Title: Geometric Design of Scroll Expanders Optimized for Small Organic Rankine Cycles

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Abstract:

**Background:** The application of organic Rankine cycles (ORC) for small scale power generation is inhibited by a lack of suitable expansion devices. Thermodynamic and mechanistic considerations suggest that scroll machines are advantageous in kilowatt-scale ORC equipment, but a method of independently selecting a geometric design optimized for high-volume-ratio ORC scroll expanders is needed.

**Method of Approach:** The generalized 8-dimensional planar curve framework of Gravesen et. al. (2001), previously developed for scroll compressors, is applied to the expansion scroll, and its useful domain limits are defined. The set of workable scroll geometries is 1) established using a generate-and-test algorithm with inclusion based on theoretical viability and engineering criteria, and 2) the corresponding parameter space is related to thermodynamically relevant metrics through an analytic ranking quantity $f_c$ (‘compactness factor’) equal to the volume ratio divided by the normalized scroll diameter.

**Results:** This method for selecting optimal scroll geometry is described and demonstrated using a 3kWe ORC specification as an example. Workable scroll geometry identification is achieved at a rate greater than 3 s$^{-1}$ with standard desktop computing, whereas the originally undefined 8-D parameter space yields an arbitrarily low success rate for determining valid scroll mating pairs. For the test case, the selection of a non-circle involute scroll results in a 5% improvement in modeled isentropic expansion efficiency compared to standard circle involute geometry. A readily computed geometric property, the compactness factor (volume expansion ratio per diameter), is shown to correlate with isentropic efficiency with $R^2=0.699$. 
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Conclusions: The rapid, computationally efficient generation and selection of complex, validated scroll geometries ranked by physically meaningful properties is demonstrated. This procedure represents an essential preliminary qualification for intensive modeling and prototyping efforts necessary to generate new high performance scroll expander designs for kilowatt scale ORC systems.

Keywords: scroll expander design, planar curves, volume ratio, compactness factor, isentropic efficiency kilowatt-scale organic Rankine cycle
Background

The Organic Rankine Cycle (ORC) is an established technology for power generation from low temperature (< 300°C) thermal sources (e.g., geothermal, solar, and industrial). ORC applications are generally more economical as the scale of thermal resource or potential load increases; however, as a result of rising energy costs and pressing environmental considerations, the minimum size for a commercially viable ORC unit is presently decreasing into the range of 1-10 kilowatts electrical output.

Whereas large ORC systems can use industrial turbomachinery similar to that widely used in common fossil-fuel-fired thermal power plants, the main challenge to developing ORC equipment in the range of 1-100kW is in the selection of a suitable expander, given the absence of commercially available turbines at this scale. Further, positive-displacement expanders may have certain advantages over small turbines, including lower rotational speeds, proportionally less windage loss, and potentially lower cost due to the availability of machines which can be adapted from HVAC applications, e.g., reversed scroll compressors. The primary drawback of the latter approach is the low intrinsic volume ratio of commercially available scroll machines (typically ~3) which limits the cycle operational temperature range or forces acceptance of under-expansion losses [1].

Development of a scroll expander optimized for the larger volume ratios encountered at higher temperature ORC applications (3-15 or higher, depending on the temperature and working fluid) would promote the viability of ORC power generation from smaller, distributed thermal resources. While several investigations of scroll expander models and validation experiments are described in the literature [2-7], discussion of choice of scroll geometry, the single feature upon which all other properties depend, is generally limited to the case of circle involutes in low volume ratio compressor applications [8,9]. In contrast, the present work explores the effect of varying the basic scroll geometry as a method for developing novel scroll machinery at the higher volume ratios needed for many ORC applications. The results demonstrate a computationally efficient process, based on thermodynamically relevant criteria, for converging on a set of near-optimal candidates for scroll geometry. Details of the algorithm, along with a specific case study, are described in this publication. The complete thermodynamic analysis of a particular
scroll expander in an ORC application, such as performed by [10,11], is not addressed in this paper but is further developed from the results of this work in [12].

Method of Approach

Development of our design tool was based on the mathematical scroll model described by Gravesen et al. [8], generalized for a wide range of scroll geometries and vectorized for implementation in Matlab. In the following sections we briefly review the planar curve mathematic framework used, discuss the importance of discovering domain limits within the framework, and relate the input parameters to relevant metrics for scroll expander design.

Equations defining scroll geometry

The geometry of the scroll, classically based on circle involutes, was more generally described by Gravesen et al. [8] where the parameterization simplifies to the circle involute as a special case, but cases with varying wall thicknesses can also be modeled. As presented in [8], the planar curves of the scroll wrap are defined by the intrinsic equation linking the arc length \( s_x \) to the tangent direction \( \Phi \):

\[
s_x = c_1 + c_2 \cdot \Phi + c_3 \cdot \Phi^2 + c_4 \cdot \Phi^3 + c_5 \cdot \Phi^4
\]  
(1)

where \( \{c_1, c_5\} \) are scalar coefficients. Based on this general fourth-order polynomial for \( s_x \), and the coordinate system used by Gravesen, Cartesian coordinates for the initial scroll wall, \( x \), can be found analytically, as shown in Eq. (2).

\[
x = (\alpha \cdot \sin(\Phi) + \beta \cdot \cos(\Phi), \beta \cdot \sin(\Phi) - \alpha \cdot \cos(\Phi))
\]  
(2)

Parameters \( \alpha \) and \( \beta \) are related to the tangent direction, \( \Phi \), and original scalar coefficients as follows:

\[
\alpha = (c_1 - 6c_3) + (2c_2 - 24c_4) \cdot \Phi + (3c_3) \cdot \Phi^2 + (4c_4) \cdot \Phi^3
\]  
(3)

\[
\beta = (2c_2 - 24c_4) + (6c_3) \cdot \Phi + (12c_4) \cdot \Phi^2
\]  
(4)
Equations for the mating curve, as well as opposite sides of these scroll walls, are found by reflection and symmetry following the method in [8]. This requires definition of two additional parameters: R, the orbital radius of the moving scroll, and d, a scalar length related to wall thickness. The range of \( \Phi \) over which these walls are considered is constrained by definition of N, the desired number of turns of the scroll spiral, such that \( \Phi_{\text{max}} - \Phi_{\text{min}} = 2\pi N \). The N consecutive points of conjugacy between the moving scroll (orbiting at radius R) and the fixed scroll, found at \{ \Phi_c, \Phi_c + 2\pi, \Phi_c + 4\pi, \ldots \} for some initial conjugacy angle \( \Phi_c \), represent the terminal points of adjacent internal chambers (“pockets”) within which working fluid expansion occurs (Fig 1). For additional details on the determination of conjugacy points and calculation of pocket volumes see [8,9].

The scroll geometry parameter space is thus 8-dimensional, defined by \{c_1..c_5\}, d, R and N. While the relationship of these parameters to the scroll wall x is analytically derived in [8], the valid domains of these parameters and their effects on important scroll characteristics are not. This work addresses this limitation by systematically exploring the parameter domains and relating the eight-dimensional space to relevant design metrics.

**Parameter domain definition**

Because the parameter space for mating scroll pairs based on the intrinsic equation is infinite, realistic simulation of scrolls to meet physical specifications is greatly aided by knowledge of the parameter domains as they relate to viable examples. To discover envelopes of ‘viability islands’ within the parameter space and assign relationships to relevant criteria, we employ a generate-and-test algorithm for expanding the parameter domains from an initial set that includes known viable scroll geometries, i.e. the archetypal cases of [8] (circle involute and two examples with increasing wall thicknesses), followed by identification of parameterizations producing unworkable scroll designs.

Scroll designs are unworkable in cases where, e.g., the intrinsic equation produces a non-monotonically increasing spiral that crosses itself or its mating curve, violating the conjugacy between orbiting and fixed
scrolls upon which the action of the machine depends; these cases are immediately discarded upon identification. To further improve the quality of resulting scrolls, however, we also include three other criteria of relevance to scroll engineering with which to evaluate potential scroll geometries: wall thickness (for mass and mechanical strength), scroll diameter (form factor, with size limited to $D_{\text{max}}$ calculated as a function of $V_{\text{in}}$), and orbital radius $R$ (i.e., the throw of the crank arm for power takeoff). The designer must choose an acceptable range for these values based on material constraints, conformation within the ORC, the mode of power transmission, and potentially on considerations related to leakage, lubrication, etc.; reasonable default values for a kilowatt-scale scroll expander are given in Table 1.

Creation of a distribution of useful scrolls proceeds as follows: random values for each parameter are generated from within the defined domains, a scroll is generated based on these parameters, the scroll size is normalized to match the specified inlet volume $V_{\text{in}}$ (as the displacement and hence power rating must be similar for a direct comparison among differing but valid scroll geometries), and finally the geometry is tested against the above criteria for viability and engineering utility (Fig. 2). This is repeated until a sufficiently large number of scrolls has been obtained (e.g., 10,000-20,000). At this point, the distribution of each parameter is examined to determine whether the range capable of producing a valid scroll has been circumscribed; if no such ‘envelope’ is detected, the domains are expanded, generating a new scroll distribution, until this condition is met. Examples of the identification envelope relationships are shown in Fig. 3. Constraining the infinite domains of the 8-D parameter space for viable scrolls thus limits unproductive simulation effort, and represents a computationally efficient means for selection of optimal scroll geometry.

Because the domain envelopes are expected to be interdependent functions of each other, we characterize them in a hierarchy corresponding to the order of operations in the search algorithm (Table 1) and related to their relative independence in determining the outcome of the scroll geometry. The result of this process is a stable valid-scroll identification success rate of approximately 0.02 (valid scrolls per random parameter set generated), or greater than 3 scrolls $s^{-1}$ using the computational resources described in Table 2 and discussed further below. This success rate is a significant improvement over the case of arbitrary domain
limits, where the success rate is very low; implementation of envelope identification provided approximately an order of magnitude improvement relative to searching arbitrarily within identified valid domains.

Table 1: Viability constraints and parameter domains for the planar curve scroll geometry framework. The lower limit on $R_{\text{norm}}$ is proposed to ensure that orbital motion can be practically translated into rotation mechanically, e.g., with a crank. The wall thickness minimum constraint is derived from an empirical correlation using ZR type scroll machines and a power law for force on the scroll wraps as a function of arc length, described in [12]. The parameter domain functions are defined through the process described in Fig. 2.

Viability Constraints

$$\begin{align*}
D_{\text{max}} & \quad D_{\text{max}} < 100 \ln(V_{\text{in}}) \\
R_{\text{norm}} \text{ (after normalization)} & \quad R_{\text{norm}} > 2\text{mm} \\
\text{Wall thickness} & \quad < \text{wall thickness} < 100 \text{mm}
\end{align*}$$

Parameter Domains (with envelopes)

$$\begin{align*}
N & \quad 3 < N < 12 \\
R & \quad (2.4241 N - 16.1332) < R < 60 \\
D & \quad 0.5 R < d < 30 \\
c_1 & \quad c_1 = 0 \\
c_2 & \quad -4 < c_2 < 4 \\
c_3 & \quad 1.0145 N - 8.8014 < c_3 < -0.40411 N + 18.0718 \\
c_4 & \quad 0.083725 d - 4.8712 < c_4 < 0.54374 d + 0.63304 \\
c_5 & \quad -0.058902 c_3 - 0.016956 < c_4 < -0.083835 c_3 + 1.1011 \\
c_6 & \quad -0.016401 c_4 - 0.0013445 < c_5 < \min(0.02, 0.38923 e^{-0.64318 N} + 0.0014341)
\end{align*}$$
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Domains for N, R_{max} and d_{max} are arbitrary and are chosen based on the experience of the researchers to include a reasonable search space. Values of N higher than 12 are possible, but the mechanical tolerances necessary to achieve compliance present an engineering challenge as N increases. Valid scroll geometries can also be found for arbitrarily high values of R and d, with the result simply scaled down during normalization. In practice, however, the uniqueness of scroll geometries is sufficiently captured at some finite domain limit for R and d. Finally, c_1 is set to zero because it has no effect on the output other than to locate the curve geometry within the coordinate system.

Table 2: Computer system hardware and software configuration

<table>
<thead>
<tr>
<th>Workstation</th>
<th>Operating System:</th>
<th>Windows Enterprise 64-bit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Processor:</td>
<td>Intel i7 LGA1366 2.67GHz</td>
</tr>
<tr>
<td></td>
<td>RAM:</td>
<td>6 GB</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Programming Environment</th>
<th>Matlab R2011b 32-bit</th>
</tr>
</thead>
</table>

| Computation Time         | 4.3 scrolls s^{-1} (σ = 0.23 scrolls s^{-1}) |

Optimization of scroll geometry

Once a viable set of scrolls has been established with the previous method, optimization is possible over a range of criteria. Ideally, candidates should be chosen for further exploration using a mechanistic and thermodynamic model based on R, or other geometric data. Gravesen suggested the leakage factor as a possible figure of merit to prioritize further investigation [8] but provided analytic treatment of only tangential leakage, whereas Chen identifies radial leakage as the dominant mode [11]. Within Graveson’s generalized framework, radial leakage, which is dependent in large part on arc length, is problematic to compute because the arc length, s_x corresponding to the scaled version of x, cannot be derived analytically. To avoid this difficulty, as well as to simplify the ranking process, an alternative figure of merit – the compactness factor - is proposed.

Rapid selection based on compactness factor
In this study we have essentially normalized tangential leakage by normalizing scroll height, $z$, using a relationship we derived empirically from measurements of ZR-type scroll compressors ($N=5$, $R^2=0.97$) manufactured by Copeland:

$$z = 7 V_{in}^{0.58}$$

(5)

where $z$ is the scroll wrap height in mm and $V_{in}$ is in units of cm$^3$. This leaves radial leakage, axial friction, and heat loss from the unit as the main relevant physical criteria to inform optimal scroll selection.

Analytical calculation of these quantities is both difficult to achieve, as noted above for the case of radial leakage, and computationally expensive given that sufficiently dense coverage of the viable domain may include tens of thousands of potential scroll geometries. We instead propose that radial leakage, axial friction, and heat loss may be captured to a first approximation by aspects of the geometry distilled into a ‘compactness factor’ ($f_c$) as a function of the volume ratio $R_v$ and the normalized scroll diameter:

$$f_c = \frac{R_v}{D_{norm}}$$

(6)

where $f_c$ has units of 1/L. **Whereas normalized diameter alone does not capture information regarding efficiency, the compactness factor as a concept is useful in comparing scrolls across ORCs of different size and temperature specifications.** For the assumption of a constant leakage gap height, $f_c$ will be inversely proportional to the radial leakage path as a function of $(s_x)$ for any given $R_v$. Similarly, by conserving area and volume for a given $R_v$, $f_c$ intrinsically captures an important scale coefficient for both axial friction losses and heat transfer between the scroll and its surroundings. Thus, searching for maximal $f_c$ enables rapid, computationally inexpensive selection of a subset of viable scroll geometries that are likely to be close to optimal when evaluating mechanistic and thermodynamic features in an intensive model such as described by [7,8], including, e.g., refinements for the suction wrap profiles [9, 13] and tradeoffs between tangential and radial leakage as a function of varying $z$ and the normalization factor.

**Results**
To illustrate the method proposed above, we provide the following case study wherein we consider the design of an expander for a 3kWe ORC having the characteristics defined in Table 3. The nominal expander isentropic efficiency combines assumed values for expander mechanical efficiency (0.8) and small induction generator efficiency (0.82) derived from test bench results using expander-generators based on reversed hermetic scroll compressors and separately tested induction machines [14, 15].

Table 3: Design specifications for a 3kW ORC using R245fa.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>3 kWe</td>
</tr>
<tr>
<td>T_{exp,su}</td>
<td>120 °C</td>
</tr>
<tr>
<td>T_{cd}</td>
<td>40 °C</td>
</tr>
<tr>
<td>Superheat</td>
<td>5 °C</td>
</tr>
<tr>
<td>Rotational speed</td>
<td>3000 RPM</td>
</tr>
<tr>
<td>Expander effectiveness</td>
<td>0.66</td>
</tr>
<tr>
<td>R_{VP}</td>
<td>8.5</td>
</tr>
<tr>
<td>V_{in}</td>
<td>24.5 cc/rev</td>
</tr>
<tr>
<td>m_dot</td>
<td>0.123 kg/s</td>
</tr>
</tbody>
</table>

The data of Table 3 show ORC parameters that bear on the expander design, namely the targeted power output [kW], the working fluid temperature differential, the rotational speed, and the working fluid (suggested by the ΔT) [16]. Pinch conditions of the heat exchangers may be considered using, e.g., the epsilon-NTU or LMTD method to arrive at actual working fluid conditions, including any superheat, from knowledge of thermal source temperatures [17].

From this data the desired volume expansion ratio, R_{VP}, of the expander can be inferred from the working fluid properties assuming 95% volumetric efficiency (reasonable for scrolls) [2,4,18] and using an imputed isentropic efficiency from, e.g., [3,4], which may be inclusive of downstream losses from a generator.
These characteristics define the initial volume displacement ($V_{in}$) of the expansion pockets formed by the scroll wraps as follows:

$$V_{in} = \frac{Power \cdot v_{in}(T_{SU}, P_{SU}) \cdot 6 \cdot 10^7}{RPM \cdot \varepsilon_{exp} \cdot (h_{SU}(T_{SU}, P_{SU}) = h_{EX}(T_{SU}, P_{EX}))} \text{ [cm}^3\text{]} \text{ (rev}^{-1})$$  \hspace{1cm} (7)

where $Power$ is in kilowatts. The initial pocket area $A = V_{in}/2z$ because two pockets form and discharge per cycle of the orbiting scroll. Using the wrap height $z$ calculated with Eq. (6), initial pocket area can in turn be described as a function of $V_{inlet}$:

$$A = 71.4 \ V_{in}^{0.42}$$ \hspace{1cm} (8)

where $A$ is in units of $\text{mm}^2$ and $V_{inlet}$ is in units of $\text{cm}^3$. The search domain is thus set from these criteria-$V_{in}$ relationships and the specified viability constraints from Table 1.

The optimization algorithm then proceeds according to Fig. 4. Scrolls are simulated, normalized to $V_{in}$, and checked for viability, with the resulting valid scrolls sub-selected for appropriate values of $R_v$ matching the $R_{VP}$ of the application and ranked based on $f_c$. Optimal results can be displayed for visual comparison; the scroll identified in this case as optimal is shown in Fig. 5. Total computation time to produce this result, along with 137 other potentially useful geometries in the appropriate range of $R_v$, was approximately 40 minutes using the workstation detailed above for a viable scroll dataset population of $n=10,000$.

To validate this approach and the use of $f_c$ as a proxy for isentropic expansion efficiency, a subset ($N=16$) of the scroll dataset for the test specifications is selected for analytical and numerical thermodynamic modeling to determine losses due to leakage, friction and heat transfer, according to parameterization (gap height, oil film thickness etc.) described in [12]. This analysis shows a robust correlation between $f_c$ and efficiency ($R^2=0.699$). Interestingly, the results with the highest compactness factor and isentropic efficiency tend not to be true circle involutes. The geometry with the highest modeled efficiency (87.3%)
outperforms the circle involute example (82.7%) by nearly 5%. Scroll geometries with varying wall thickness have the potential for higher isentropic expansion efficiency at reduced mass compared with circle involutes by executing a larger volume expansion per revolution so that fewer wraps (N) are needed for a given RVP application and the corresponding mating arc length (and the potential leakage path) is shorter. Beyond the circle involute case, a scroll designed with planar curve geometry also presents an opportunity for distributing wall thickness in accordance with the dynamic forces on the wrap walls (i.e., wider at the inlet, decreasing thickness towards the outlet). These observations raise the possibility that unexploited potential for scroll geometry optimization exists that could increase the performance of scroll devices in a variety of applications.

**Conclusions**

The generalized scroll geometry framework of Gravesen [8] was employed to develop a design tool for ORC scroll expanders using an algorithm based on the generate-and-test selection process. Computational efficiency is optimized by identifying key domain boundaries within the 8-dimensional parameter space and implementing physically relevant constraints. The success rate for identifying valid scrolls by this method was approximately 2%, enabling computation of valid scroll geometries for a given design specification in <1 hour using standard desktop computing available in 2012. A figure of merit from analytic geometric data, the 'compactness factor’ $f_c$, is proposed and validated as a positive correlate for isentropic efficiency and the objective function for the identification of scrolls with advantageous properties regarding leakage, friction, and heat losses. These procedures are combined into a design tool and its use is demonstrated through selection of an optimal scroll geometry for an example ORC specification. This approach provides a generalizable, rapid and physically meaningful basis for selecting scroll geometries for subsequent, more computationally intensive modeling. The results of this method should promote improved outcomes with detailed mechanistic and thermodynamic models and ultimately support the development of high performance scroll machines for various applications, including expanders in kilowatt-scale ORC systems.
Acknowledgements

This work was supported by the William E. Leonhard Professorship to Harold Hemond; the MIT Energy Initiative Seed Fund [award number 015728-020]; and the United States Environmental Protection Agency through a P3 Phase II [award number SU 83436701] (This work does not necessarily reflect the views of the Agency and no official endorsement should be inferred).

Nomenclature

A area (m²)
c coefficient (-)
D diameter (mm)
d wall thickness offset (mm)
f_c compactness factor (mm⁻¹)
h specific enthalpy (J kg⁻¹ K⁻¹)
m_dot mass flow rate (kg s⁻¹)
n scroll sample set size
N number of wraps
P pressure
R radius (mm)
R ratio (-)
RPM revolutions per minute (RPM)
T temperature (°C)
v specific volume (m³ kg⁻¹)
V volume (cm³)
z height (mm)
Greek symbols

\( \phi \)  tangent direction

\( \varepsilon \)  effectiveness

Subscripts and superscripts

\( c \)  compactness

\( cd \)  condenser

\( ex \)  exhaust

\( exp \)  expander

\( in \)  inlet

\( norm \)  normalized

\( su \)  supply

\( V \)  volume

\( VP \)  volume (process)

References


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University, Corvallis, OR.


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Figure 1
Figure 2
Figure 3
**Figure 4**

1. **Start**
2. Specify ORC parameters: kW, RPM, ΔT, fluid
3. Calculate $V_{\text{inlet}}, R_v$
4. Filter on $R_v$
5. Generate Scroll Distribution Function
6. Optionally update default constraints for $DIA_{\text{norm}}, R_{\text{norm}}$, and min/max wall thickness
7. Order by compactness factor $f_c$
8. Display/Save results
9. **END**
Figure 5

Figure 6
List of Figures:

Figure 1: The expansion action of the scroll device works via a series of chambers defined by adjacent conjugate points. High pressure vapor enters at the inlet and expands against the orbiting scroll in an expanding chamber following the spiral. The orbit of radius R translates to rotation with a crank. The mating pairs of scroll curves are formed by reflection across the center point C, accounting for the wall thickness scalar d.

Figure 2: Flow diagram for design to development method.

Figure 3: Four example scroll distributions are plotted from within the 8-D planar curve parameter space. The color bar represents the value of the proposed 'compactness factor' (Volume ratio divided by normalized diameter), and gradients within the domains reveal the relationships of input parameters to this metric. White space indicates non-viability or practical constraint violation. The delineation of these domain envelopes through the algorithm of figure 2 forms the basis for the equations in table 1. The resulting avoidance of non-productive parameter combinations conserves computation effort and accelerates selection of optimal scroll geometries.

Figure 4: Flow diagram for use of design tool in an actual ORC application.

Figure 5: Right: A high 'compactness factor' design for a non-circle involute scroll proposed for an ORC case study based on a \( R_{VP} = 8.5 \). Planar curve parameters chosen are: \( c1=0, c2=-.44, c3=3.8, c4=0.3, c5=-0.0027, N=7.25, R=28.8, d=25.8 \). The scaling factor to normalize to \( V_{in} = 24.5 \text{ cm}^3 \) is 3.1. Left: A standard constant wall thickness (circle involute) scroll achieving the same volume expansion.

Figure 6: Modeled [12] isentropic expander efficiency correlation to compactness factor, \( f_c \), for the test case ORC (\( R_{VP} = 8.5 \)) dataset. \( N=16, R^2=0.699 \). The circle involute case is highlighted in red. Selection of a variable wall thickness scroll geometry indicates a potential ~5% improvement in efficiency.
List of Tables:

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