Solution Method of the Poisson Equation for the Electric Field with a Thin Sheath

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In the PIC/ MC simulation of materials processing plasmas, [(1)] one has to solve the Poisson equation in each electron timestep that is very small. Usually this is the rate determining process of numerical simulation. The Poisson equation has been solved using its finite difference form. Since there exists always a thin sheath near a solid boundary, a very fine mesh should be introduced in the sheath, which results in excessive computation time.

In the present paper, we propose a very quick solution method of the Poisson equation without using the finite difference equation. We use the boundary-layer theory[(2)] in fluid dynamics and the variational principle to find an accurate solution of the Poisson equation. The present method drastically reduces the computational load of the PIC/ MC simulation of materials processing plasmas.

Our idea is best explained for the case of the electric field in an infinitely long cylinder with radius a. The potential $\phi(r,\theta)$ satisfies the Poisson equation, where (r,θ) is the polar coordinate system. The solution of the Poisson equation minimized the integral

$$I = \int_0^a dr \int_{-\pi}^{\pi} d\theta \left[\left(\frac{\partial \phi}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial \phi}{\partial \theta}\right)^2 - \frac{2}{\epsilon_0}\varrho\phi \right],$$

where $\varrho(r,\theta)$ is the charge density in the sheath and ϵ_0 is the dielectric constant of vacuum. We assume

$$\phi(=\phi_{\rm b}) = A_0 + \sum_{n=1}^{2} \left(\frac{r}{a}\right)^n (A_n \cos n\theta + B_n \sin n\theta)$$

in the plasma bulk and

$$\phi(=\phi_{\rm s}) = \phi_{\rm b}(a-\delta(\theta),\theta)(2\eta-\eta^2) + \Lambda(\eta - 2\eta^2 + \eta^3)$$

in the sheath, where $\eta = (a-r)/\,\delta(\theta)$, $\delta(\theta)$ is the sheath thickness, and Λ is the shape factor in the boundary-layer theory[(2)]. We determined the six parameters A_0, A_1, A_2, B_1, B_2, and Λ using the variational principle. We have found that the relative error of the potential obtained using the present method is at most $1-2\%$.

References

1. K. Nanbu, IEEE Trans. Plasma Science **28**(2000) 971-990.
2. H. Schlichting, Boundary-Layer Theory, 7th ed., McGraw-Hill, New York, 1979.