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## 2 Transient cooling of thermoelectric coolers and its 3 applications for microdevices

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### 12 Abstract

13 If a current pulse with a magnitude several times higher than the steady state optimum current is applied  
14 to a thermoelectric cooler, an instantaneously lower temperature than that reachable at the steady state can  
15 be obtained. Most previous studies of this transient cooling effect focus on the minimum temperature  
16 achievable for free standing thermoelectric (TE) elements. In this work, we systematically study the tran-  
17 sient response of thermoelectric coolers with and without mass loads through examination of both the min-  
18 imum temperature reached and the time constants involved in the cooling and the recovering stages. For  
19 integrated thermoelectric-passive mass load systems, two distinguishable cooling regimes, uniform cooling  
20 and interfacial cooling, are identified, and the criterion for utilization of the transient cooling effect is estab-  
21 lished based on the time constants. Although the results of this work are generally applicable, the discus-  
22 sions are geared towards cooling of microdevices that are of current interests.

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24 *Keywords:* Thermoelectric; Thermal management; Transient cooling; Microdevice; Thermoelectric cooler

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## Nomenclature

$A$	cross-sectional area ( $\text{m}^2$ )
$A_0$	constant introduced in Eq. (5)
$B$	constant introduced in Eq. (15)
$\rho c_p$	volumetric heat capacity ( $\text{J}/\text{m}^3 \text{K}$ )
$j$	applied current density ( $\text{A}/\text{m}^2$ )
$j_0$	optimum applied current density ( $\text{A}/\text{m}^2$ )
$k$	thermal conductivity ( $\text{W}/\text{m K}$ )
$l$	length (m)
$l_c$	critical dimension of cooling target for using interfacial cooling (m)
$P$	normalized pulse magnitude
$S$	Seebeck coefficient ( $\text{V}/\text{K}$ )
$t$	time (s)
$t_x$	diffusion time constant (s)
$t_m$	time to reach minimum temperature (TRM) (s)
$t_h$	holding time (s)
$T$	temperature (K)
$T_1$	minimum steady state temperature at cold junction (K)
$x$	coordinate
$Z$	figure of merit ( $\text{K}^{-1}$ )
$Z'$	reduced figure of merit of integrated system as defined in Eq. (15) ( $\text{K}^{-1}$ )

### Greeks

$\alpha$	thermal diffusivity ( $\text{m}^2/\text{s}$ )
$\rho$	electrical resistivity ( $\Omega \text{m}$ )
$\xi$	effusivity ratio [ $= (k\rho C_p)_L / (k\rho C_p)$ ]
$\zeta$	constant introduced in Eq. (16)

### Subscripts

c	cold end of TE cooler
h	hot end of TE cooler
l	cooling object of integrated system
SS	steady state
t	transient

## 26 1. Introduction

27 The Peltier effect in a thermoelectric (TE) device is a local effect confined to the junctions of the  
28 thermoelectric elements while the Joule heating occurs volumetrically over the thermoelectric

29 elements. At steady state conditions, these two effects, combined with heat conduction from the  
30 hot end to the cold end, determine the cold side temperature. The cooling coefficient of perform-  
31 ance and maximum temperature drop depend on the properties of the thermoelectric materials  
32 through the figure of merit,  $Z = S^2/\rho k$ , where  $S$  is the Seebeck coefficient,  $\rho$  is the electrical resis-  
33 tivity and  $k$  is the thermal conductivity. If a current pulse with a magnitude several times higher  
34 than the steady state optimum current  $j_0$ , which is the current to obtain the minimum steady state  
35 cold side temperature, is applied to the element, an instantaneously lower temperature than that  
36 reachable at the steady state can be achieved at the cold end because of the delay of the thermal  
37 diffusion of the volumetric Joule heat. This phenomenon is referred to as the transient thermoe-  
38 lectric effect [1].

39 After Stilbans and Fedorovich [2] first reported the transient cooling effect in thermoelectric  
40 (TE) elements, the phenomenon has been extensively investigated [3–14]. To obtain larger tran-  
41 sient cooling temperature differences, measured by the additional temperature drop at the cold  
42 junction caused by the transient current, various approaches have been taken, such as applying  
43 a non-square transient current [4], using thermoelectric elements with variable cross-sectional area  
44 [11] and surface junction [12].

45 Recent developments in the fabrication of thermoelectric microcoolers make it possible to place  
46 the TE microcoolers near the high heat flux producing regions of electronic or optoelectronic de-  
47 vices that need to be cooled [15–18]. This will enable compact thermal systems for device and  
48 package level cooling. The transient cooling effect in thermoelectric coolers might be employed  
49 to improve the performance of these devices further [10,13,14]. The results of the previously men-  
50 tioned studies on transient cooling are not directly applicable to the microcoolers because those  
51 studies are extensively for free standing bulk thermoelectric elements and most of them focused  
52 only on the maximum transient temperature difference, i.e. the minimum temperature achievable.  
53 Several issues that are of particular importance for microdevices need to be addressed. First of all,  
54 several time constants, the time to reach minimum temperature (TRM) and the time to remain at  
55 minimum temperature (holding time), are very important for characterization and utilization of  
56 the transient cooling effect since the effect can only be sustained for a limited time. In parallel  
57 of this work, Snyder et al. [10] established theoretically and experimentally the essential param-  
58 eters that describe the transient cooling effect using a square pulse, such as the minimum temper-  
59 ature achieved, the maximum temperature overshoot, the TRM, the holding time and the time  
60 between pulses. Semi-empirical relationships are established for the dependence of these param-  
61 eters on the current pulse amplitude, thermoelectric element length, thermoelectric figure of merit  
62 and thermal diffusivity. In Section 2 of this paper, we systematically studied the dependence of the  
63 minimum achievable temperature and the time to reach minimum temperature on the current  
64 pulse amplitude, thermoelectric element length, applied current shape and the TE element geom-  
65 etry using the finite difference method for a free standing TE element. Since the object to be cooled  
66 is a passive mass load for the thermoelectric coolers, it affects the minimum temperature achiev-  
67 able, particularly when the object to be cooled is comparable to the TE coolers in size, as is often  
68 the case for microcoolers. In Section 3, we present the performance analysis of the cooling object  
69 and microthermoelectric cooler integrated system. Two distinguishable cooling regimes (uniform  
70 cooling and interfacial cooling) are identified, and the criterion for utilization of the transient  
71 cooling effect is established based on the time constants.

## 72 2. Free standing thermoelectric element

73 In this section, we evaluate the performance of a freestanding TE element. This serves as a basis  
 74 for analyzing the minimum temperature that can be obtained during the transient mode operation  
 75 and also sets the limit for an integrated TE-load system discussed in Section 3. In steady state  
 76 operation of the TE device, the effect of the applied current magnitude on the device performance  
 77 is well understood. However, during the transient mode, operation parameters such as applied  
 78 current pulse shape and pulse amplitude severely affect the performance of the device. Another  
 79 constraint that affects operation of the TE device during transient operation is the geometry of  
 80 the TE element. Geometry parameters like the length of the element and cross-sectional area  
 81 of the cold and hot ends particularly affect the thermal diffusion to and away from the ends of  
 82 the TE element.

### 83 2.1. Finite length thermoelectric element with square pulse

84 The theoretical analysis of the transient cooling of a free standing TE element can be approx-  
 85 imated into a one-dimensional problem as shown in Fig. 1(a) by assuming the n-type and p-type  
 86 thermoelectric elements have exactly the same properties except for the opposite sign of the See-  
 87 beck coefficient. The differential equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{j^2 \rho}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}, \quad (1)$$

91 where  $a$  is the thermal diffusivity,  $\rho$  is the electrical resistivity,  $k$  is the thermal conductivity,  $j$  is the  
 92 applied current density and  $T$  is temperature. The lowest temperature is achieved when there is no  
 93 external heat load onto the TE element, i.e. at  $x = 0$ ,

$$-k \frac{\partial T}{\partial x} + S j T = 0. \quad (2)$$

96 We further assume that the hot side is maintained at a constant heat sink temperature, i.e. at  $x = l$ ,  
 97 where  $l$  is the length of the thermoelectric elements,

$$T(x = l, t) = T_h. \quad (3)$$

100 When the right-hand side of Eq. (1) equals 0, we obtain the steady state temperature distribution  
 101  $T_{SS}(x)$

$$T_{SS}(x) = T_c + (T_h - T_c)(2x/l - x^2/l^2), \quad (4)$$

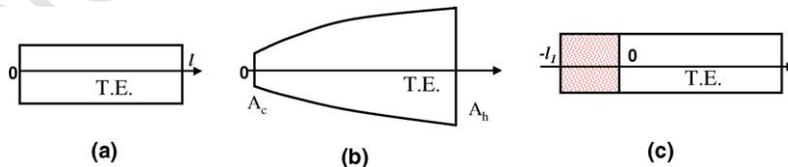


Fig. 1. Schematic drawing: (a) free standing TE element, (b) axisymmetric TE element with variable cross-sectional area (cf. Section 2.3) and (c) cooling object and micro TE cooler integrated system (cf. Section 3).

104 where  $T_c$  is the cold junction temperature. The solution to the steady state problem for a material  
 105 with  $Z$  independent of temperature [1] leads to the maximum steady state temperature difference  
 106  $T_{\max} = T_h - T_1 = ZT_1^2/2$ , where  $T_1$  is the minimum steady state cold side temperature at  $j_0 = \alpha T_1 /$   
 107  $\rho l$ . After the initial steady state temperature distribution is obtained, the current is suddenly in-  
 108 creased to  $j_t$  to obtain the transient cooling. We define a normalized pulse magnitude  $P$  as  
 109  $P = j_t/j_0$  for a square pulse. After the cold junction temperature increases back to its steady state  
 110 value  $T_1$ , the current is switched back to its optimum steady state value  $j_0$ . Fig. 2 shows the numer-  
 111 ical simulation of the cold junction temperature in a typical transient cycle. Also shown are the  
 112 definitions of several time constants. The holding time, which is the period to keep the cold side  
 113 temperature below a certain temperature, depends on its application limit. The recovery period is  
 114 the time required for the TE element to reach its steady state temperature after removing the ap-  
 115 plied transient current, and the steady state optimum current is applied as shown in Fig. 2.

116 Babin and Iordanishvili [6] analyzed the transient response of free standing TE elements and  
 117 found that for currents that are at least twice as large as the steady state optimum current  $j_0$ ,  
 118 it is a reasonable approximation to treat the TE element as a semi-infinite body because the  
 119 TRM is small compared to the diffusion time constant for a large transient pulse  $t_\alpha = l^2/\alpha$ . They  
 120 showed that the transient cooling effect  $\Delta T_t = T_1 - T_2$ , where  $T_2$  is the minimum transient tem-  
 121 perature of the cold side, does not depend on the TE element length when the TE element is longer  
 122 than  $3\sqrt{\alpha t_m}$ . The time to reach the minimum temperature (TRM)  $t_m$  can be approximated as

$$t_m = A_0^2(k\rho c_p)/(j^2 S^2), \quad (5)$$

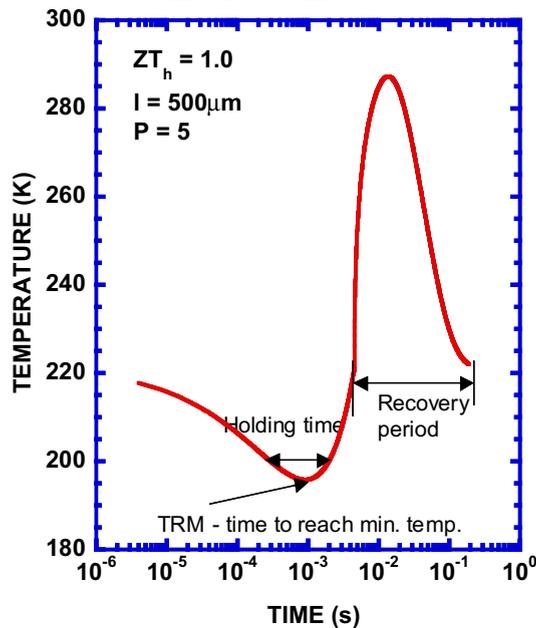


Fig. 2. The change of the cold junction temperature with time in a typical transient cycle and the definition of time constants.

126 where  $A_0$  is determined by the properties of the TE elements and can be determined by:

$$ZT_1 = \frac{1}{\gamma} \frac{\sqrt{\pi} A_0 \exp A_0^2 \operatorname{erfc} A_0}{1 - \sqrt{\pi} A_0 \exp A_0^2 \operatorname{erfc} A_0} \quad (6)$$

129 Our numerical simulation confirms that the transient cooling effect  $\Delta T_t = T_1 - T_2$  (where  $T_2$  is  
130 the minimum transient temperature) does not depend on the TE element length. Fig. 3(a) com-  
131 pares the numerical solution of the maximum cooling effect with the model by Babin and Iordan-  
132 ishvili [6]. The model agrees well with the numerical solution for large current pulse. A simplified  
133 model by linearization of the transient term in Eq. (1) has been established and documented in  
134 [10]. Although the length does not have much effect on the minimum transient temperature for  
135 free standing TE elements for a given transient pulse as shown in Fig. 4(a), it determines the ther-  
136 mal inertia of the TE elements. This indicates that the length of a TE element affects the holding  
137 time as well as the recovery period. For easy comparison, we have defined the holding time as the  
138 time period over which it is possible to maintain the temperature within the range of 2 K (the  
139 choice of 2 K is arbitrary as stated before) from the lowest temperature possible in this section.  
140 Fig. 4 shows the holding time (Fig. 4(b)) as a function of TE element length for different normal-  
141 ized pulse magnitudes and the duty cycle (Fig. 4(c)), which is defined as the percentage of the hold-  
142 ing time over the recovery period. As shown in Figs. 4(a) and (b), the holding time is longer as the  
143 length of the element increases. This is because the applied current flux is much larger for shorter  
144 elements, and thus, there is high heat dissipation density close to the cold junction, for example the  
145 applied current pulse ( $5 \times j_0$ ) for the 50  $\mu\text{m}$  element is ten times larger than that of the 500  $\mu\text{m}$

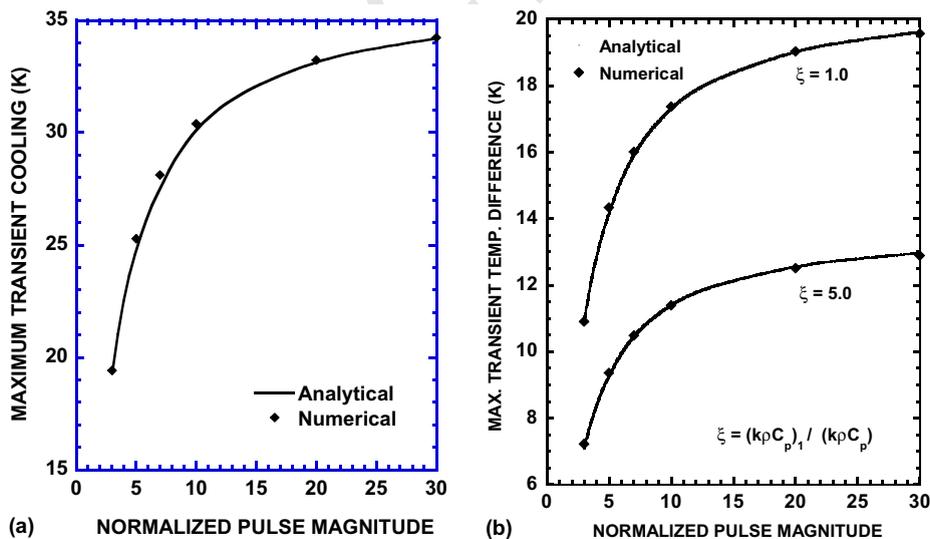


Fig. 3. (a) Comparison of the maximum transient temperature difference obtained by numerical simulation with the model by Babin and Iordanishvili [6] and (b) the maximum transient temperature difference of the semi-infinite integrated system as a function of the normalized pulse magnitude of applied transient current and the effusivity ratio  $\xi = (k\rho C_p)_1 / (k\rho C_p)$  (cf. Section 3.1) Results of analytical model are compared with numerical simulation.

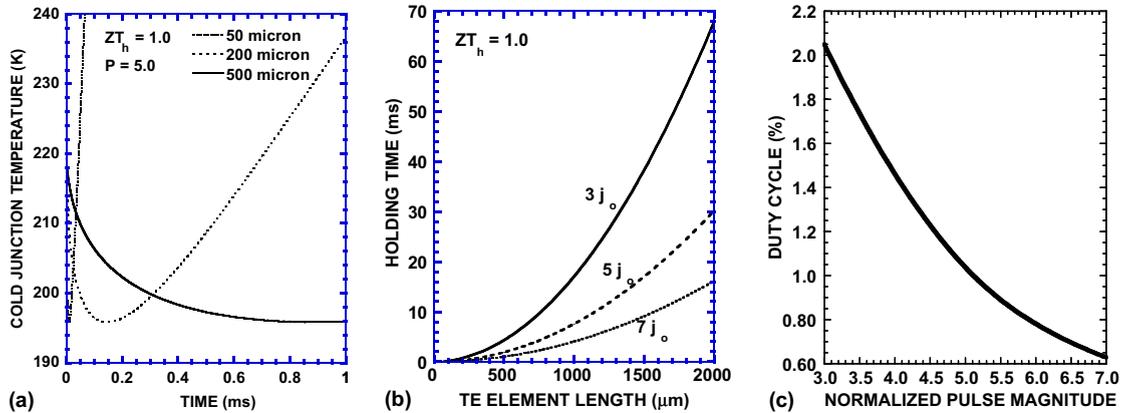


Fig. 4. (a) Cold junction temperature of different thermoelectric element length under same applied transient current shows that the minimum cold side temperature achievable is approximately the same for various TE element lengths, (b) the holding time as a function of TE element length for different normalized pulse magnitudes and (c) the duty cycle. Here, the holding time is defined as the time period over which it is possible to maintain the temperature within the range of 2 K from the lowest temperature possible.

146 element. However, the recovery period will be longer for the longer element and vice versa. This  
 147 results in the duty cycle not being a function of the TE element length but being a function of the  
 148 magnitude of the transient pulse only.

### 149 2.2. Pulse shape effect

150 The current pulse shape also affects the transient response. Using the variational method, Lan-  
 151 decker and Findlay [4] concluded that the transient temperature would approach absolute zero  
 152 with temperature independent thermal and thermoelectric properties, provided that the current  
 153 were allowed to rise indefinitely. No one has been able experimentally to demonstrate this by  
 154 far. Fig. 5 shows the temperature response for three different pulse shapes for a 0.5 mm TE ele-  
 155 ment obtained from numerical simulations. We found that the lowest temperature that can be ob-  
 156 tained is approximately the same for any pulse shape, but the holding time differs for different  
 157 pulse shapes. In order to take advantage of the spatial difference between the Peltier and Joule  
 158 effects, a better approach would be that the applied current pulse should be higher at the begin-  
 159 ning, and subsequently, it should be reduced, similar to the pulse  $j = t^{-0.5}$ . This will enable a long-  
 160 er holding time compared to a ramp pulse or the square pulse, as shown in Fig. 5.

### 161 2.3. Effects of TE element shape

162 It has been known that the minimum temperature achievable by a TE device in steady state  
 163 does not depend on the shape, but it has been conjectured that the TE element shape might have  
 164 an effect on the transient performance of thermoelectric devices [11]. The cross-sectional areas of  
 165 the hot end  $A_h$  and cold end  $A_c$  determine the thermal resistance for the Joule heat. By increasing  
 166 the ratio of  $A_h$  to  $A_c$ , Hoyos et al. [7] experimentally showed that it is possible to obtain better

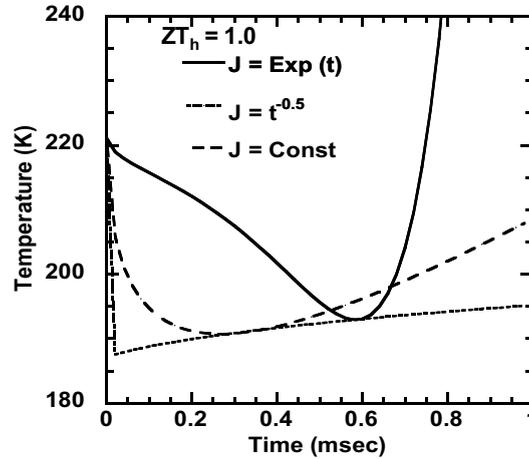


Fig. 5. Cold junction temperature response for three different pulse shapes for a 0.5 mm TE element.

167 transient performance, i.e. lower temperature and shorter recovery time after turning off the tran-  
 168 sient pulse, compared to TE elements having equal cross-sectional area. In microfabricated de-  
 169 vices, the TE legs might not be straightly vertical, as in electrodeposition of thermoelectric  
 170 microdevices [16]. However, rigorous theoretical study on the TE element shape effect on the tran-  
 171 sient cooling effect has yet to be reported. This section presents numerical results on the effects of  
 172 leg shape on the transient cooling performance. Again, the analysis here is based on the assump-  
 173 tion that the thermoelectric properties are independent of temperature and the contact resistance  
 174 is negligible.

175 For axisymmetric TE elements with variable cross-sectional area as shown in Fig. 1(b), the gov-  
 176 erning differential equation should be written as

$$\frac{1}{\alpha} \frac{dT}{dt} = \frac{I^2 \rho}{kA^2(x)} + \frac{1}{A(x)} \frac{dA(x)}{dx} \frac{dT}{dx} + \frac{d^2T}{dx^2}, \quad (7)$$

180 where  $I$  is the total current flow through cross-sectional area  $A(x)$ . The shape effect is reflected by  
 181 the second term of the right-hand side of Eq. (7). The results presented below are for tapered axi-  
 182 symmetric TE legs with the cross-sectional area changing linearly with the TE leg length,

$$A(x) = A_c \left( 1 + \frac{A_h - A_c}{A_c} \frac{x}{l} \right), \quad (8)$$

185 where  $A_c$  is the cross-sectional area at the cold end  $x = 0$  and  $A_h$  is the cross-sectional area at the  
 186 hot end  $x = l$ . Similar to the cylindrical TE elements, the maximum transient temperature differ-  
 187 ence and the holding time for the tapered axisymmetric TE elements also do not depend on the  
 188 absolute value of the cross-sectional area. Fig. 6(a) shows the normalized holding time and the  
 189 minimum cold side temperature with different area ratio of the cold end and the hot end. The  
 190 holding time is normalized to that of the cylindrical TE leg whose cross-sectional area is the same  
 191 as the average area of the tapered element. The tapered leg makes the thermal resistance asymmet-  
 192 ric, and the Joule heat will preferentially be conducted towards the end that has the larger cross-

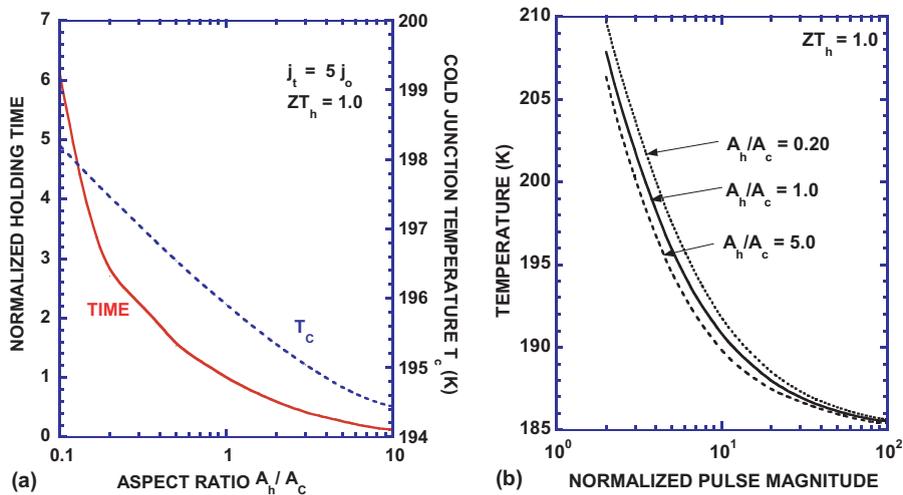


Fig. 6. Transient performance of tapered axisymmetric thermoelectric legs: (a) normalized holding time and minimum temperature as a function of the area ratio, (b) the applied current effect on the minimum temperature for various area ratios of shaped TE cooler.

193 sectional area. However, more Joule heat is generated close to the end that has the smaller cross-  
 194 sectional area. The competition between these two effects results in a lower minimum transient  
 195 temperature for the tapered axisymmetric thermoelectric legs with smaller cross-sectional area  
 196 at the cold end. However, the holding time is decreased by several times for such tapered thermo-  
 197 electric legs with the smaller cross-sectional area at the cold end. The increase of holding time of  
 198 those TE legs with larger cross-sectional area at the cold end can potentially be useful for the de-  
 199 vice to be operated for a longer time. Fig. 6(b) shows the minimum transient temperature as a  
 200 function of the magnitude of the applied transient current and the area ratio.

### 201 3. Response of integrated ththermoelectric-passive load system

202 In applications, it is expected that an additional mass (i.e, the object to be cooled) be attached  
 203 to the TE element. The performance of such an integrated system differs from that of the stand  
 204 alone TE elements. The thermal properties of the attached mass can severely affect the transient  
 205 performance of the integrated system. Here, we focus discussion particularly on important prop-  
 206 erties such as thermal conductivity, heat capacitance and density. We begin our analysis with a  
 207 system that consists of a semi-infinite object to be cooled that is attached to a semi-infinite TE  
 208 element. Later, the discussion is extended to a more realistic situation where a finite length object  
 209 is attached to a finite length TE element. No active heat generation is assumed in the load.

210 The object to be cooled can be treated as a passive mass load attached to a thermoelectric ele-  
 211 ment when contact resistance is neglected. Fig. 1(c) shows the schematic configuration of an inte-  
 212 grated TE-load system. The governing equation for heat conduction in the loaded mass  
 213 ( $-l_1 < x < 0$ ) is

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$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha_1} \frac{\partial T}{\partial t}. \quad (9)$$

216 The subscript 1 denotes the cooling object. The boundary conditions at the interface between the  
217 cooling object and the TE element are

$$-k_1 \frac{\partial T}{\partial x} = -k \frac{\partial T}{\partial x} + SjT, \quad (10)$$

$$T(0^-, t) = T(0^+, t). \quad (11)$$

222 The other end of the attached mass is insulated, thus

$$\frac{\partial T}{\partial x} \Big|_{x=-l_1} = 0. \quad (12)$$

### 225 3.1. Transient response of semi-infinite integrated systems

226 In a previous study [13], we analyzed the transient temperature difference based on the assump-  
227 tion that both the TE element and the object to be cooled are semi-infinite and maintained at  
228 room temperature. In a real situation, the TE element must be maintained at its optimum steady  
229 state before an additional transient current is applied. The initial temperature distribution of the  
230 TE element degrades the transient temperature difference predicted in [13] and should be taken  
231 into account. That is, the new initial condition should be written as

$$\begin{cases} T(x) = T_h - (T_h - T_1) \left(1 - \frac{x}{l}\right)^2, & l > x > 0, \\ T(x) = T_1, & -l_1 < x < 0. \end{cases} \quad (13)$$

234 Following the same technique as in [13] and taking into account the initial steady state temper-  
235 ature distribution, we obtain the following analytical solution for the transient temperature differ-  
236 ence  $\Delta T_t$  for a square pulse:

$$\begin{aligned} \Delta T_t = & \left[1 - \frac{\gamma'}{P}\right] \cdot \left\{ T_1 \left[ (1 - \exp B^2 \operatorname{erfc} B) \cdot \left( \frac{1}{Z'T_1} + \frac{1 - \gamma'}{1 - \gamma'/P} \right) - \frac{2B}{\sqrt{\pi} Z'T_1} \right] \right. \\ & \left. - \frac{k_1/\sqrt{\alpha_1}}{k_1/\sqrt{\alpha_1} + k/\sqrt{\alpha}} \cdot \frac{1}{Z'} \left[ 1 - \exp B^2 \operatorname{erfc} B - \frac{2B}{\sqrt{\pi}} \right] \right\}, \end{aligned} \quad (14)$$

240 where

$$\gamma' = \frac{2(T_h - T_1)}{T_1(\sqrt{1 + 2ZT_h} - 1)P}, \quad Z' = Z \frac{k^2/\alpha}{(k/\sqrt{\alpha} + k_1/\sqrt{\alpha_1})^2}, \quad B = \frac{Sj\sqrt{t}}{k/\sqrt{\alpha} + k_1/\sqrt{\alpha_1}}. \quad (15)$$

244 The condition for maximum  $\Delta T_t$  is obtained when  $B = B_0$  satisfies

$$Z'T_1 = \frac{[1 - (k_1/\sqrt{\alpha_1})/(k/\sqrt{\alpha} + k_1/\sqrt{\alpha_1})]}{\zeta} \frac{\sqrt{\pi} B_0 \exp B_0^2 \operatorname{erfc} B_0}{1 - \sqrt{\pi} B_0 \exp B_0^2 \operatorname{erfc} B_0}, \quad (16)$$

248 where  $\zeta = \frac{1 - \gamma'}{1 - \gamma'/P}$ .

249 The effect of the initial temperature distribution is reflected in  $\gamma'$ . Eq. (14) shows that the initial  
250 temperature distribution indeed reduces the additional temperature drop derived in [13].

251 Fig. 3(b) shows the maximum transient temperature difference of the semi-infinite integrated  
252 system as a function of the magnitude of the applied transient current and the effusivity ratio  
253  $\xi = (k\rho C_p)_1 / (k\rho C_p)$ . With the mass attached at the cold end, the cooling power produced diffuses  
254 into the cooling object, which degrades the additional temperature drop compared to the free  
255 standing TE element. It shows that a decrease in the value of the effusivity of the object to be  
256 cooled helps in decreasing the thermal diffusion into the object and, hence, achieving a larger max-  
257 imum transient temperature difference. This partial cooling of the object might be attractive for  
258 some applications such as cooling the active region of a semiconductor laser rather than the whole  
259 substrate. The model is also compared with numerical simulation results. The analytical model  
260 agrees well with the numerical simulation in the semi-infinite regime.

### 261 3.2. Integrated system with finite length TE element

262 In practical applications, the cooling object and the TE element integrated system might behave  
263 neither like a free standing TE element nor like a semi-infinite integrated system. The transient  
264 temperature response of a practical integrated system with finite length depends on not only  
265 the transient current but also the length and the thermal properties of the object to be cooled  
266 and TE elements. Since general analytical solutions for such cases are difficult, a numerical meth-  
267 od is used to study the transient effect for a practical integrated load and TE cooler system as  
268 shown in Fig. 1(c). In addition, experiments were conducted on a system as shown in the inset  
269 of Fig. 7. It consists of two  $1\text{ mm} \times 1\text{ mm} \times 6\text{ mm}$   $\text{Bi}_2\text{Te}_3$  thermoelectric legs soldered by a  $3.5$   
270  $\text{mm} \times 2.5\text{ mm}$  copper sheet that is  $35\text{ }\mu\text{m}$  thick. The details of such experimental studies are re-  
271 ported in [10]. Fig. 7 compares the numerical simulation results with experimental data. Copper  
272 is treated as the cooling object and the properties used were obtained by fitting the steady state  
273 response of the TE element. To treat the system as 1D problem, the length of the copper stub  
274 is equivalent to a length of  $153\text{ }\mu\text{m}$  with the same cross-sectional area while considering the high  
275 thermal conductivity of copper.  $k_1 = 350\text{ W/(m K)}$  and  $(\rho c_p)_1 = 1.20 \times 10^6\text{ J/m}^3\text{ K}$  are used for the  
276 copper properties. The properties of  $\text{Bi}_2\text{Te}_3$  are fitted as:  $(\rho c_p) = 1.20 \times 10^6\text{ J/m}^3\text{ K}$ ,  $k = 1.20\text{ W/}$   
277  $(\text{m K})$ ,  $S = 235\text{ }\mu\text{V/K}$ ,  $ZT_{300\text{ K}} = 0.706$ . Fig. 7 shows that the numerical model developed here  
278 can be used to predict the transient response. The deviation between the experimental data and  
279 the simulation results in Fig. 7 after the transient pulses are turned off is due to the thermal resist-  
280 ance of the hot side heat sink, which is not considered in our current model.

281 Fig. 8(a) shows the effect of thermal conductivity on the maximum transient temperature dif-  
282 ference. It shows that the thermal conductivity does not have much effect on the maximum tran-  
283 sient temperature difference when  $l_1/l$  is small, where  $l_1$  is the length of the cooling object and  $l$  is  
284 the length of the TE element. This means that the attached cooling object is cooled uniformly be-  
285 cause the transient cooling power diffuses effectively to the whole cooling object. As the length  
286 ratio  $l_1/l$  increases, the maximum transient temperature difference becomes independent of the  
287 length ratio because both sides can be treated as semi-infinite. In this case, however, the thermal  
288 conductivity of the load affects the maximum temperature difference. A high load thermal conduc-  
289 tivity leads to a smaller transient cooling effect due to the larger heat spreading in the load side.  
290 Fig. 8(b) shows the effect of the volumetric heat capacity on the maximum transient temperature

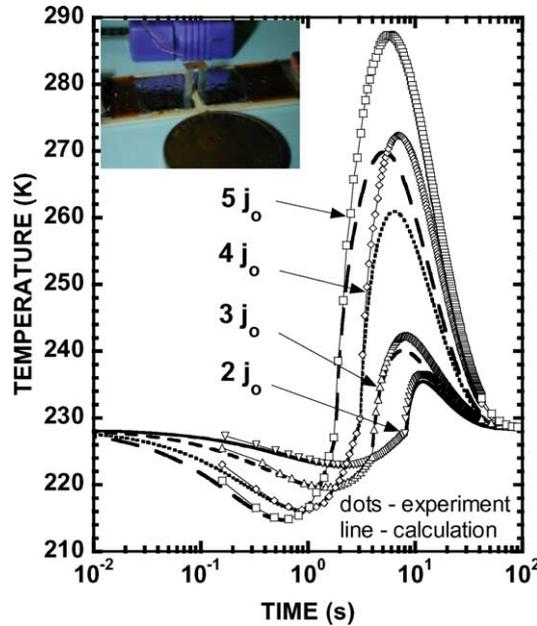


Fig. 7. Comparison of numerical simulation and experimental results (line—numerical results, dots—experimental results). The inset shows the configuration of the experimented TE cooler. It consists of two 1 mm × 1 mm × 6 mm Bi<sub>2</sub>Te<sub>3</sub> thermoelectric legs soldered by a 3.5 mm × 2.5 mm copper sheet that is 35 μm thick.

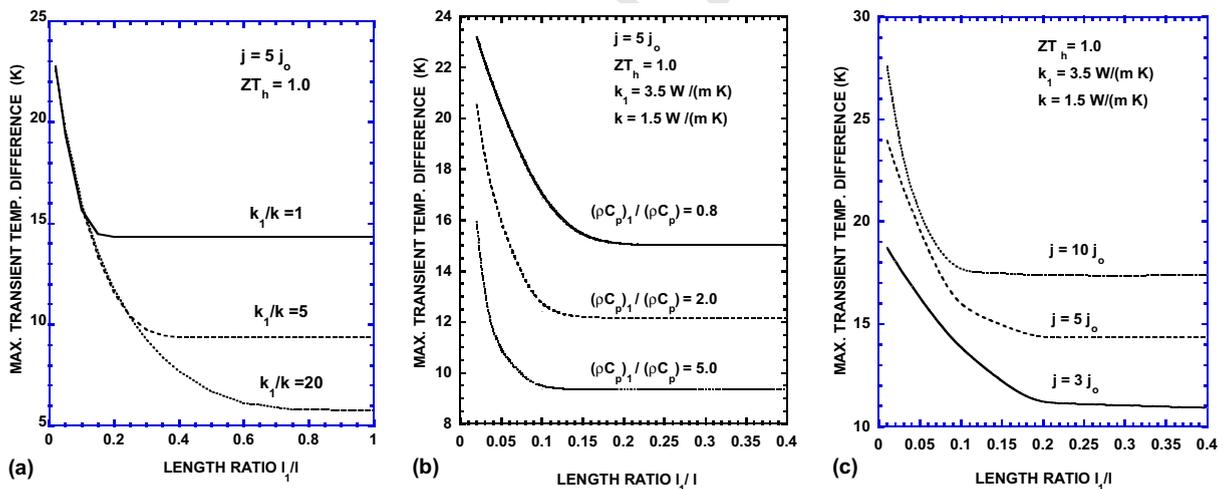


Fig. 8. The maximum transient temperature difference of integrated systems as a function of the length ratio: (a) thermal conductivity effect, (b) heat capacity effect and (c) applied current effect.

291 difference. The maximum transient temperature difference increases when decreasing the heat  
 292 capacity of the cooling object for any length ratio. Fig. 8(c) shows the effect of the applied tran-  
 293 sient current on the maximum transient temperature difference. All three figures show a flat region

294 of the maximum transient temperature difference when  $l_1/l$  is larger than a certain value. This is  
 295 the interfacial cooling region. In this regime, the thermoelectric legs and the passive load can  
 296 be treated as semi-infinite. The maximum transient temperature difference does not change with  
 297 the length ratio but depends on the thermal conductivity or thermal effusivity ratio. In the other  
 298 limit, the uniform cooling regime, the thermal conductivity of the load does not affect the maxi-  
 299 mum transient temperature difference. The maximum transient temperature difference is a func-  
 300 tion of the thermal mass ratio of the load to the thermoelectric elements. In the uniform  
 301 cooling regime, the transient response can be modeled as a small attached mass system.

302 Fig. 9(a) shows that different cooling objectmicro TE element ( $\text{Bi}_2\text{Te}_3$ ) integrated systems might  
 303 fall in different cooling regimes under the transient pulse. The conductivity of the cooling object  
 304 has been chosen to be  $k_1 = 3.5 \text{ W/(m K)}$ , which mimics the active region of the InAs/AlSb mid-IR  
 305 laser. For a given integrated system, the transient cooling might change from uniform cooling to  
 306 interfacial cooling if the applied current increases substantially. For a small normalized pulse  
 307 magnitude  $P$ , the cooling object is cooled uniformly and the cooling effect follows similarly to  
 308 3(a), i.e. the system can be treated as a free standing TE element with a very small attached mass,  
 309 if the TE element is long enough that the holding time is much larger than the thermal diffusion  
 310 time to the other end of the cooling object. The holding time decreases with the normalized pulse  
 311 magnitude  $P$ . For a large normalized pulse magnitude  $P$ , the holding time is shorter than the ther-  
 312 mal diffusion time to the other end of the cooling object, and the transient effect is confined to the  
 313 interface region. In both extremes, the cold junction temperature decreases with increasing pulse  
 314 magnitude. In the transition regime, the competition between the localized cooling at the interface  
 315 and the cooling power diffused into the loaded mass results in the increase of the cold junction  
 316 temperature with increasing pulse. Fig. 9(b) is to illustrate these arguments. It shows the transient  
 317 cooling effect in a  $75 \mu\text{m}$  cooling object— $1000 \mu\text{m}$   $\text{Bi}_2\text{Te}_3$  integrated system. Uniform cooling

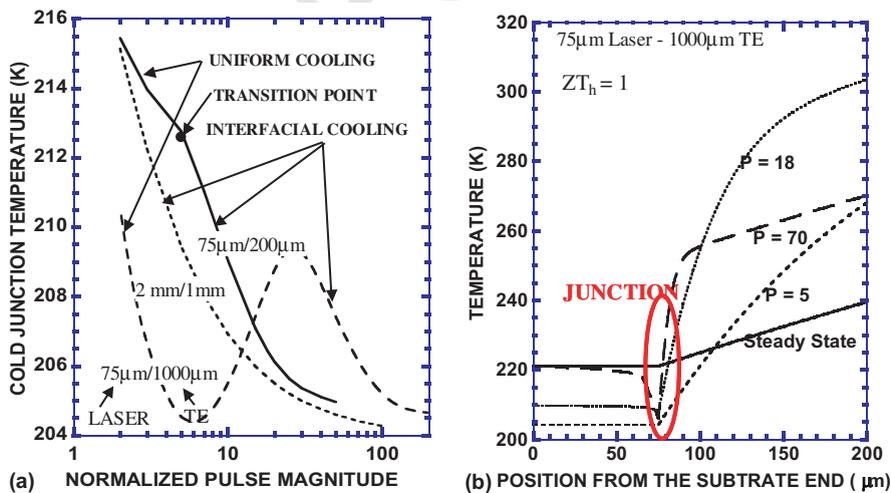


Fig. 9. (a) Two cooling regimes are observed depending on the length ratio of the cooling object micro TE element ( $\text{Bi}_2\text{Te}_3$ ) integrated system and the applied transient current and (b) the transient cooling effect in a  $75 \mu\text{m}$  cooling object— $1000 \mu\text{m}$   $\text{Bi}_2\text{Te}_3$  integrated system: Uniform cooling occurs at  $P = 5$ , and interfacial cooling occurs at  $P = 70$ .

318 occurs at  $P = 5$ , and interfacial cooling occurs at  $P = 70$ . The temperature profile for  $P = 18$  shows  
319 the competition of localized cooling at the interface and the cooling power diffusion into the  
320 loaded mass.

321 Lumped analysis shows that the holding time  $t_h$  of an integrated system, which is defined as the  
322 time that the cold junction is maintained below the steady state minimum temperature, is

$$t_h = \frac{1}{(P+1)^2} \frac{l^2}{\alpha} \quad (17)$$

325 which is around four times as long as the TRM [10]. This holding time expression is approxi-  
326 mately valid for both free standing TE elements and integrated cooling systems. To utilize the  
327 transient cooling effect in a uniform cooling mode, the holding time must be larger than the dif-  
328 fusion time, which is the time required for the transient cooling effect to diffuse from the interface  
329 to the other end of the object to be cooled. Comparing the scale of the holding time and the dif-  
330 fusion time, we found that the criterion for utilizing the transient cooling effect is

$$\frac{l_1}{l} < \frac{1}{P+1} \sqrt{\frac{\alpha_1}{\alpha}}. \quad (18)$$

333 In other words, when  $l_1 > \frac{l}{P+1} \sqrt{\frac{\alpha_1}{\alpha}}$ , the integrated system can be treated as a semi-infinite system,  
334 and the maximum transient temperature difference can be predicted by Eq. (1). To utilize interfa-  
335 cial cooling, the critical dimension  $l_c$  of the cooling target should satisfy  $l_c < \frac{l}{P+1} \sqrt{\frac{\alpha_1}{\alpha}} < l$ .

#### 336 4. Conclusions

337 The transient cooling effect should be characterized by both the minimum temperature and sev-  
338 eral time constants, such as the time to reach the minimum temperature (TRM) and the holding  
339 time. We systematically studied the effects on the transient cooling performance of the current  
340 pulse amplitude, thermoelectric element length, applied current shape and the TE element geom-  
341 etry using primarily the finite difference method. Because the cooling object is a passive mass load,  
342 it will affect the transient performance. The performance of the cooling object and micro TE ele-  
343 ment integrated system are analyzed. Two distinctive cooling regimes (uniform cooling and inter-  
344 facial cooling) are identified, and the criterion for utilization of the transient cooling effect is  
345 established based on the time constants.

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#### 348 References

349 [1] Goldsmid HJ. Electronic refrigeration. New York: Plenum Press; 1986.

- 350 [2] Stilbans LS, Fedorovich NA. Cooling of thermoelectric cells under nonstationary conditions. *Sov Phys Tech Phys*  
351 1958;3:460–3.
- 352 [3] Parrott JE. Interpretation of stationary and transient behaviour of refrigerating thermocouples. *Solid State*  
353 *Electron* 1960;1:135–43.
- 354 [4] Landecker K, Findlay AW. Study of fast transient behaviour of Peltier junctions. *Solid State Electron*  
355 1961;2:239–60.
- 356 [5] Idnurm M, Landecker K. Experiments with Peltier junctions pulsed with high transient currents. *J Appl Phys*  
357 1963;34:1806–10.
- 358 [6] Babin VP, Iordanishvili EK. Enhancement of thermoelectric cooling in nonstationary operation. *Sov Phys Tech*  
359 *Phys* 1969;14:293–8.
- 360 [7] Hoyos GE, Rao KR, Jerger D. Fast transient response of novel Peltier junctions. *Energy Conversion*  
361 1977;17:23–9.
- 362 [8] Field RL, Blum HA. Fast transient behavior of thermoelectric coolers with high current pulse and finite cold  
363 junction. *Energy Conversion* 1979;19:159–65.
- 364 [9] Miner A, Majumdar A, Ghoshal U. Thermoelectromechanical refrigeration based on transient thermoelectric  
365 effects. *Appl Phys Lett* 1999;75:1176–8.
- 366 [10] Snyder GJ, Fleurial J-P, Caillat T, Yang RG, Chen G. Supercooling of Peltier cooler using a current pulse. *J Appl*  
367 *Phys* 2002;92:1564–9.
- 368 [11] Hoyos GE, Rao KR, Jerger D. Numerical analysis of transient behavior of thermoelectric coolers. *Energy*  
369 *Conversion* 1977;17:45–54.
- 370 [12] Landecker K. Some further remarks on the improvements of Peltier junctions for thermoelectric cooling. *Energy*  
371 *Conversion* 1974;14:21–33.
- 372 [13] Kumar R, Yang RG, Chen G, et al. Transient thermoelectric cooling of thin film devices. In: *Proceedings of the*  
373 *MRS Spring meeting; 2000.*
- 374 [14] Yang RG, Chen G, Snyder GJ, et al. Geometric effect on the transient cooling of thermoelectric coolers. In:  
375 *Proceedings of the MRS Fall meeting; 2001.*
- 376 [15] Volklein F, Blumers M, Schmitt L. Thermoelectric microsensors and microactuators (MEMS) fabricated by thin  
377 film technology and micromachining. In: *Proceedings of the 18th ICT; 1999.* p. 285–93.
- 378 [16] Fleurial J-P, Snyder GJ, Herman JA, et al. Thick-film thermoelectric microdevices. In: *Proceedings of the 18th*  
379 *ICT; 1999.* p. 294–300.
- 380 [17] Min G, Rowe DM, Volklein F. Integrated thin film thermoelectric cooler. *Electron Lett* 1998;34:222–3.
- 381 [18] Yao DJ, Kim CJ, Chen G. Design of thermoelectric thin film coolers. In: *Proceedings of the international*  
382 *mechanical engineering congress and exhibition, Orlando, FL; 2000.*
- 383