Fully Implicit, Nonlinear 3D Extended Magnetohydrodynamics

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The extended magnetohydrodynamics (XMHD) formalism includes nonideal effects such as resistivity, viscosity, nonlinear anisotropic transport, and two-fluid (Hall) effects. Such a model supports a variety of fast time scales (including dispersive waves of the form $\omega \sim k^2$) that make explicit time differencing approaches extremely inefficient. While an implicit implementation promises efficiency without sacrificing numerical accuracy [1], the nonlinear nature of the XMHD system and the numerical stiffness associated with the fast waves make this endeavor difficult.

Newton-Krylov methods are, however, ideally suited for such a task. These synergistically combine Newton's method for nonlinear convergence, and Krylov techniques to solve the associated Jacobian (linear) algebraic systems. Since Krylov methods only require the product of the system matrix with a vector, the Jacobian system can be implemented Jacobian-free, thus avoiding forming and storing it. Furthermore, Krylov methods can be preconditioned for efficiency.

The XMHD model is generally nonsymmetric, and hence a robust nonsymmetric solver such as GMRES is employed. However, the CPU cost associated with GMRES increases quadratically with the number of iterations, and hence preconditioning becomes essential for efficiency. Successful proof-of-principle preconditioning strategies have been developed for 2D incompressible resistive [2] and Hall [3] MHD models. These strategies are based on "physics-based" ideas, in which intimate knowledge of the physics of the system at hand is exploited to derive inexpensive, well-conditioned (diagonally-dominant) approximations to the original system that are amenable to optimal solver technologies such as multigrid.

In this work, we will describe the status of the extension of the preconditioning ideas in [2,3] in two main directions: 1) the addition of nonlinear, anisotropic transport effects in 2D XMHD, 2) the development of a suitable preconditioning strategy for a 3D compressible, single-fluid XMHD model.

[1] D. A. Knoll et al., *J. Comput. Phys.* **185** (2), 583-611 (2003)

[2] L. Chacón, D. A. Knoll, and J. M. Finn, *J. Comput. Phys*. **178** (1), 15- 36 (2002)

[3] L. Chacón and D. A. Knoll, *J. Comput. Phys.*, accepted (2003)