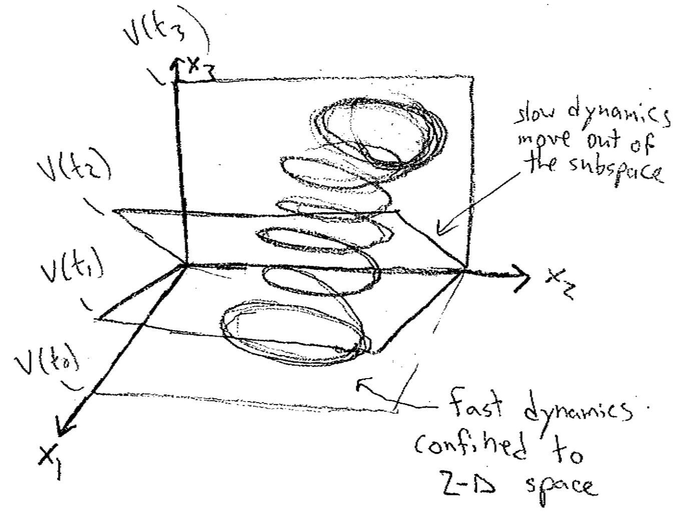
Time-Varying Basis

2009-04-13

① Why time-varying basis?



$$x(t) = V(t)\, x_r(t)$$

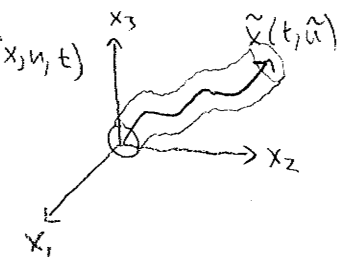
- More coverage of original space
- Necessary for preserving stability of LTV systems

② LTV Systems

$$\dot{x} = A(t)\,x + B(t)\,u$$

- Useful for local analysis of nonlinear systems around some nominal trajectory
- Much more expensive to store and simulate than LTI systems

$$\dot{x} = f(x, u, t)$$



$$\longrightarrow \quad \Delta = x - \tilde{x}$$

$$\delta = u - \tilde{u}$$

$$\dot{\Delta} = \underbrace{\left.\frac{\partial f}{\partial x}\right|_{\tilde{x}(t),\tilde{u}}}_{A(t)} \Delta + \underbrace{\left.\frac{\partial f}{\partial u}\right|_{\tilde{x},\tilde{u}}}_{\tilde{B}(t)} \delta$$

③ Projection

$$\underbrace{\frac{d}{dt}\left(V(t)\, x_r\right)}_{\dot{V}(t) x_r + V(t) \dot{x}_r} = A(t) V(t)\, x_r + B(t)\, u$$

Note: $V(t)$ cannot pass through time derivative due to time-dependence

- Left-multiply by $U(t)^T$ to force residual to be orthogonal to $U(t)$

$$\underbrace{U(t)^T V(t)}_{E_r(t)}\, \dot{x}_r = \underbrace{\left[U(t)^T A(t) V(t) - U(t)^T \dot{V}(t)\right]}_{A_r(t)} x_r + \underbrace{U(t)^T B(t)}_{B_r(t)}\, u$$

Note the extra term in $A_r(t)$

④ Selecting $U(t)$, $V(t)$

- As always, want accuracy & stability
- Time-Varying Balanced Truncation preserves stability and provides error bound, but is very expensive
- Need continuity between subspaces
- Dimensions of subspace can change with time.