Semi-Lagrangian scheme for the Vlasov on an unstructured mesh of phase space

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The numerical resolution of the Vlasov equation is most of the time performed by Lagrangian methods like particle in cell methods (PIC) which consist of approximating the plasma by a finite number of macro-particles. The trajectories of these particles are computed from characteristic curves given by the Vlasov equation, whereas self-consistent fields are computed by gathering the charge and current densities of the the particles on a mesh of the physical space (see Birdsall and Langdon for more details [2]). Although this method allows to obtain satisfying results with a small number of particles, it is well known that the numerical noise inherent to the particle method becomes too signifcant to allow a precise description of the tail of the distribution function which plays an important role in charged particle beams. As another option, Eulerian methods, which consist in discretizing the Vlasov equation on a mesh of phase space have been proposed. A comparaison of Euleriangrid-based Vlasov solvers can be found in [1].

A new scheme for solving the Vlasov equation using an unstructured mesh for the phase space is proposed. The algorithm is based on the semi-Lagrangian method which exploits the fact that the distribution function is constant along the characteristic curves. We use different local interpolation operators to reconstruct the distribution function f, some of which need the knowledge of the gradient of f that we obtain by advecting them. We can use limiter coefficients to maintain the positivity and the $L\infty$ bound of f and optimize these coefficients to ensure the conservation of the L^1 norm, that is to say the mass by solving a linear programming problem. Several numerical results are presented in two $(x; v_x)$ and three (axisymetric case, $(r; v_r; v_\theta)$) and four $(x; y; v_x; v_y)$ dimensional phase space, in the field of collisionless plasma and charged particle beams. The local interpolation technique is well suited for parallel computation.

References
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