

Aliasing of fields in stair-step boundaries*

J. P. Verboncoeur
University of California, Berkeley

Aliasing occurs in mesh-based algorithms such as those employing finite difference solutions of the fields in PIC and fluid models of plasmas. The literature describes the effect for configurations in which the boundaries are orthogonal to the mesh, and align on mesh edges¹. However, when the boundaries are not mesh-aligned, such as in diagonal boundaries or curvilinear boundaries represented on a Cartesian mesh, the boundaries may be represented as stair steps. The stair step representation of the boundaries alters the spectral content for both finite difference calculations on the mesh, as well as spectral and modal diagnostics such as determining the strenghts of electromagnetic modes in a waveguiding structure.

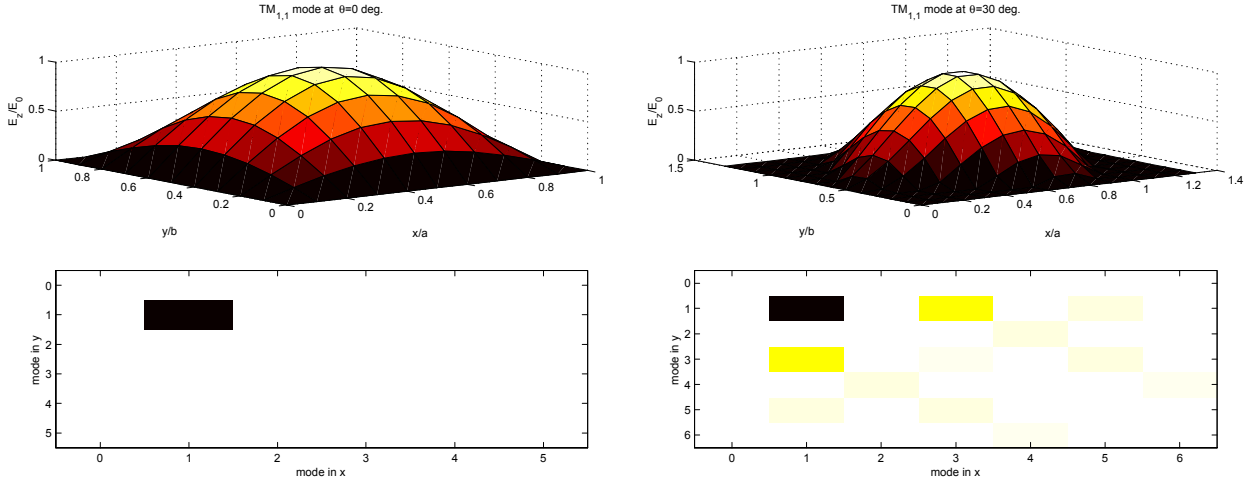


Figure 1: *Fundamental mode in a rectangular waveguide on a uniform mesh.*

Consider the illustrative case of waveguide modes represented in a rectangular waveguide. Fig. 1 (left) shows a transverse electric field component for the TE_{11} mode and its spatial transform. The transform is a single point of amplitude $A_{1,1} = 1$ at $k_x a/\pi = k_y b/\pi = 1$. Next, the same waveguide is rotated by 30 degrees with respect to the mesh, resulting in stair-stepped boundaries, as shown in Fig. 1 (right). The spatial transform results in many modes, with the peak amplitude for the fundamental mode given by $A_{1,1} = 0.7205$, and aliased amplitudes for other modes including $A_{1,3} = A_{3,1} = 0.1817$, $A_{2,4} = A_{4,2} = 0.0328$, etc.

A number of solutions to stair-step aliasing are evaluated. Filtering in k-space filtering can be used to eliminate short wavelengths in some situations where the spectral transform is practical to obtain. Digital filtering can be used to reduce short wavelengths in most cases, regardless of geometric complexity. We also consider a set of natural modes which can exist on a specific mesh-boundary system as the natural modes of the system, and means of approximating continuum modes with the discrete set on the mesh.

*Work supported in part by the Air Force Research Laboratory-Kirtland.

¹C.K. Birdsall and A.B. Langdon, *Plasma Physics via Computer Simulation*, IOP Publishing Ltd. (1991).