolate the Θ_i on $\mathcal{T}_{\mathcal{S}_0}(\mathcal{G})$ for $\tilde{\Theta}(\tilde{M}_\infty)$.
4. Map $\tilde{\Theta}$ with $Exp_{\mathcal{S}_0}$ to $\tilde{\Phi}$

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   For every $i = 1, \ldots, N_R$, perform SVD

$$(I - \Phi_0 \Phi_0^{\mathrm{T}}) \Phi_i (\Phi_0^{\mathrm{T}} \Phi_i)^{-1} = U_i \Sigma_i V_i^{\mathrm{T}}$$

   and compute

$$\Theta_i = U_i \tan^{-1} \Sigma_i V_i^{\mathrm{T}}$$

3. Interpolate the Θ_i on $\mathcal{T}_{\mathcal{S}_0}(\mathcal{G})$ for $\tilde{\Theta}(\tilde{M}_\infty)$.
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3. Interpolate the Θ_i on $\mathcal{T}_{\mathcal{S}_0}(\mathcal{G})$ for $\tilde{\Theta}(\tilde{M}_\infty)$.

$$\tilde{\Theta}(\tilde{M}_\infty) = \sum_{i=1}^{N_R} \left(\prod_{j \neq i} \frac{\tilde{M}_\infty - M_{\infty,j}}{M_{\infty,i} - M_{\infty,j}} \right) \Theta_i(M_{\infty,i})$$

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Perform SVD

$$\tilde{\Theta}(\tilde{M}_\infty) = \tilde{U}\tilde{\Sigma}\tilde{V}^{\mathrm{T}}$$

and compute

$$\tilde{\Phi}(\tilde{M}_\infty) = \Phi_0 \tilde{V} \cos \tilde{\Sigma} + \tilde{U} \sin \tilde{\Sigma}$$

Limitations

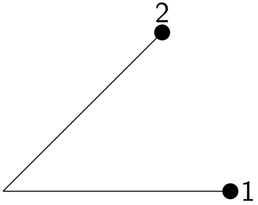
- Φ_i must have same dimension
- Only valid in neighborhood of $M_{\infty,0}$

Other techniques

- Direct interpolation: interpolate the basis vectors constructed for different values of M_∞
- Subspace angle interpolation: interpolate the subspace angles between two sets of basis vectors

Subspace angle interpolation

- Interpolate between two sets of basis vectors
- Sets can contain different number of basis vectors



$$\theta(1, 1.3) = \frac{1.3 - 1}{2 - 1}\theta(1, 2)$$

Subspace angle interpolation

- Interpolate between two sets of basis vectors
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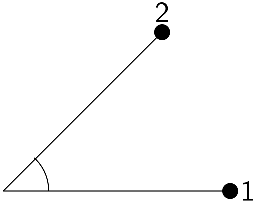

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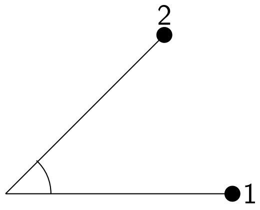

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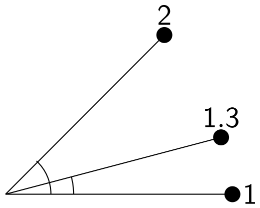

$$\theta(1, 1.3) = \frac{1.3 - 1}{2 - 1}\theta(1, 2)$$

Subspace angle interpolation

Calculate $\tilde{\Phi}(\tilde{M}_\infty)$ from the orthogonal matrices $\Phi_1(M_{\infty,1})$ and $\Phi_2(M_{\infty,2})$ with dimensions $n \times k_1$ and $n \times k_2$ $(k_2 \leq k_1)$.

1. Calculate the principal angles and principal vectors

   Perform SVD

   $$\Phi_1^{\mathrm{T}}\Phi_2 = Y\Sigma Z$$

   Principal angles: $\cos\theta_i = \sigma_i$ $(\sigma_i \leq 1,\ i = 1, \ldots, k_2)$
   Principal vectors: $U = \Phi_1 Y$, $V = \Phi_2 Z$

2. Linear interpolation of the θ_i

3. $u_i \in U$ is rotated towards $v_i \in V$ through $\tilde{\theta}_i(M_{\infty,1}, \tilde{M}_\infty)$

Subspace angle interpolation

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1. Calculate the principal angles and principal vectors
2. Linear interpolation of the θ_i

$$\tilde{\theta}_i(M_{\infty,1}, \tilde{M}_\infty) = \left(\frac{\tilde{M}_\infty - M_{\infty,1}}{M_{\infty,2} - M_{\infty,1}}\right)\theta_i(M_{\infty,1}, M_{\infty,2})$$

θ_i vary linearly with M_∞, contrary to the components of the basis vectors ($i = 1, \ldots, k_2$)

3. $u_i \in U$ is rotated towards $v_i \in V$ through $\tilde{\theta}_i(M_{\infty,1}, \tilde{M}_\infty)$

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1. Calculate the principal angles and principal vectors
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3. $u_i \in U$ is rotated towards $v_i \in V$ through $\tilde{\theta}_i(M_{\infty,1}, \tilde{M}_\infty)$

$$w_i(\tilde{M}_\infty) = u_i \cos\tilde{\theta}_i(M_{\infty,1}, \tilde{M}_\infty) + \frac{v_i - (u_i^{\mathrm{T}} v_i)u_i}{||v_i - (u_i^{\mathrm{T}} v_i)u_i||_2} \sin\tilde{\theta}_i(M_{\infty,1}, \tilde{M}_\infty)$$

w_i are new basis at $\tilde{M}_\infty$ $(i = 1, \ldots, k_2)$

F-16

- Basis vectors for fluid obtained with POD
- $M_{\infty,1} = 0.923$ and $M_{\infty,2} = 1.114 \Rightarrow$ fails
- $M_{\infty,3} = 1.031$ is necessary to obtain a stable ROM between $M_{\infty,1}$ and $M_{\infty,2}$
- Lift time-history and damping coefficient of the first torsional mode very close to non-linear, better than subspace angle interpolation.

F-18/A

- Basis vectors for fluid obtained with POD
- $M_{\infty,1} = 0.6$, $M_{\infty,2} = 0.7$, $M_{\infty,3} = 0.75$ and $M_{\infty,4} = 0.8$
- Direct interpolation unstable
- 4 subspaces: outperforms subspace angle interpolation.
- 2 subspaces: nearly identical to subspace angle interpolation.

→ suggests that interpolation on a tangent space to a Grassmann manifold is a generalization of the subspace angle method

Conclusion

- Interpolate multiple sets of basis vectors rapidly
- Successfully applied on large problem in transonic range
- All Φ_i must have the same size
- Only Φ is interpolated. Steady-state, linearization and projection are recalculated.

References I

Amsallem, D., Farhat, C., and Lieu, T. (2007).
Aeroelastic analysis of f-16 and f-18/a configurations using adapted cfd-based reduced-order models.
In *48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, volume AIAA 2007-2364, Honolulu, Hawaii.

Begelfor, E. and Werman, M. (2006).
Affine invariance revisited.
In *Proceedings of the 2006 IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, New York, NY.

References II

Björck, A. and Golub, G. (1973).
Numerical methods for computing angles between linear subspaces.
Mathematics of Computation, 27(123):579–594.

Lieu, T. and Farhat, C. (2005).
Adaptation of pod-based aeroelastic roms for varying mach number and angle of attack: Application to a complete f-16 configuration.
AIAA Paper, 2005-7666.

References III

Vierendeels, J., Lanoye, L., Degroote, J., and Verdonck, P. (2007).
Implicit coupling of partitioned fluid-structure interaction problems with reduced order models.
Computers and Structures, 85(11–14):970–976.