MODEL REDUCTION GROUP MEETING 3/30/09

ASSUME: LINEAR SYSTEMS

ERROR ESTIMATION:

input/parameter: $u \in \mathcal{U}$

state: $x \in \mathcal{X}$ $\quad x_m \in \mathcal{X}_m$

outputs: $y \in \mathcal{Y}$ $\quad y_m \in \mathcal{Y}$

full-order model: $\mathcal{F} : \mathcal{U} \rightarrow \mathcal{X} \rightarrow \mathcal{Y}$

reduced-order model: $\mathcal{R} : \mathcal{U} \rightarrow \mathcal{X}_m \rightarrow \mathcal{Y}$

let $r \in \mathcal{R}$ s.t. $r : \mathcal{U} \rightarrow \mathcal{X}_m \rightarrow \mathcal{Y} \ni y_r$

and $f \in \mathcal{F}$ s.t. $f : \mathcal{U} \rightarrow \mathcal{X} \rightarrow \mathcal{Y} \ni y$

define <u>absolute error</u>:

$$\boxed{e(r) = \begin{matrix} \max\limits_{u \in \mathcal{U}_T} \\ \int_{\mathcal{U}_T} \end{matrix} \left\{ \| y(u) - y_r(u) \|_{\mathcal{Y}}^2 \right\} du}$$

where $\mathcal{U}_T$ is a test set s.t. $\mathcal{U}_T \subset \mathcal{U}$.

Also of interest may be <u>relative error</u>.

Let $\tilde{m}$ be fixed positive integer.

define $e^*(r) = e(r^*)$ and $\boxed{r^* = \arg \min\limits_{\substack{r \in \mathcal{R}, \\ m = \tilde{m}}} e(r)}$

then,

$$\boxed{e_{rel}(r) = \frac{e(r)}{e^*(r)}}$$