A High-Resolution Constrained Transport Method with Adaptive Mesh Refinement for Ideal MHD

James A. Rossmanith
Department of Mathematics, University of Michigan, USA
E-mail: rossmani@umich.edu

The ideal magnetohydrodynamic (MHD) equations are important in modeling phenomena in a wide range of applications including space weather, solar physics, laboratory plasmas, and astrophysical fluid flows. In vector form these equations can be written as

$$\partial_t \begin{bmatrix} \rho \\ \rho\vec{u} \\ \mathcal{E} \\ \vec{B} \end{bmatrix} + \nabla \cdot \begin{bmatrix} \rho\vec{u} \\ \rho\vec{u}\otimes\vec{u} + \left(p + \frac{1}{2}|\vec{B}|^2\right)\mathcal{I} - \vec{B}\otimes\vec{B} \\ \vec{u}\left(\mathcal{E} + p + \frac{1}{2}|\vec{B}|^2\right) - \vec{B}(\vec{u}\cdot\vec{B}) \\ \vec{u}\otimes\vec{B} - \vec{B}\otimes\vec{u} \end{bmatrix} = 0 \tag{1}$$

$$\nabla \cdot \vec{B} = 0\,, \tag{2}$$

where ρ is the density, $\vec{u} = (u_1, u_2, u_3)^t$ is the fluid velocity, $\mathcal{E}$ is the total energy, $\vec{B} = (B_1, B_2, B_3)^t$ is the magnetic field, p is the thermal pressure, and $(1/2)|\vec{B}|^2$ is the magnetic pressure. Specifying an *equation of state* closes the system by relating the thermal pressure to the other unknowns. Equations (1) and (2) model the dynamics of an electrically conducting fluid and are a combination of gas dynamics and Maxwell's equations [T.I. Gombosi, *Physics of the Space Environment*, Cambridge University Press (1998)].

The ideal MHD equations form a hyperbolic system of conservation laws. In general, shock waves and other types of discontinuities will develop even from C^∞ initial data. Furthermore, physically relevant solutions to (1) must also satisfy the divergence-free constraint (2). Therefore, a numerical method for solving the ideal MHD equations is considered accurate if it has all of the following properties:

1. Smooth solutions are approximated to second order accuracy in space and time.
2. The method is numerically conservative.
3. The magnetic field satisfies a discrete divergence-free condition.
4. Shock waves and other discontinuities are accurately captured.

Failure to satisfy the above conditions can lead to unphysical solutions. Many of the currently available methods for solving the ideal MHD equations do not satisfy all of these conditions [G. Tóth, *J. Comp. Phys.* **161**, 605-652 (2000)].

In this work, we present a numerical method based on the high-resolution wave propagation method first introduced by LeVeque [*J. Comp. Phys.* **131**, 327-353 (1997)]. In order to keep the magnetic field divergence-free, this method is coupled to a variant of the constrained transport technique introduced by Evans and Hawley [*Astrophys. J.* **332**, 659 (1998)]. We develop a novel constrained transport technique that is based on directly solving a hyperbolic equation for the magnetic potential in conjunction with a new limiting strategy to obtain a non-oscillatory magnetic field. We provide evidence that the new method satisfies conditions (1) – (4).

We present numerical results on regular Cartesian grids as well as on adaptively refined Cartesian grids. The CLAWPACK software package written by Randall J. LeVeque and the BEARCLAW software package written by Sorin Mitran are used to obtain these results.