Finite difference scheme for solving general 3D convective-diffusion equation

R.Zagórski[1], N. McTaggart[2], X. Bonnin[2], A. Runov[2], R. Schneider[2]
[1]*Institute of Plasma Physics and Laser Microfusion, P.O.Box 49, Warsaw, Poland*
[2]Max-Planck-Institut fur Plasmaphysik, Teilinstitut Greifswald,
EURATOM Association, Wendelsteinstraβe 1, D-17491 Greifswald, Germany

The effect of ergodisation (either by additional coils like in TEXTOR-DED or by intrinsic plasma effects like in W7-X) defines the need for transport models being able to describe this properly. A prerequisite for this is the concept of local magnetic coordinates allowing a correct discretization with minimized numerical errors [1]. For these coordinates the full respective metric tensor has to be known.

In the present paper we employ the finite difference discretization method which allows the numerical simulation of energy transport in complex 3D edge geometries (in particular for W7-X) using a custom-tailored unstructured grid in local magnetic coordinates. This grid is generated by field-line tracing to guarantee an exact discretization of the dominant parallel transport. Therefore the radial and parallel fluxes are almost perfectly separated which minimizes the numerical diffusion connected with strong anisotropy of the system ($\Gamma_{||} / \Gamma_{\perp} \sim 10^7$). In addition, the parallel and radial directions can be treated independently in the numerical method.

Along the magnetic field lines we use the standard Patankar concept [2] to discretize the convective-diffusion equation.

In order to solve a quasi-isotropic problem in a plane (toroidal cut), we modified the finite volume method to obtain its finite difference representation. The fluxes in the cut are interpolated on the mesh using a constrained Delaunay triangulation (keeping the structural information for magnetic surfaces if they exist). The general concept of an upwind scheme [2] is used to discretize the convective terms.

The first tests for W7-X geometry were successfully performed.

[1] A.M.Runov, et al., Physics of Plasmas, vol.8, No.3 (2001) 916
[2] S.V. Patankar, Numerical Heat Transfer and Fluid Flow, McGraw-Hill, New York (1980)