

# Extra-Solar Planetary Microlensing

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## I. Introduction

Throughout history, people have debated the existence of life elsewhere in the universe, prompting researchers to seek out planets in other systems that could give rise to life. The current search for extra-solar planets has scientists asking whether the structure of our solar system is unique or different from other systems. By investigating the dynamics of various planetary systems, researchers can theorize how our solar system and other systems formed, what conditions gave rise to our solar system being similar to or different than other systems, and what the evolution of various planetary systems will be.

Before 1990, investigation of planetary systems was limited to looking at our own. Researchers have since developed many methods for detecting extra-solar planets and have discovered dozens of them.<sup>1</sup> Each method is sensitive to certain types of planets based on the planet's size and orbital motion. However, because distant planets are very faint, these methods are indirect and planets are detected mainly by their influence on nearby stars.

## II. Extra-solar Planet Detection

The classical methods for detecting planets include radial velocity measurements, astrometry, and photometric detection of transits. Radial velocity measurements can be gained by fitting the Doppler shift of a large number of features within a star's spectrum and removing the motion of the observer relative to the barycenter of the star system and other known motions. Astrometry detects planets by observing the wobble they induce in the motion of their stars projected onto the plane of the sky. Photometric detection of transits can spot a planet passing in front of the disk of the star once per orbit when viewed from Earth.<sup>1</sup> Although the classical methods are sensitive to certain types of planets, they complement one another because they measure a similar range of semi major axes and parameters.

However, there are some drawbacks to using classical methods. First, they are limited to nearby systems because they rely on light from either the parent star or the planet itself. Second, they are not sensitive to low-mass planets: Essentially, they can only detect giant planets, brown dwarfs, and low-mass stars. As Guadi (2002) showed, the systematic floor of radial velocity surveys implies that extra-solar planet Earth, Uranus, and Neptune analogs are probably inaccessible to radial velocity measurements. Third, these methods require that a system be observed for at least one full period of its companion.<sup>2</sup>

Microlensing is a new and alternative way for detecting extra-solar-planets and was first proposed in 1991 by Mao and Pacynski. Their work was expanded upon in 1992 by Gould and Loeb.<sup>2</sup> Microlensing is a general relativistic effect that occurs when light from a distant star is bent by a massive object (i.e., the lens) as the light passes near the object on its way from a source to an observer. Microlensing seeks to overcome some of the drawbacks associated with the classical methods.

## III. Gravitational Lensing

### A. Deflection of Light

Here are some of the basic equations and ideas in gravitational lensing. One of the most famous results of general relativity is a ray of light deflected as it passes by a massive object. In Newtonian mechanics this deflection

angle ( $\alpha$ ) can be calculated as follows. A ray of light passes by a massive object at a distance  $b$  (i.e., the impact parameter). Here, we shall assume the massive object is a point mass that exerts an acceleration perpendicular to the direction of the motion of the light ray:<sup>3</sup>

$$g = \frac{GMb}{(b^2 + z^2)^{3/2}}, \quad (1)$$

where  $G$  is the gravitational constant,  $M$  is the mass of the point object, and  $z$  is distance. The deflection angle equals the total integrated velocity ( $v$ ) divided by the speed of light ( $c$ ):

$$v = \int g \cdot dt = \int g \cdot \frac{dz}{c} = \frac{2GM}{bc}, \quad (2)$$

so:

$$\alpha_{Newtonian} = \frac{v}{c} = \frac{2GM}{bc^2}. \quad (3)$$

In general relativity (GR), this deflection angle is multiplied by a factor of 2, which results from the Schwartzchild metric:<sup>4</sup>

$$\alpha_{GR} = \frac{4GM}{bc^2}. \quad (4)$$

From this result, one can calculate the Einstein ring, which is one of the most important equations in gravitational lensing, as we shall see later.  $\theta_{observer}$  is the angle made by a ray of light coming from the source that has been bent and is currently incident on the observer's line of sight with the line connecting the observer and the source. Also,  $D_{OL}$  is the distance from the observer to the lens (i.e., the point mass in this case);  $D_{LS}$  is the distance from the lens to the source; and  $D_{OS}$  is the distance from the observer to the source. Therefore:

$$\theta_{observer} = \frac{b}{D_{OL}} \quad (5)$$

and

$$\alpha = \frac{b}{D_{LS}} + \theta_{observer}. \quad (6)$$

If one solves for  $\theta_{observer}$  and eliminates the impact parameter, then:

$$\theta_{observer} = \theta_{Einstein} = \left( \frac{4GM D_{SL}}{D_{OL} (D_{OL} + D_{LS}) c^2} \right)^{\frac{1}{2}}. \quad (7)$$

This can be rewritten as:

$$\theta_{Einstein} = \left( \frac{4GM}{Dc^2} \right)^{\frac{1}{2}} = \left( \frac{2R_{Schwartzchild}}{D} \right)^{\frac{1}{2}}, \quad (8)$$

where  $D \equiv \frac{D_{OS} D_{OL}}{D_{LS}}$ ;  $R = \frac{2GM}{c^2}$  is the Schwartzchild radius of the lens.<sup>5</sup>

For a microlensing event, this equation can be approximated as:<sup>2</sup>

$$\theta_{Microlensing} \cong 3 \times 10^{-7} \left( \frac{M}{0.3 M_{Sun}} \right)^{\frac{1}{2}} \text{ arc seconds} \cong 4 \times 10^{-22} (M)^{1/2} \text{ arc seconds}, \quad (9)$$

$M$  (kilograms) is the mass of the extra-solar object and  $M_{Sun}$  is the mass of the Sun ( $1.99 \times 10^{30}$  kg). As one can see,  $\theta_{Microlensing}$  produces a very small result, especially when Earth, Uranus, or Neptune extra-solar analogs have a mass between  $1 \times 10^{24}$  kg– $1 \times 10^{26}$  kg. Despite the small separation in the images (i.e., the small Einstein ring), an extra-solar microlensing event is detectable and will be discussed shortly.

## B. Time Delays

Because the trajectory for which a deflected light traverses is curved, the amount of time it takes a light ray to propagate from a source to an observer increases. This increase in time is known as the time delay. To calculate the time delay and other lensing effects, one must work in two dimensions. The source is described by an angular position  $\vec{u}$ . The source emits a ray of light that passes by a massive object (i.e., the lens) in the foreground. The massive object has an impact parameter with the angular position  $\vec{x}$ . Here the ray of light is deflected by the gravitational field of the lens. Similar to how we assumed in the Einstein ring calculation that the massive object was point like, we must now assume that the lens occupies only a small fraction of the total path length (i.e., the thin lens approximation). The time delay can be viewed as equaling a geometric delay plus a gravitational delay, where the gravitational potential roughly corresponds to a lens's mass distribution:

$$\tau(\vec{x}) = \frac{1 + z_L}{c} \frac{D_{OL} D_{OS}}{D_{LS}} \left[ \frac{1}{2} |\vec{x} - \vec{u}|^2 - \varphi(\vec{x}) \right]^{\frac{1}{2}}, \quad (10)$$

where  $z_L$  is the red shift of the lens and  $\theta(\vec{x})$  is the two-dimensional gravitational potential.<sup>6</sup> The time delay equation is important when modeling an observed gravitational lens and hypothesizing how, why, and where images will appear.

## C. Lens Equation and Magnification Tensor

The geometry of lensing (i.e., the positions where images will be observed after light is deflected by a lens) can be described by Fermat's Principle: Rays of light follow paths that form images, which represent stationary points in arrival time.<sup>3,6</sup> This principle also corresponds to coherent phases that result in positive interference. The stationary points where images form represent solutions to the lens equation:

$$\vec{u} = \vec{x} - \nabla \varphi(\vec{x}). \quad (11)$$

This equation relates the source position ( $\vec{u}$ ) to the image positions ( $\vec{x}$ ) via the deflection angle:<sup>6</sup>

$$\alpha = \nabla \varphi. \quad (12)$$

Solutions to the lens equation can be related to the time delay equation, such that:

$$\nabla \tau = 0 \Rightarrow \bar{x} - \bar{u} - \nabla \phi(\bar{x}) = 0. \quad (13)$$

In addition to the lens deflecting light from a distant source to create multiple images, the lens magnifies each image differently. The magnification ( $\mu$ ) is described by the determinant of the Jacobian of the lens mapping:<sup>3, 6</sup>

$$\mu = \det(M) = \det\left[\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)^{-1}\right] = \det\left[\begin{bmatrix} 1 - \phi_{xx} & -\phi_{xy} \\ -\phi_{xy} & 1 - \phi_{yy} \end{bmatrix}\right]^{-1}. \quad (14)$$

The subscripts following the gravitational potential denote partial derivatives with respect to  $x$  and  $y$ . As shown earlier, typically the separation in the images of an extra-solar microlensing event is small. In such a case, it is important to observe image magnification rather than image separation. Moreover, when individual images are indistinguishable, one must observe the combined differential image magnification as a function of time. This idea will be further investigated in following sections.

#### D. Caustic and Critical Curves

In some cases, a gravitationally lensed image may become very intensely magnified. This occurs because the source has a unique position with respect to a caustic curve and subsequently a critical curve. Critical curves exist in the image plane and are locations where images become infinitely magnified (theoretically). However, no gravitationally lensed image is ever really infinitely magnified because no source is point like. Critical curves are defined as:<sup>6</sup>

$$\det(M^{-1}) = 0. \quad (15)$$

These critical curves are where one would observe very bright images (i.e., highly magnified sources). By mapping the critical curves back to the source plane, one forms caustic curves—locations in the source plane where multiple light rays collect. Any source lying near a caustic is highly magnified. If a source crosses a caustic, the number of images changes by two (i.e., two images are created or annihilated).<sup>3, 6</sup>

#### IV. Extra-Solar Microlensing

Extra-solar microlensing is one form of gravitational lensing that occurs when compact objects such as low-mass stars and planets pass close to an observer's line of sight of a distant star. Because microlensing occurs with compact objects, microlenses are modeled as point like masses. This point like structure dictates the formalism of the lens's gravitational potential and causes the lens to split the source into two images separated by  $\sim 2\theta_{Einstein}^2$

Because the observer, source, and lens undergo independent motion, the angular separation of the images and the image magnification are functions of time. Usually, the images produced by a microlens are difficult to resolve. Therefore, the only observable element is the sum of the magnified flux of the two images. Due to the low resolution, microlensing can only be detected when the source-lens-observer alignment changes, which

alters the flux of the images.<sup>3</sup> Therefore, in order to detect a microlensing event one needs to monitor sources for variation in brightness. (See Appendix for diagrams illustrating a microlensing event).

In a microlensing event, three parameters define observables. These parameters include the time of maximum magnification, the impact parameter, and the characteristic time scale for the event. As we saw earlier, a microlensing event is a function of time, so the characteristic time scale for this event follows:

$$t_{event} = \frac{\theta_{Einstein}}{\delta}, \quad (16)$$

where  $\delta$  is the proper motion of the lens-source.<sup>2</sup> For a microlensing case, this equation can be approximated as:<sup>2</sup>

$$t_{Microlensing} \cong 20 \left(\frac{M}{0.3M_{Sun}}\right)^{\frac{1}{2}} days \cong 3 \times 10^{-14} (M)^{1/2} days, \quad (17)$$

where the values for the mass of the extra-solar object and the mass of the Sun are consistent with earlier calculations.

The primary observables that are defined by the above parameters are the mass ratio ( $q$ ) and the instantaneous projected separation ( $d$ ) between the planet and star.<sup>2</sup> The mass ratio is the most important observable because it gives direct information about the planet-star system. The instantaneous projected separation only gives statistical information about the semi major axis ( $a$ ). As Guadi (2002) illustrated, the planet must be near one or both of the images in order to be detected. Moreover, the images are near the Einstein ring radius while the source is significantly magnified. Therefore, microlensing is sensitive to planets with  $d \sim 1$ . This corresponds to:

$$a \sim D_{OL} \theta_{Einstein}, \quad (18)$$

and for the microlensing case:<sup>2</sup>

$$a_{Microlensing} \sim 2AU \left(\frac{M}{M_{Sun}}\right)^{\frac{1}{2}} \cong 1 \times 10^{-15} (M)^{1/2} AU. \quad (19)$$

#### V. Microlensing Advantages and Disadvantages

Microlensing has several advantages over classical extra-solar planetary detection methods. First, microlensing enables detection of planets at distances up to several tens of kiloparsecs, because no flux is needed from either the lens or the source.<sup>2, 5</sup> Second, the strength of the planet's signal weakly depends on the planet/primary mass ratio, where the signal drops as  $\sqrt{q}$ . This advantage makes microlensing the only method capable of detecting Earth-mass planets.<sup>7</sup> Third, microlensing is sensitive to planets at separations of  $d = 1 - 10 AU$  without having to monitor the entire orbital period.<sup>2</sup>

However, there are some disadvantages in using microlensing to detect extra-solar planets. First,

microlensing events are essentially non repeatable and occur for only a short time, which makes follow-up observations very difficult.<sup>2, 5</sup> Second, in gravitational lensing, it is nearly impossible to completely deduce the lensing potential from observational data because the lens cannot be removed to observe an undisturbed source plane. As a result, one is unable to completely model the system due to degeneracies. However, degeneracies can be broken if there is independent knowledge of the scale of the source plane.<sup>3</sup>

## VI. Conclusion

Gravitational lensing has become an important observational method in astrophysics and planetary physics. It has allowed researchers to probe cosmology and discover the structure of nearby planetary systems. Monitoring many microlensing events would provide information on the distribution of planets within our galaxy. Although the many parameters involved in monitoring microlensing events results in mainly statistical data, microlensing remains the only method for detecting extra-solar planetary Earth analogs, which could harbor life.

## Appendix

Figure 1 is a diagram taken from Kuijken (2003) that illustrates a microlensing event as a function of time. In the lower diagram, the lens (i.e., possibly a low-mass star or planet) is designated by an x. The Einstein ring, which is the critical curve in this case, is shown by a dotted circle. The upper graph demonstrates how the combined flux of the two images (the darkened objects) changes as the source (the small, complete circle) moves with respect to the planet.

Figure 2 is a computer simulation taken from the March 6, 2002, Astronomy Picture of the Day (<http://antwrp.gsfc.nasa.gov/apod/ap020306.html>). The computer simulation is credited to Rychard Bouwens at the University of California Observatories/Lick Observatory, the Advanced Camera for Surveys Team, and NASA. The diagram illustrates what one would observe if light from a distant non point like source or sources (e.g., a luminous galaxy or galaxies) passed through and was

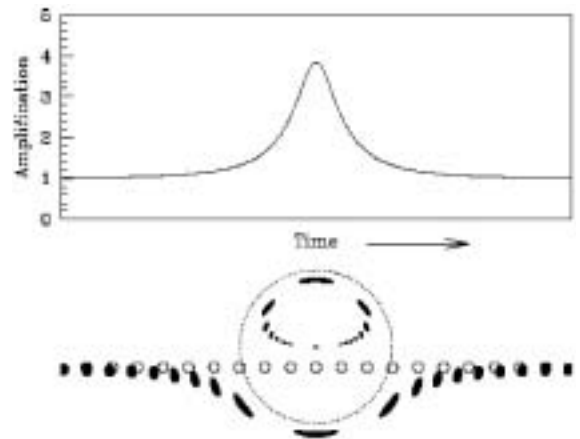


Figure 1. A microlensing event as a function of time.

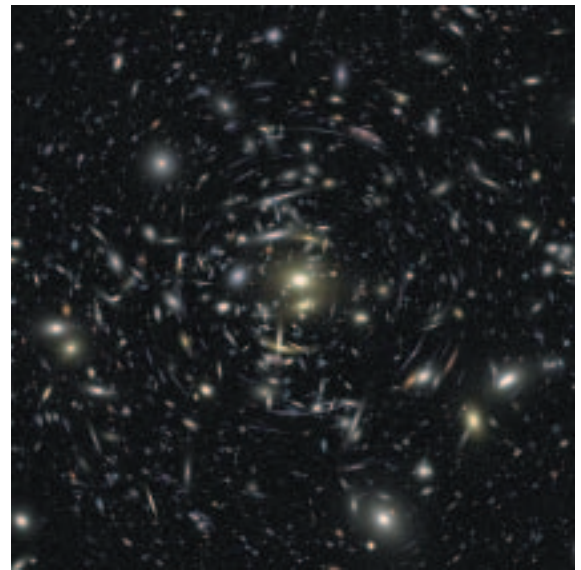



Figure 2. what one would observe if light from a distant non point like source or sources passed through and was deflected by another galaxy.

deflected by another galaxy. The concentric patterns are known as archlets and are another manifestation of gravitational lensing. 

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