A Grid-Free Treecode Field Solver for Plasma Simulations with Application to a Confined Electron Column in a Penning-Malmberg Trap

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The most expensive step in a Lagrangian plasma simulation is the self-consistent computation of the force on each particle. In a system of charged particles x_i, the electrostatic force F_i on particle i due to all the other particles is given by

$$F_i \;=\; q_i \sum_{j=1\,,\,j\neq i}^{N} q_j \left(-\nabla G(x_i, x_j)\right) \;, \tag{1}$$

where q_i is the charge on particle i, $G(x_i, x_j)$ is the Coulomb potential at particle i due to particle j, and N is the total number of particles. Since the sum must be computed for each particle in the system, the direct calculation is an $O(N^2)$ process which is prohibitively expensive when N is large, as often occurs in systems of practical interest. Particle-in-cell (PIC) methods reduce the computational cost of the force calculation to $O(N \log N)$ by solving a Poisson equation for the electrostatic potential on a regular grid [1]. Treecode algorithms represent a grid-free $O(N \log N)$ alternative in which the particles are grouped into a hierarchy of clusters and the particle-cluster interactions are evaluated using a multipole approximation [2]. Previous researchers have concluded that the treecode approach is very effective for periodic or unbounded plasmas, but that the method was either too difficult or impractical to extend to bounded domains [3]. On the other hand, Lagrangian particle methods have been successfully applied to incompressible fluid dynamics on bounded domains by using boundary integral techniques to satisfy the appropriate boundary conditions [4].

In the present work we combine a treecode approach with a boundary integral technique to compute the electrostatic particle forces in a plasma confined to a bounded domain with conducting walls. The particle-cluster interactions are evaluated using multi-dimensional Taylor expansions in Cartesian coordinates and the necessary Taylor coefficients are evaluated using a recurrence relation [5]. The method is first described and then applied to study the dynamical behavior of a confined electron column in a Penning-Malmberg trap. This system was chosen as a test case because it is accurately modeled by the Vlasov-Poisson equations, and has been previously studied numerically via a PIC simulation and experimentally [6]. We explore the relative advantages and disadvantages of the treecode versus the PIC simulation in terms of efficiency, stability, accuracy, and ease of implementation in the context of the confined electron column.

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