A Reprint from

INFORMATION THEORY

Papers read at a Symposium on 'Information Theory' held at the Royal Institution, London, August 29th to September 2nd 1960

Edited by

COLIN CHERRY

Published by

BUTTERWORTH & CO. (PUBLISHERS) LIMITED
88 KINGSWAY, LONDON, W.C.2

THE CHARACTERIZATION OF CURSIVE WRITING†

Murray Eden‡ and Morris Halle§
Research Laboratory of Electronics
Massachusetts Institute of Technology
Cambridge 39, Massachusetts, U.S.A.

The aim of this study is to give a scientific account of cursive writing as practised in the United States and, with relatively minor modifications, in all parts of the world where a Latin script is used. We have chosen to study writing not only because it is an intrinsically interesting form of human communicative behaviour, but also because it is our hope that what is learned about writing and about reading of handwritten texts will throw new light on other forms of communicative behaviour as well, in particular, on the acoustical analogon of script, speech.

We regard our task as that of discovering a simple and complete set of statements that characterizes any specimen written in the script under study, or alternatively as that of describing an algorithm that generates any such specimen. The algorithm must, therefore, contain the essence of what a child learns in the first years of his schooling. It can, however, not be identical with the instructions given to children in elementary grades, for being primarily interested in a scientific account of writing rather than in effective ways of teaching it, we cannot, unlike the teacher, take advantage of the pupils' considerable intelligence that allows them to learn many things which are not expressly taught or which are incompletely or inconsistently presented. A scientific account must contain both the overt instruction as well as the covert, intuitively learned, contribution of the pupil.

Since we are interested in writing as an instance of human communicative behaviour, our description must be devised so as to account also for all facets of behaviour that are concomitant with man's ability to write. This forces us to pay close attention to the structure of the proposed algorithm and makes it impossible for us to be satisfied with *ad hoc* solutions. The algorithm must, for instance, provide an explanation for the fact that readers judge certain handwritten specimens as replications of a single text in apparent disregard of striking graphic differences among the specimens, while specimens that are graphically much more alike are judged to represent different texts. To account for this it is necessary to assume that the (quasi) continuous line of a handwritten specimen is a representation of discrete entities in terms of which the reader performs his identification. Accordingly our algorithm must be

† This work was supported in part by the U.S. Army (Signal Corps), the U.S. Navy (Office of Naval Research), and the U.S. Air Force (Office of Scientific Research, Air Research and Development Command); and in part by the National Science Foundation.
‡ Department of Electrical Engineering, M.I.T.
§ Department of Modern Languages, M.I.T.

constructed so as to generate its output from a finite set of discrete symbols†.

The proposed algorithm has a hierarchical structure of fair complexity. It contains only a small number of primitives, from which by a group of transformations (reflection about a horizontal or a vertical axis), a larger number of intermediate elements is generated. We now select a subset of these intermediate elements and subject them to a further transformation (translation), thereby generating a yet larger number of entities. From the latter we again select a particular subset, which we call *strokes*. We next designate certain strings of strokes as *letters* and delimit each such string by a special punctuation mark. Every handwritten specimen is in turn represented by a string of letters, which is a string of strokes interspersed with punctuation marks delimiting the individual letter. To generate the (quasi) continuous line which actually issues from under the hand of the writer the algorithm includes special rules that describe the manner in which strokes are collated and traced.

We consider the hierarchical structure of the algorithm as one of its most significant features, for behaviour of any complexity is inconceivable without an elaborate hierarchical organization‡. We find such a hierarchical organization in speech§, which is hardly surprising since speech and writing are obviously closely related activities. But even much simpler forms of behaviour such as walking, grasping, or the rhythmic beating of a drum must be organized in a complex hierarchical fashion[2]. The rather special structure of our algorithm is, therefore, not an arbitrary complication; it is rather dictated by our need to satisfy the condition that our description should account for writing as a form of human behaviour.

The primitive notion underlying the present description¶ is that of a pair of points located in the plane. The following relations are predicated among the points: *vertical ordering* i.e. one point may be higher than the other; and *horizontal ordering* i.e. one point may be to the right of the other. We distinguish moreover, two degrees of horizontal ordering, a more proximate ordering, designated by ε, and a less proximate ordering, designated by 1.

† The fact that no generally valid procedure can be stated for discovering in a handwritten specimen the point where one letter ends and the next letter begins is no argument against regarding writing as generated from discrete elements. It only means that the operations involved in the generation of the specimen are not generally reversible. While we do not know of any published argument against regarding writing as consisting of discrete entities, such arguments have been advanced with regard to speech. The impossibility of finding a generally workable procedure for segmenting the quasi-continuous speech wave has led some students[1] to reject the concept of the (discrete) phoneme as an empirically unsupportable hypostatization. Needless to say, we consider such scruples as due to a simple misunderstanding of the nature of the problem.

‡ See reference 2. This point has recently been re-emphasized with great eloquence and force by G. A. Miller, 'The magical number seven, plus or minus two: Some limits on our capacity for processing information', *Psychol. Rev.*, 63 (1956) 81, and elsewhere.

§ For a description of the hierarchical structure of speech see reference 3. In this work the acoustical speech event is regarded as a physical representation of a linear sequence of symbols, the phonemes, which themselves are simultaneous complexes of more elementary entities, the distinctive features. This close resemblance between speech and the model of writing proposed here must not, however, obscure some important differences between the two. Perhaps the most important of these is that in the case of writing there are elements intermediate in the hierarchy between the primitives and the letters, whereas in the case of speech no such intermediate elements are found between the distinctive features and the phoneme. The strokes of the writing algorithm are therefore not exactly analogous to the distinctive features of speech, nor are the letters exact structural analogues of the phonemes.

¶ A formal account of the following algorithm can be found in the appendix.

Vertical ordering will be designated by 1 and lack of ordering in either dimension by 0. These relations define six distinct point pairs, of which, however, only four are significant for the handwriting under consideration: with vertical ordering 0, horizontal ordering 1 alone is relevant.

Each of the four significant point-pairs is connected by a line in accordance with the following rules: (*a*) point-pairs with horizontal ordering 0, are connected with a straight line; (*b*) point-pairs with horizontal ordering other than 0 are connected with a curved line sweeping through an angle of

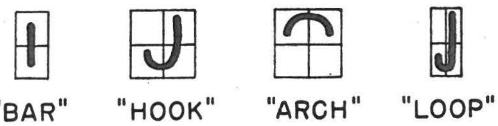

Figure 1. A graphic portrayal of the form line segments

180°. A point-pair together with its appropriate connecting rule defines a *line-segment*. The four line-segments, which we have called respectively bar, hook, arch and loop, can be pictured graphically as shown in *Figure 1*.

The four line-segments can be transformed by reflection about either a horizontal or a vertical axis, yielding eleven distinguishably different segments, of which, however, only nine are utilized in our script.

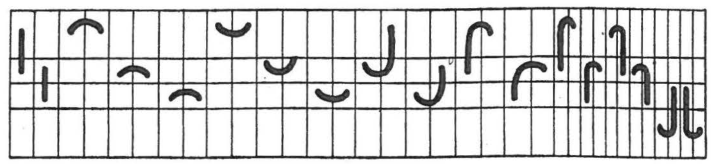

Figure 2. A graphic portrayal of the strokes

The reflected line-segments are located in one of three partially overlapping horizontal *fields*, which together constitute the region in which the written line appears (see *Figure 2*). In placing a line-segment into a particular field it is required that the two terminals of a segment be contained within the field in question and that its maximum or minimum (if such exists) define the upper or lower bound of the field respectively. Only eighteen of the reflected and appropriately positioned line-segments are utilized in the handwriting under consideration. We shall call these 18 line-segments, the *strokes;* they may be pictured as in *Figure 2*. Being a line-segment, every stroke has two termini or *nodes*. We designate as the *initial node*, the terminus that is located higher or, if both the nodes are on the same level, as is the case in the arch, the one that is farthest to the left. The other node is the *terminal node*. In segments in which the nodes are not on the same horizontal level—i.e. in the hook and loop—rotation about a horizontal axis will cause different nodes to be designated as initial and final.

In addition, we shall be interested in the *sense* (clockwise or counterclockwise) of the stroke as it is being traced from its initial to its final node. The bar, which has no inherent sense, will be assigned the sense of the adjacent stroke or strokes with which it forms a continuous curve. It follows from this convention that a bar preceded by a convex arch and followed by a concave arch will have both a clockwise and a counterclockwise sense. All higher level entities of the script are constructed of the strokes.

The following sequences of strokes define *letters* in the script of interest here. Letters that have non-trivial variants like *y z* are represented by more than one entry in our alphabet, which is given in *Figure 3*.

It is not necessary to specify the various diacritic marks, like the dots on the 'i' and 'j' and the crosses of the 't' and 'x', since in the above code all letters are distinct without diacritics. We need, however, a letter boundary symbol, for a sequence of strokes specifying a particular letter may be an initial subsequence in a sequence specifying a different letter; e.g. *c* is contained in *a* ; or *l* is contained in *b*. We need also a word boundary to specify the place where letters are separated by a major space, as well as various punctuation marks which, however, need not be discussed here.

A *word* is completely specified as far as our system is concerned by the stroke sequence composing the letters of the word†. It is the image of a mapping of a finite sequence of letters, into the set of continuous (not necessarily continuous in their derivatives) functions, the mapping being specified by collating and tracing rules applied in a specified order which will be reflected in the numbering system used. Each rule will be designated by a number and a letter, of which the former reflects the ordering of the rule with respect to other rules.

Rule 1a: Within a letter, two consecutive strokes are collated so that the abscissa of the neighbourhood containing the terminal node of the first stroke is made to coincide with the abscissa of the neighbourhood containing the initial node of the second stroke. (Note that in the alphabet (*Figure 3*) there is no letter consisting of a single stroke.)

Rule 1b: Across a letter boundary, the abscissa of the neighbourhood containing the leftmost node of the stroke following the boundary is placed so as to be one unit to the right of the abscissa of the neighbourhood containing the rightmost node of the *penultimate* stroke before the letter boundary. It is to be noted that an immediate consequence of *Rule 1b* is the elimination of letter boundaries. After *Rule 1b* has been applied it is, in general, no longer possible to recover the boundaries between letters in a sequence.

Next the collated stroke representation is modified in accordance with the following conventions.

Convention (a) 'i' and 'j' are dotted.
Convention (b) 't' and 'x' are crossed.
Convention (c) Capital 'T', 'F' and 'A_2' are crossed.
Convention (d) Capital 'D' is provided with a backward hook and flourish.
Convention (e) In the letter 'k' the node common to the hook and the lower concave arch is raised to the middle of the central band, thereby reducing the size of the hook and increasing that of the arch[8].

† It might be noted at this point that the statement that the strokes completely specify the word is true not only of the script examined here but also of Chinese writing, where the placement of the strokes and their specific execution, e. g. their relative size with respect to other strokes, is determined by general rules, so that any Chinese character can be uniquely identified by the strokes composing it. This fact has been utilized in a linotype machine for Chinese designed by Professor S. H. Caldwell[4].

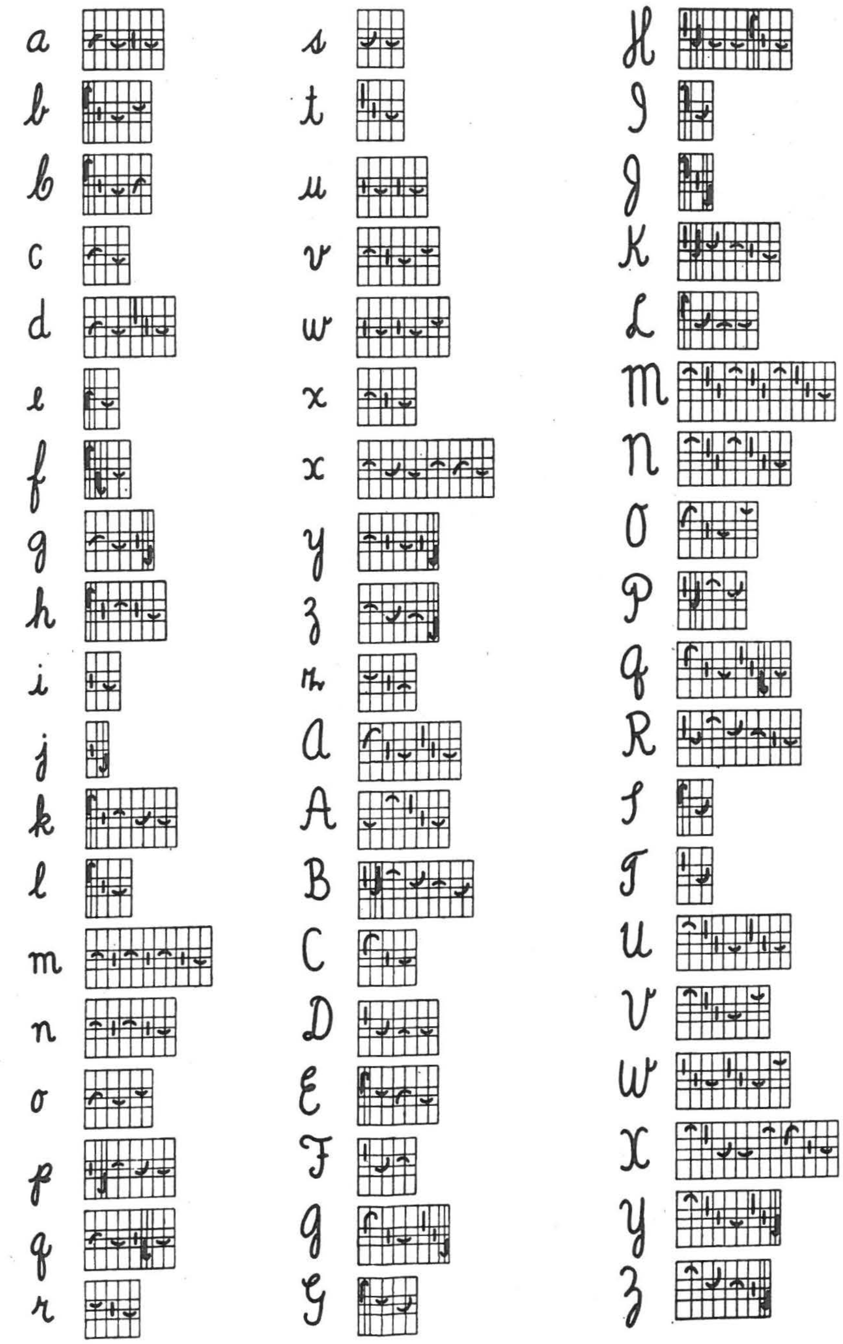

Figure 3. A stroke representation of the English alphabet

Convention (*f*) The nonfinal lower concave arches are eliminated in 'x_2' 'X' 'H'†.

An illustration of the application of *Rules 1a* and *1b*, and of some of the conventions is given in *Figure 4*.

This figure is printed upside down

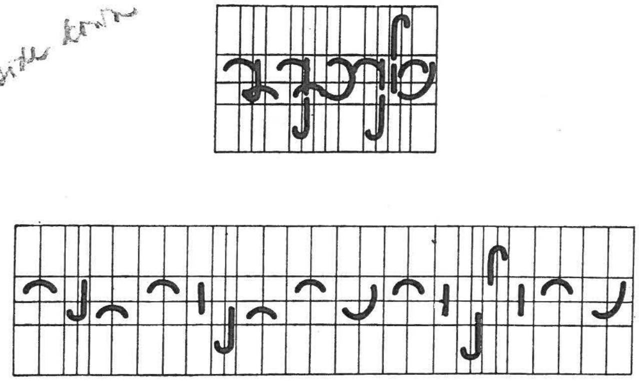

Figure 4. The stroke representation of the word 'globe' and the result of applying *Collating Rules* (*1a* and *1b*) to the former

Rule 2: The strokes are traced in the order in which they appear in the sequential representation of the word. In tracing a stroke we start at its initial node and draw a line so as to maintain the sense and direction of the stroke.

Finally we must detail the manner in which strokes are joined.

Rule 3: The terminal node of the first of two consecutive strokes is joined to the initial node of the following stroke. If the adjacent nodes of the two strokes are not located in the same neighbourhood, then the respective strokes are joined by a ligature; e.g.  ⟶ *b*. The manner of joining the strokes is further determined by the direction of the strokes at the adjacent nodes. If there is a change in direction, the nodes are joined in a singularity. If there is no change in direction, the nodes are joined in a smooth curve, e.g.  ⟶ *s*  ⟶ *l*

Convention (*g*) If the sequence begins with a lower-case letter, the trace is normally preceded by a ligature which connects a point on the bottom of the second field and somewhat to the left of the leftmost node of the first stroke, with the initial node of the first stroke.

Convention (*h*) Ligatures must not cross hooks, e.g. *la*, *le*.
(The ligature in 'la' must, therefore, go above the letter.) Except for this limitation, ligatures are the shortest lines connecting two adjacent nodes.

† These last conventions may seem somewhat arbitrary. They allow us, however, to represent the four letters without in any way modifying the collation and tracing rules. Furthermore, in terms of the descriptive framework adopted here, the best way of characterizing the exceedingly common hands in which 'u' and 'n' are systematically confused is by assuming that the arches are eliminated after applying *Rules 1a* and *1b*.

Because the formalization as presented here puts so few constraints on the actual curves drawn to represent a word, it seemed desirable to try certain functions as generators of the physical curves.

The words shown in *Figure 5* were generated by one such procedure. The particular procedure used can be regarded as a Markov process generating a path on a square lattice. The first point of the first stroke was assigned arbitrarily. A set of adjacent points was specified and a probability assigned to each of these points. Thus the second point was chosen in accordance with the distribution function defined on this set. A new set of adjacent points was chosen taking into account the constraints imposed by the particular stroke which was being formed. A stopping rule was provided which indicated when one stroke had been completed and another should start.

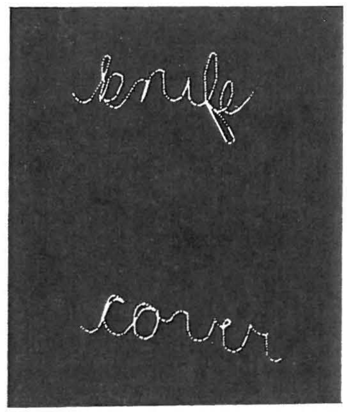

Figure 5. Two words generated by a Markov process constrained by the formal rules specified in the text

We have recently been informed about the work of Van der Gon and Thüring (see Discussion, p. 298) in simulating handwritten words written at high speed by an analogue device which embodies a simple physical description of a theory concerning the muscular forces used in writing.

The specific motivation behind the experiments of these workers and our own are quite different. We have attempted to describe the structure of the handwritten word whereas they have attempted no structural analysis but rather an empirical matching. The words simulated by Van der Gon and Thuring are presumably written with virtually no feedback from the physical signal to the brain. The specimens we have derived from the Markov process are produced with point by point feedback. It is likely that an adequate physical model for handwriting will require both a suitable structural formalism and a functional description that requires a feedback somewhere between these two extremes.

The script characterized up to this point is a sort of idealized norm. It is, of course, obvious that individual writers deviate to a greater or lesser extent from this norm. We must now examine these deviations. Our discussion will be exclusively concerned with deviations in the lower-case letters, for it is among them that the overwhelming majority of all interesting cases are found.

The most common deviations in actual hands are found in the confusion of bars and loops in the noncentral bands. We find a nonsystematic replacement of loops by bars and vice versa. In the lower noncentral band, where bars do not appear distinctively, the bar usually replaces a counterclockwise loop—as in f or q —rather than a clockwise loop. The latter substitution, however, is not infrequent either. It is important to note that though the strokes are confused in the noncentral bands, the bands themselves are never collapsed into the central bands.

The major deviations in the central bands consist in the treatment of the arches, and in the manner in which strokes are joined to one another. The most important of these apparently have a common cause: an attempt to avoid having to draw a line in which there is a change of sense without there being at the same time a change of direction. This situation occurs when there is a concave arch followed by a convex arch; e.g. [on] or in [in] or when a concave arch is followed by a convex hook; e.g. [ec]

The desired simplification is achieved in the following manner:

(1) The concave arch is omitted: [on]
(2) The convex arch is omitted: [on]
(3) *Convention* (*h*), forbidding the intersection of preceding hooks is violated: [ea]
(4) The bend of the hook is eliminated: [ea]
(5) The pen is lifted off the paper within a word: [e a]

Not infrequently one encounters much more radical changes; e.g. all letter initial concave arches are eliminated from the alphabet. In addition arches are eliminated from the collated representation before tracing so that the connection between the remaining strokes becomes simply a consequence of the tracing rules: [summery]

In certain less radical cases, the upper convex arches are replaced by lower concave arches, thereby converting an 'n' into a 'u', etc. [summery]

In some fairly rare instances the distinctions between the three 'vertical' strokes (bar, hook and loop) is not maintained. Such hands appear essentially as a sequence of vertical lines connected by ligatures and a few arches. It must be noted, however, that in all cases that would be considered non-pathological (within the bounds of allowable script), vertical strokes are never omitted.

It is the object of this study to describe cursive English writing. It will be seen that a good deal of the results are applicable to other scripts as well, certainly to those languages written in a Latin script, but it also seems likely that with rather natural modifications the analytic procedure may be used for Gothic, Cyrillic and perhaps Arabic and Sanskrit.

APPENDIX†

A handwritten specimen of English, or any language for that matter, may be regarded as a bounded function in the plane, defined on some interval of

† This appendix will appear as a paper entitled 'On the Formalization of Handwriting', in the record of 'A Symposium on the Structure of Language and its Mathematical Aspects', April 14–16, 1960, New York, N.Y., by the American Mathematical Society.

the real line and continuous almost everywhere. It seems natural to disregard the fact that the written line has finite thickness, microscopic irregularities, microscopic discontinuities and the like†.

Within a particular natural language it is obvious that one handwritten specimen will be judged by a person, literate in that language, to be equivalent to a number of other specimens as well as to other physical signals. The name our observer will give to the equivalence class will be a word in the language. (The name of the class of equivalence classes is 'word'.) He will also say that each *word* is characterized by a finite linear sequence or string of letters.

However, the letters used in a word have no unique representation when they are thought of as functions. In fact it is obvious that the number of representations for a letter are uncountable. This follows immediately from the fact that the partition of two letters is not uniquely defined, nor is it clear how such a partition could be defined. In addition, consider an arbitrary finite sequence of letters rather than dictionary words known to the observer; once such a word has been represented by a particular hand-written specimen, the specimen can be partioned into several different letter sequences, rather than uniquely into the sequence that gave it rise. For example, the letter *c* is embedded in the letter *d* or *l* in *b*.

We regard as our task that of generating the class of representations of an arbitrary word given a finite set of symbols and rules for operating on these symbols. We define a set of objects called *strokes* exhibiting the following properties. Each stroke is a pair of points, the points being ordered in at least one of two (not necessarily orthogonal) coordinates‡, and a real number. The motivation for the real number is essentially as follows.

Associate a unit vector with one of the points. (The initial direction of the vector and the specification as to which is the 'first' point will be uniquely specified by rules to be found in the text.) Imagine the vector tangent to some path terminating at the second point. The magnitude of the real number mentioned above is identified with the angular rotation of the vector and the sign refers to the sense of rotation, i.e. positive if rotation is counterclockwise.

The strokes are generated from a subset of four strokes, called *segments*. A representation of these segments is given in *Figure 1*. A representation of the set of 18 strokes that are sufficient to describe all English upper and lower case letters§ are given in *Figure 2*. Every handwritten letter in English can be described as a unique sequence of these strokes. A table of English letters is given in *Figure 3*¶.

† We are obviously not concerned with those aspects of the physical signal that arise from the fact that a pencil line is actually a collection of a finite number of carbon particles or that the paper is uneven. The problem of identifying an abstract function with a physical line has been considered by J. Perkal, see references 5 and 6.

‡ Two types of 'horizontal' ordering are required; they differ by a condition on the relative ordering to the next stroke in the string of a word.

§ Certain diacritic marks e.g. the dot of the 'i' and the cross of the 't' are not considered. They are in any case redundant in the handwriting under discussion.

¶ We wish to call attention to certain analogies between the structure presented here and that proposed for spoken language. Thus the segments are analogous to the distinctive features[3] the strokes are analogous to phonemes, the letters to morphemes and the words to words. There is no counterpart in linguistics to our primitive notions of point-pair ordering and angular rotation.

We first define a set Σ of elements called segments of the form

$$\sigma_j = (\alpha_{j1}, \beta_{j1})(\alpha_{j2}, \beta_{j2}), \theta_j$$

We specify four elements of Σ which we shall call *bar*, *hook*, *arch* and *loop* respectively:

$$\sigma_1 = [(1, 0), (0, 0), 0] \quad \textit{bar}$$
$$\sigma_2 = [(1, 1), (0, 0), \pi] \quad \textit{hook}$$
$$\sigma_3 = [(1, 0), (1, 1), \pi] \quad \textit{arch}$$
$$\sigma_4 = [(1, \varepsilon), (0, 0), \pi] \quad \textit{loop} \quad 0 < \varepsilon < 1$$

The set Σ is generated by a group $\mathfrak{G}$ of transformations, ρ_j acting on the σ_{ji}.

$$[\rho_1(\sigma_j) = (\alpha_{j2}, \beta_{j1})(\alpha_{j1}, \beta_{j2})\theta]$$
$$[\rho_2(\sigma_j) = (\alpha_{j1}, \beta_{j2})(\alpha_{j2}, \beta_{j1})\theta]$$
$$[\rho_3(\sigma_j) = (\alpha_{j1}, \beta_{j1})(\alpha_{j2}, \beta_{j2})\theta]$$

The set S of *strokes* (of English script) are obtained by applying an additional set F of transformations:

$$f_+(\sigma_j) = [(\alpha_{j1} + 1, \beta_{j1})(\alpha_{j2} + 1, \beta_{j2})\theta_j]$$
$$f_-(\sigma_j) = [(\alpha_{j1} - 1)(\beta_{j1})(\alpha_{j2} - 1, \beta_{j2})\theta_j]$$

and the restriction

$$f(\sigma) = s \in S \text{ if and only if}$$

$$\alpha_1(\sigma) > \alpha_2(\sigma) \quad \text{or} \quad [\alpha_1(\sigma) = \alpha_2(\sigma) \quad \text{and} \quad \beta_2(\sigma) > \beta_1(\sigma)]$$

Thus S is restricted to those elements of $f(\Sigma)$ for which the *initial* mode is *above* the *terminal* mode or if these modes are not ordered then the initial mode is to the *left* of the terminal mode.

Associated with each stroke there is a property we shall call direction. If for any s_j, $[(\beta_{j2} - \beta_{j1}) - \theta_j] > 0$, then the *initial* direction, $D_{j1} = \pi/2$ and is read 'the initial direction is up'. Otherwise, $D_{j1} = -\pi/2$ i.e. 'down'. The *final* direction $D_{j2} = (D_{j1} + \theta_j)(\text{mod } 2\pi)$.

A *letter* is defined as a unique n-tuple of $s_j \in S$. That is, every letter in English script will be identified with a particular $\lambda_\alpha = (s_{\alpha L}, s_{\alpha 2}, s_{\alpha 3}, \ldots, s_{\alpha n})$.

Given a sequence W of λ_j; i.e. $(s_{11}, s_{12}, \ldots s_{1n})(s_{21}, s_{22}, \ldots, s_{2n}) \ldots (s_{k1}, s_{k2}, \ldots, s_{kl})$, compute recursively a new sequence W^*. W^* is called the *collation* of W. Two collation rules are required; the first holds for the concatenation of $s_{h,i-1}, s_{h,i}$; the second holds for $s_{h-1,n}, s_{h,1}$

$$\beta_1^*(s_{11}) = \beta_1(s_{11})$$
$$\beta_s^*(s_{11}) = \beta_2(s_{11})$$
$$\beta_1^*(s_{h,i}) = [\beta_2^*(s_{h,i-1})]\dagger \; h, i \neq 1$$
$$\beta_2^*(s_{h,i}) = \beta_1^*(s_{h,i}) + (\beta_2 - \beta_1)(s_{h,i})$$
$$\text{Min}\{\beta_1^*(s_{h,1}), \beta_2^*(s_{h,1})\} = \text{Max}\{\beta_1^*(s_{h-1,n-1}), \beta_2^*(s_{h-1,n-1})\} + 1$$

Note that W^* does not exhibit letter parentheses.

† The symbol x taken to mean the largest integer less than x.

A word, W', is defined as a class of continuous (finitely many-valued) bounded functions (not necessarily continuous in their first derivatives) on a closed interval of the real line with the following properties:

(1) W' is obtained from W^* by substituting

$$\theta'_j = (\theta^*_j + \delta_j) \quad \text{for} \quad \theta^*_j$$
$$\theta'_{j+1} = (\theta^*_{j+1} - \delta_j) \quad \text{for} \quad \theta^*_{j+1}$$

Thus, in general $\quad \theta'_j = \theta^*_j + \delta_j - \delta_{j-1}; \quad -\frac{\pi}{2} < \delta_i < \frac{\pi}{2}$

(2) $\psi \in W'$ is a concatenation of continuous functions, continuous in their first derivatives, i.e.

$$\psi(s'_1, s'_2, \ldots, s'_k) = \psi_1(s'_1), \psi_2(s'_2), \ldots, \psi_k(s'_k)$$

let $(\xi,\psi)_i$ be the Cartesian coordinate variables of $\psi_i(s'_i)$.

Define:

$$d\tau = \sqrt{(d\xi)^2 + (d\psi)^2}$$

τ_i will thus be a single-valued continuous function of $\psi_i s_i$.

Define: $\phi = \text{arc tan} \dfrac{d\psi}{d\xi}$

Then for each $\psi_i(s_i)$: $\quad -sgn \dfrac{d\phi}{d\xi} = sgn\ \theta_i'$

$$\phi(\tau_i = 0) = D_1(s_1)$$

Define: $\tau_i = n$ by $\displaystyle\int_{\tau_i=0}^{\tau_i=n} d\theta = \theta_i$

If we denote the values of $\psi(s_i)$ at $\tau_i = n_i$ by $(\xi_n, \psi_n)_i$

Then $\quad (\xi_n, \psi_n)_i = (\xi_0, \psi_0)_i$

I. If $\quad (\alpha_{i2}, \beta_{i2}) = (\alpha_{1+i,1}\beta_{1+i,1})$

and (*a*) if $\beta_{i2} < \beta_{i1}$ then $(\xi_n)_i < (\xi_0)_i$

(*b*) if $\alpha_{i2} < \alpha_{i1}$ then $(\psi_n)_i < (\psi_0)_i$ and the converse.

II. Otherwise

$$\exists\ \tau_i, \tau_i = y, \quad 0 < y < n_i,$$

if $\beta_{i2} < \beta_{i1}$, then $(\xi_y)_i < (\xi_0)_i$

if $\alpha_{i2} < \alpha_{i1}$, then $(\psi_y)_i < (\psi_0)_i$ and the converse.

Also,

$$\exists\ \tau_{i+1}, \tau_{i+1} = z, \quad 0 < z < n_{i+1}$$

if $\beta_{1+i}, 2 < \beta_{1+i,1}$, then $(\xi_n)_{i+1} < (\xi_z)_{i+1}$

if $\alpha_{1+i}, 2 < \alpha_{1+i,1}$, then $(\psi_n)_{i+1} < (\psi_z)_{i+1}$

and the converse.

It will be noted that the class of functions equivalent to some word will include some that are rather bizarre. Whether they would be legible or not (and this is the ultimate test of equivalence for a natural language) is open to empirical investigation. There are also a large number of specimens that are legible and that will be associated with a particular equivalence class even though this specimen cannot be generated from the appropriate W' for that class. It is our contention that the literate observer needs a good deal less than the complete W' description in order to identify a specimen so long as he has reason to believe it is a word in the language. As we have already stated, if it is not a word in the language, the specimen may well be generated by several W'_j. In reading, the literate person will reject the alternative readings because they are not words in the language so far as he knows; or if two or more are words in his vocabulary he will have to make his decision by criteria involving the context of the word, i.e. the string of words in which it is embedded.

REFERENCES

[1] LADEFOGED, P. 'The Perception of Speech', Symp. on the Mech. of Thought Processes, Teddington, Great Britain, 1958

[2] LASHLEY, K. S. 'The Problem of Serial Order in Behaviour', in *Cerebral Mechanisms in Behaviour* (ed. L. A. Jeffress), pp. 112–136 (New York, 1951)

[3] JAKOBSON, R., FANT, C. G. M. and HALLE, M. 'Preliminaries to Speech Analysis', M.I.T. Acoustics Laboratory Technical Report 13, 1952

[4] CALDWELL, S. H. 'The Sino-type—A Machine for the Composition of Chinese from a Keyboard', *Jour. Franklin Inst.*, 267 (1959) 471–502

[5] PERKAL, J. 'On the ε Length', *Bull. de l'Acad. Polon des Sciences*, CL.III (1956) pp. 339–403

[6] PERKAL, J. 'Sur les Ensembles ε-Convexes', *Colloqu. Math.* 4 (1956) 1–10

DISCUSSION

J. DENIER VAN DER GON: As Mr. Eden has already mentioned, my co-worker J. P. Thüring and I are also interested in handwriting. We were especially interested in the coding and, together with it, the generation of high-speed uninterrupted handwriting. Therefore we analysed the mechanism of handwriting and, as a result, we constructed a rather simple analogue machine producing high-speed handwriting. The machine was completed just before I left for this Conference; therefore I can only show you one result of what it can write. But perhaps you are interested in the principle and why we think it might work on the same lines as we ourselves write.

Most people use two groups of muscles for writing, one group producing horizontal forces and one the vertical forces. Now if we investigate the vertical forces used for high-speed writing the letters *elelele* without interruption it appears that the magnitude of the force needed for the long stroke of the *l* is roughly the same as the magnitude of the force used for the short stroke of the *e*. As a matter of fact we did not measure forces but accelerations; but these are more or less proportional to the forces. The longer stroke of the *l* results from a longer duration of the force and not from a stronger force.

Therefore our supposition for high-speed handwriting is that the letters are coded only in time, it is the duration of the muscle contraction which is coded, not the force. The main reasons for this supposition are that:

(1) There is no time for an instantaneous control in high-speed handwriting. We do not believe that letters are produced using a sort of position feedback.

(2) Thus letters can only be coded in force and time where the forces appear to be the same in their magnitude.

(3) It is such a very simple coding.

As for the results, *Figure 6* shows (twice) the word 'Hans'; the lower version as written by my co-worker, the other one as calculated from a time coding. (They are not very smooth because I copied them!) As you see, the general features are reproduced rather well. *Figure 7* shows the time coding used for the calculation; it codes the moments at which the muscles or forces start to work upon a mass, including a small amount of friction.

Finally we made a writing simulator, using the foregoing principles, and *Figure 8* shows how it produces, for instance, *jan* if the appropriate coding is inserted.

Figure 6

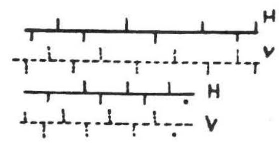

Figure 7

Figure 8

R. A. FAIRTHORNE: The authors choice of invariants is supported by the empirical success of the Cancerellesca hand of the XVI century. Under pressure of correspondence, without benefit of typewriters, secretaries of the day had to evolve letter-shapes that would remain recognizable under deformation due to speed. The basic shapes corresponded to basic motions, which vary little in character when speeded up. These correspond with the authors' elements.