Self-Organization of Plasmas with Flows

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Self-organization in a two-fluid plasma has been studied by nonlinear 3D simulation. The theory[1] predicts creation of the “double Beltrami (DB) field” represented by,

\[ B = a(V - \epsilon \nabla \times B), B + \epsilon \nabla \times V = bB, \] (1)

where \( B, V \) are a magnetic field and an ion flow velocity, \( \epsilon \equiv \lambda / L \) (\( \lambda \) : ion skin depth, \( L \) : system size), and \( a, b \) are constants. The Hall term in the two-fluid model leads to a singular perturbation that enables the formation of an equilibrium given by a pair of two different Beltrami fields (eigenfunctions of the curl operator). The DB states can span a richer set of plasma conditions than the single Beltrami states (the Taylor states).

The compressible Hall-MHD equations,

\[ \frac{\partial n}{\partial t} + \nabla \cdot (nV) = 0, \] (2)
\[ \frac{\partial V}{\partial t} + (V \cdot \nabla)V = -\nabla p_i + j \times B + \nabla \cdot \Pi, \] (3)
\[ \nabla \cdot \Pi = \nu(\nabla^2 V + \frac{1}{3} \nabla(\nabla \cdot V)), \] (4)
\[ \frac{\partial B}{\partial t} = -\nabla \times E, \] (5)
\[ E = -V \times B + \frac{\kappa}{n}(j \times B - \nabla p_e) + \eta j, \] (6)
\[ \frac{\partial p}{\partial t} + (V \cdot \nabla)p + \gamma p \nabla \cdot V = 0, \] (7)

(\( n \) : number density, \( j \) : current density, \( E \) : electric field, \( p \) : pressure, \( \nu \) : viscosity, \( \eta \) : resistivity, \( \gamma \) : ratio of specific heat) are solved by the finite difference and the Runge-Kutta-Gill methods in a 3 dimensional rectangular domain. The quasi-neutrality \( n = n_i = n_e \) are assumed. Figure shows isosurfaces of the relaxed toroidal magnetic field. We see that, compared with the single fluid case (a)[2], fine structures are imposed on global modes in the two-fluid case (b). Variational principles in the two-fluid system will be discussed.