Electronic Companion for "Fairness, Efficiency and Flexibility in Organ Allocation for Kidney Transplantation" by Bertsimas, Farias and Trichakis

A. Simulation Results

Table 2 includes the simulated average percentage distributions (and sample deviations) of recipients across different race, dialysis time, blood type, sensitization, diagnosis and age patient groups for the policies designed in the case studies in Section 4, alongside the KTC proposed policy. The results are for an out-of-sample period of 6 months in 2008.

B. Additional Case Studies

B.1. Varying Planning Horizon

We repeat Case Study 1 (see Section 4.3) for different planning horizons. In particular, in Case Study 1 we considered a planning horizon of 6 months (primarily because of data availability). To study the effect of the length of the planning horizon, we present here results for planning horizons of 3 months and 4.5 months.

Table 3 reports the weights that our methodology assigns to each of the score components of the policy, for the different planning horizons we consider. Similarly, Table 4 reports the average organ wastage, *i.e.*, the percentage of procured organs that were discarded due to successive rejections by patients, and the average life years from transplant. Finally, the distributions of recipients among different groups are depicted in Figure 4 and reported in Table 5.

C. Category-based Policies

As discussed in Section 5, OPTN policymakers have proposed policies in which patients and/or organs are categorized into different groups according to some criteria. Then, specific groups receive priority in the allocation of kidneys. For instance, a proposal presented in OPTNKTC (2007) suggests to categorize patients in 5 different groups according to their expected life year gains (LYFT): top 20% goes to the first group, bottom 20% goes to the last group, etc. Similarly,

	KTC		Case Study 1		Case Study 2	
	mean	std	mean	std	mean	std
race						
African American	37.2	0.45	35.6	0.5	35.5	0.45
Hispanic	15.1	0.3	15	0.35	14.8	0.3
Caucasian	39.7	0.5	41.1	0.5	41.5	0.45
Other	8	0.3	8.3	0.25	8.2	0.25
dialysis time						
0-5 yr	54	0.4	56.3	0.4	53.3	0.45
5-10 yrs	29.5	0.5	27.1	0.4	28.8	0.45
10-15 yrs	11.1	0.3	11.6	0.3	12.3	0.25
>15 yrs	5.4	0.15	5	0.2	5.6	0.2
blood type						
A	32.1	0.4	31.9	0.35	32	0.3
AB	6.1	0.2	6.1	0.2	6	0.2
B	13.8	0.3	13.6	0.3	13.8	0.35
O	48	0.35	48.4	0.35	48.2	0.45
sensitization						
CPRA 0-10	54.8	0.5	56.3	0.5	54.5	0.5
CPRA 10-80	22.6	0.5	22.6	0.45	23	0.45
CPRA 80-100	22.6	0.4	21.1	0.4	22.5	0.4
diagnosis						
Nephritis	25.8	0.5	25.8	0.45	22.7	0.4
Hypertension	21	0.4	18.4	0.4	19.4	0.4
Polycystic	7.2	0.3	8.6	0.3	9.2	0.25
Other	28.2	0.4	29.6	0.5	28.4	0.5
Diabetes	17.8	0.5	17.6	0.4	20.3	0.4
age						
<18 yr	5	0.2	4.7	0.2	5	0.2
18-35 yr	20.4	0.4	21.7	0.35	15	0.35
35-50 yr	32.4	0.4	32.6	0.45	23	0.45
50-65 yr	30.7	0.5	30.8	0.45	39.7	0.6
>65 yr	11.5	0.3	10.2	0.35	17.3	0.4

Table 2: Simulated average percentage distributions (and sample deviations) of recipients across different race, dialysis time, blood type, sensitization, diagnosis and age patient groups for the policies considered in Section 4, for an out-of-sample period of 6 months in 2008.

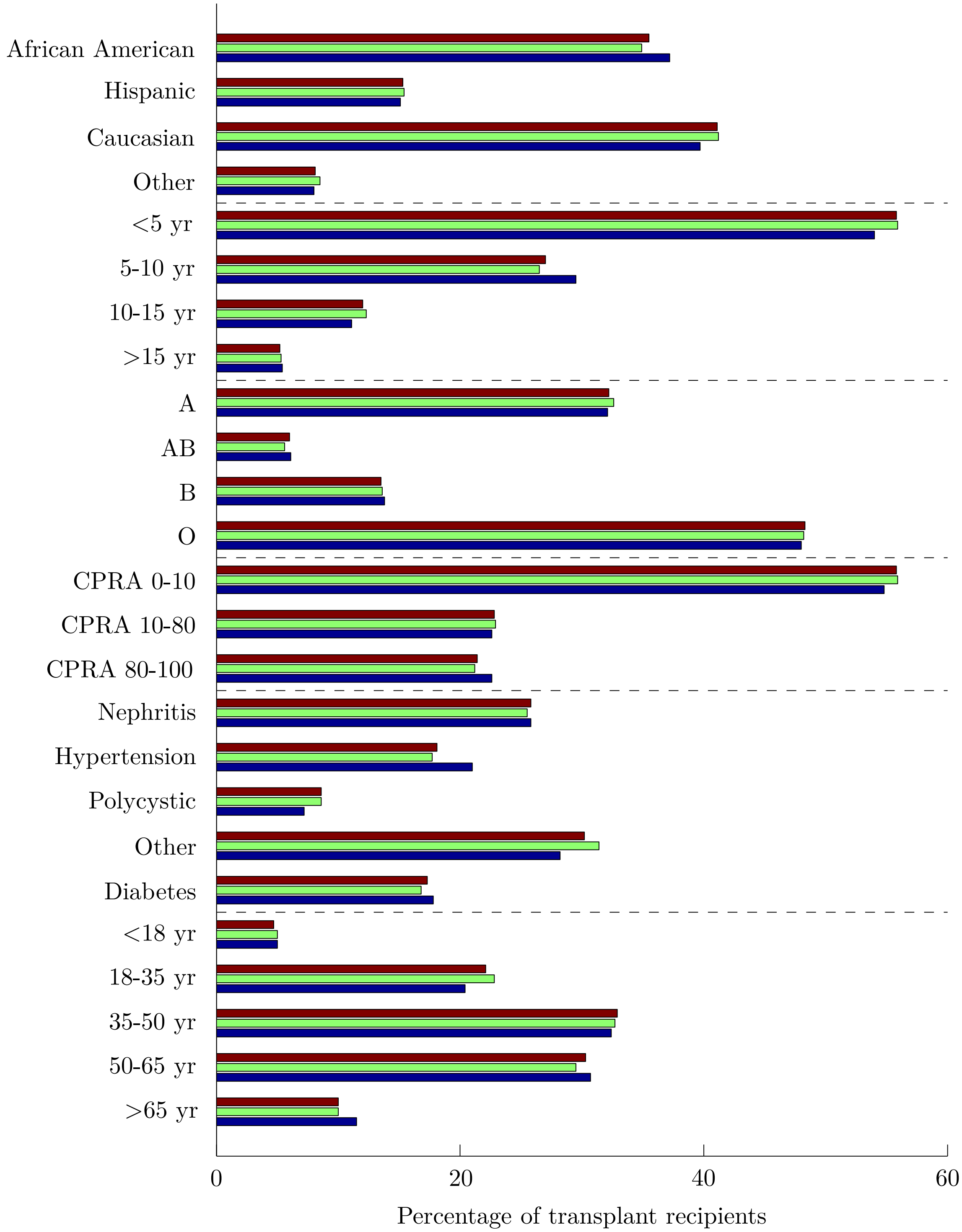

Figure 4: Simulated average percentage distributions of recipients across different race, dialysis time, blood type, sensitization, diagnosis and age patient groups for the policies designed in Section B.1, for different planning horizons: 3 months (green) and 4.5 months (red). The target distributions of the KTC policy are also depicted (blue). The results are for out-of-sample periods of 3 (or 4.5) months in 2008.

	3 months	4.5 months	6 months
points per lyft	1	1	1
points per year on dialysis 0-5	0.6	0.65	0.65
points per year on dialysis 5-10	0.9	0.95	1
points per year on dialysis 10-15	0.1	0.15	0.2
points per year cpra score	0.08	0.08	0.08
points if aged above 50	0.5	0.5	0.5

Table 3: Output weights for the policies designed as in Case Study 1 in Section 4.3, for different planning horizons; see Section B.1.

	KTC policy	3 months	4.5 months	6 months
organ wastage (std)	14.6% (0.3%)	22.2% (0.4%)	15% (0.3%)	14.5% (0.3%)
avg lyft (std)	5.9 (0.03)	6.48 (0.06)	6.38 (0.04)	6.35 (0.04)

Table 4: Simulation results for the KTC policy and policies designed as in Case Study 1 in Section 4.3, for different planning horizons; see Section B.1.

organs are categorized according to their quality (DPI). In the allocation then, group 1 patients are given priority for group 1 organs, groups 2 patients are given priority for group 2 organs and so on. In this section we illustrate how one can use a modified version of the framework developed in Section 3 to decide how one should partition patients and/or organs to such groups.

Suppose we want to design a policy that categorizes patients and organs into groups A and B, according to their (average) LYFT and DPI scores. Specifically, the top q% of patients are categorized into group A and the remaining into group B, where q is a policy parameter that needs to be determined. Similarly we categorize the top q% of organs into group A and the remaining into group B. Then, once an organ of group A is procured, patients from group A have priority over group B patients. Patients are then ranked by the time they have spent on dialysis (DT).

If $q = 0$, the allocation policy is essentially a first-come first-serve policy (FCFS), with respect to dialysis time. As q increases and we depart from a pure FCFS policy, more emphasis is given to efficiency as high quality organs are first offered to patients who are likely to benefit more from them (in terms of life year gains). Thus, one expects higher LYFT gains. As q approaches 100 however, we again recover FCFS. Below we discuss a way of guiding the selection of q based on our framework.

For the case of FCFS, *i.e.*, $q = 0$ or $q = 100$, one can solve problem (1) without any fairness constraints and with objective coefficients equal to $\text{DT}(p)$ for every (p, o) pair, to obtain an "ap-

	KTC		3 months		4.5 months	
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race						
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50-65 yr	30.7	0.5	29.5	0.75	30.3	0.55
>65 yr	11.5	0.3	10	0.5	10	0.35

Table 5: Simulated average percentage distributions (and sample deviations) of recipients across different race, dialysis time, blood type, sensitization, diagnosis and age patient groups for the policies considered in Section B.1, for different planning horizons.

proximate" expected allocation. Suppose we now consider $0 < q < 100$. Let $A_P(q)$ and $A_O(q)$ be the sets of all group A patients and organs respectively. We then define an adjusted set of eligible patient-organ pairs $\mathcal{C}(q)$ (see Section 3.1), in which group A organs are only allocated to group A patients (according to the selected value of q), *i.e.*,

$$\mathcal{C}(q) = \{(p, o) : (p, o) \in \mathcal{C} \text{ and } p \in A_P(q) \text{ if } o \in A_O(q)\}\,.$$

One can then solve the following problem to obtain an "approximate" expected allocation for any selection of q:

$$\begin{array}{ll} \text{maximize} & \displaystyle\sum_{(p,o)\in\mathcal{C}(q)} \mathrm{DT}(p) x_{(p,o)} \\ \text{subject to} & \displaystyle\sum_{o:(p,o)\in\mathcal{C}(q)} x_{(p,o)} \leq 1, \quad \forall p \\ & \displaystyle\sum_{p:(p,o)\in\mathcal{C}(q)} x_{(p,o)} \leq 1, \quad \forall o \\ & x \geq 0. \end{array} \tag{5}$$

Let $x(q)$ denote an optimal solution of (5) for any selection of q.

To quantitatively characterize the efficiency of each of those policies (as we vary q), one can calculate the quantity

$$\sum_{(p,o)\in\mathcal{C}(q)} \mathrm{LYFT}(p, o) x_{(p,o)}.$$

In some sense, the above quantity is associated with the anticipated life year gains in case organs are allocated using such a policy. Figure 5 depicts the (normalized) values we obtain for the anticipated life year gains for different values of q, by using our training data of the 2008 SRTR dataset (see Section 4.2). Such a graph can be used by policymakers to decide on the value of q, as it characterizes which values are expected to provide high life year gains.

To evaluate the performance of the method we described, we use the out-of-sample 6 month data of the 2008 dataset to simulate policies for various values of q. For each value of q, we record the number of life year gains. The (normalized) results are depicted in Figure 5. One can observe that our methodology accurately predicts the qualitative dependence of life year gains on the parameter q.

Finally, note that in a realistic setting, points would also be given to patients not just based

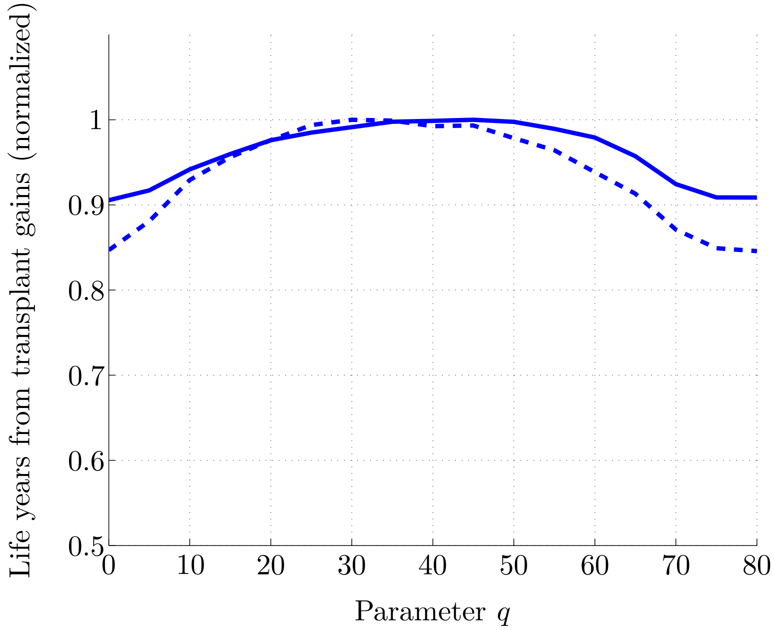

Figure 5: Predicted (solid) and simulated (dashed) life years from transplant gains for different values of the parameter q for the case study in Section C.

on their dialysis or waiting time, but also based on sensitization and perhaps other criteria, so as to enforce particular fairness constraints. In that case, one can modify the methodology described above to include those fairness constraints in Problem (5).