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Dynamic Pricing Under Debt: Spiraling Distortions and Efficiency Losses

Omar Besbes,[a] Dan A. Iancu,[b] Nikolaos Trichakis[c]

[a] Graduate School of Business, Columbia University, New York, New York 10027 [b] Graduate School of Business, Stanford University, Stanford, California 94305 [c] MIT Sloan School of Management, Massachusetts Institute of Technology, Cambridge, Massachusetts 02142

Contact: ob2105@columbia.edu, http://orcid.org/0000-0002-2982-3794 (OB); daniancu@stanford.edu, http://orcid.org/0000-0001-9890-0928 (DAI); ntrichakis@mit.edu, http://orcid.org/0000-0002-8324-9148 (NT)

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Abstract. Firms often finance their inventory through debt and subsequently sell it to generate profits and service the debt. Pricing of products is consequently driven by inventory and debt servicing considerations. We show that limited liability under debt induces sellers to charge higher prices and to discount products at a slower pace. We find that these distortions result in revenue losses that compound over time, leading to some form of performance spiral down. We quantify the extent to which these inefficiencies can be mitigated by practical debt contract terms that emerge as natural remedies from our analysis, and find debt amortization or financial covenants to be the most effective, followed by debt relief and early repayment options.

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1. Introduction

Recent years have witnessed a surge in the adoption of dynamic pricing practices across a wide range of industries, fueled in no small part by the increased availability of data and inexpensive computing, and by a growing body of focused research. The canonical models studied in the academic literature consider a firm endowed with inventory that is dynamically adjusting prices to maximize revenues from sales. Although such an approach is well aligned in spirit with the typical *material flows* of many firms, whereby inventory is built up and subsequently sold to customers, it remains agnostic to the associated *financial flows*, whereby inventory is predominantly financed through debt that is then serviced using sales revenues. This could lead to potentially misleading conclusions, as it is well known that the presence of debt and its associated limited liability can significantly distort a decision maker's incentives and actions, leading to efficiency losses (Jensen and Meckling 1976, Myers 1977). This raises the natural question of how debt would affect pricing decisions and their efficiency, as measured through expected sales revenues. Also, given the inherent dynamic nature of pricing decisions, how would these distortions and losses evolve dynamically over time?

For example, consider a real-estate firm that undertakes a new development project. As is common in this industry, the firm would finance the large upfront costs through a loan, which would be repaid through the sales of individual units. Construction loans for real-estate (tract) development in the United States usually cover up to 80% of project costs (OCC 2013, pp. 10–12), and currently amount to a total of \$1.1 trillion, with \$201.7 billion raised during 2015 alone (Board of Governors of the Federal Reserve System 2016). Because it is customary to presell only a subset of the units before securing the loan (OCC 2013, pp. 28–30), the firm would then have considerable flexibility in setting subsequent prices, and this would be done under a heavy debt burden. Would this lead to lower prices (and a boost in sales), higher prices (and larger margins) or distortions of no particular pattern? What would that imply for the total sales revenues and the efficiency of the entire channel?

Similarly, consider a liquidation house that undertakes a going-out-of-business (GOB) sale for a bankrupt retailer, whereby remaining inventory is liquidated through successive markdowns over a fixed period of time. The liquidator would commonly purchase the inventory upfront through a large, debt-financed transaction (Craig and Raman 2016). For instance, one week before administering the liquidation of Borders Group in 2011, two liquidation houses, Hilco and Gordon Brothers, joined forces to purchase the book retailer's entire store assets for approximately \$270 million (Foley et al. 2012). Similar transactions occurred in several other liquidations, including those of CompUSA, Dots Stores, and Sports Authority. Would debt then induce more or less aggressive markdowns over time?

To address these questions, we anchor our analysis around the dynamic pricing formulation of Gallego and van Ryzin (1994). Although this classical model ignores several practical considerations that can shape pricing policies, e.g., competition or customers' strategic behavior, it allows us to isolate the effect of debt on pricing policies and benchmark against a well understood setting. More precisely, we consider a seller endowed with a given inventory of a single product who is dynamically adjusting prices in discrete time over a fixed planning horizon. The seller is faced with a debt payment at the end of the horizon, and collects only the residual revenues remaining after the debt is paid off. We compare the pricing policy and the revenues achieved by the seller with those of a seller without debt, who would follow a classical revenue-maximizing policy.

1.1. Our Findings

To highlight the main insights, we first analyze the case in which the seller faces two periods and a linear demand function, and then generalize the results.

1. We formulate the seller's decision problem as a dynamic program, where the value function depends on two state variables: remaining inventory and outstanding debt. Although the state-space extension is natural, the basic structural properties of the value function differ drastically from the classical dynamic pricing problem. More precisely, we find that the value function is convex in the debt level and can become locally *convex in inventory*, i.e., the marginal value of an extra inventory unit can be increasing. This highlights the subtle but key impact that debt has on the nature of the problem and underscores the analytical challenges it introduces.

2. We show that a seller under debt sets higher prices than a revenue-maximizing seller. This occurs even when the debt amounts are small, and becomes more pronounced as debt levels increase. Although prices generally *tend* to grow with the debt, we find that this increase need not be monotonic, due to a subtle interplay between debt and inventory. In particular, when faced with a larger debt burden, a seller may prefer relying on more sales at lower prices to pay off the debt, whereas with a lower debt, he may rely on fewer sales at a higher price.

3. We show that time dynamics *maintain and amplify* these effects in a spiraling fashion. More precisely, the pricing policy under debt recommends less steep markdowns than the revenue-maximizing policy (to the extent that it marks up prices in expectation when inventory is ample, whereas the revenue-maximizing policy would maintain constant prices). In turn, this compounding distortion of the pricing policy gives rise to a degradation in efficiency, as relative revenue losses strictly increase in expectation over time.

4. We quantify the extent to which efficiency losses can be mitigated by practical debt contract terms that emerge as natural remedies from our analysis, i.e., early payment discounts, debt relief, debt amortization or financial covenants.[1] We find an ordering, with early payment options being less effective that debt relief, which in turn is strictly less effective than debt amortization or financial covenants. None of the terms can fully restore efficiency.

We establish the robustness of our findings through several modeling extensions: by confirming some of the analytical results for the asset selling problem over an arbitrary number of periods and general demand functions (Section 5.1); by conducting numerical experiments for the general version of the problem (Section 5.2); by considering loans where borrowers are responsible for some of the losses when debt is not fully repaid (Online Appendix A); and by studying a setting where debt is endogenously determined (Online Appendix C).

Our results are consistent with anecdotal evidence in the popular press that documents underwhelming discounts or even price markups during GOB sales at Circuit City, Borders or Linens 'n Things (see, e.g., Chang 2009, Sakraida 2011, White 2016).[2] They are also aligned with the empirical study by Genesove and Mayer (1997), who find that homeowners with larger debt loads set higher asking prices for their houses, have them listed for a longer time on the market, and receive a higher price upon an eventual sale than owners with smaller debt loads (see our discussion in Section 1.3).

1.2. Managerial Implications

Our finding of increased prices highlights the importance of controlling for slow markdowns in the presence of debt. Although this could in principle be enforced through price controls, these would be difficult to implement in practice.[3] Instead, incentives to lower prices so as to counterbalance the distortions could be provided through various other means, for example, by allowing for repayments at lower prices—such as early payment discounts in trade credit, or by limiting the amount of available credit if sales fall short of projections (OCC 2013, pp. 20 and 28).

To limit potential revenue losses caused by increased leverage, lenders can control the size of the outstanding debt throughout the loan's tenor. Under slow sales, all parties could benefit from debt restructuring through, e.g., a reduction in principal or interest (i.e., a "bondholder haircut") or a tenor extension. For revolving real estate loans, leverage could also be controlled by enforcing borrowing limits tied to the sales rate[4] or by directly limiting the investment size, e.g., by financing large developments in phases or by constraining the number of unsold units financed at any point of time (OCC 2013, p. 29). To the latter point, however, our findings also issue a cautious warning *against* enforcing strict limits, since borrowers with larger amounts of inventory may actually prefer relying on more sales (at lower prices) to cover their debt, thus reducing distortions and improving efficiency.

To alleviate the compounding of distortions, lenders can frequently monitor sales performance, and require repayments that track the selling of units. This could be achieved through, e.g., suitable repayment schedules (debt amortization) or financial covenants requiring minimum levels of cashflow generated throughout the loan's tenor (OCC 2013).

With regard to the efficacy of the aforementioned countermeasures, our analysis suggests that debt amortization or financial covenants tend to be more effective ways of improving efficiency than debt relief, which in turn could be more effective than early payment discounts. However, even these mechanisms may fail to fully alleviate distortions. This implies that, unlike flexibility in adjusting inventory levels (Iancu et al. 2017), flexibility in adjusting prices under debt can induce inefficiencies that are more difficult to mitigate through the design of common contractual terms and conditions.

1.3. Literature Review

Our paper is related to the extensive literature on pricing and revenue management, which is surveyed in Talluri and van Ryzin (2005) and Phillips (2005), among others. We build on a discrete-time counterpart of the classical dynamic pricing model of Gallego and van Ryzin (1994), by changing the decision maker's objective to reflect the presence of debt. This relates our work to Levin et al. (2008), who consider a pricing problem in continuous-time with a risk-averse objective that mixes expected revenues with the probability of meeting a revenue target. They find that the optimal risk-averse policy involves *discounts* relative to the risk-neutral (i.e., revenue-maximizing) policy when revenues are immediately below the target. Besbes and Maglaras (2012) also consider revenue targets, but as *constraints*, with the classical objective of maximizing expected revenue; they derive the structure of the optimal policy for a deterministic problem, and argue that the prescription obtained also performs well even under a model with limited demand information. Our paper differs from these in focus and model: Our objective corresponds to the residual revenues in excess of the (debt payment) target, and we analyze the distortions in the pricing policy, as well as the resulting efficiency losses. Furthermore, our insights are qualitatively different, as the optimal policy becomes *risk-seeking* in our case, always pricing *above* the revenue-maximizing policy while the target is not met. Our work is also related to papers studying the dynamic evolution of (bid) prices and revenues. Pang et al. (2015) study the inter-temporal behavior of bid prices, finding an upward (downward) trend in time when the seller has multiple (a single) unit(s) in inventory. Cooper et al. (2006) document a downward spiral in revenues driven by incorrect assumptions about customer behavior. The present paper differs from these through its focus on debt.

Our work is also related to several papers in the operations management literature documenting operating distortions caused by debt. For instance, Xu and Birge (2004), Buzacott and Zhang (2004), Dada and Hu (2008), and Boyabatlı and Toktay (2011) extend the newsvendor model to include financing considerations, and show how these can affect the firm's optimal order quantity or choice of (flexible) capacity. Closer to our work, Chod (2017) shows how a firm that has already secured debt financing can engage in risk-shifting by ordering riskier products from suppliers, and studies the role of trade credit in alleviating the distortions. Such models are typically cast in a static setting, and are not focused on pricing decisions or on quantifying the dynamics of efficiency losses.

Several papers in the operations literature have also considered dynamic models with alternative objective functions motivated by financial considerations, e.g., Porteus (1972) (optimizing inventory policies while maintaining a cash safety level), Archibald et al. (2002) (maximizing the probability of survival of start-up firms), Possani et al. (2003) (maximizing survival probabilities of start-ups in manufacturing), Babich and Sobel (2004) (maximizing initial public offering cash flows), Swinney et al. (2011) (optimizing capacity investments for start-ups and established firms), Gong et al. (2014) (examining inventory control under leverage), Li et al. (2013) (maximizing discounted dividends), and others. Closest to our work, Iancu et al. (2017) consider a two-period model of a firm endowed with inventory management capabilities, in the form of replenishments and partial liquidations (at fixed prices). They show that extra flexibility in managing inventory can lead to significant efficiency losses when the firm is financed through debt, but that such losses can be fully alleviated by common covenants present in debt agreements. By contrast to these papers, which deal almost exclusively with inventory management, we focus on pricing decisions, and on quantifying the time evolution of efficiency losses. We find that such losses persist in our setting, even under the types of covenants considered in Iancu et al. (2017), and become more pronounced in time due to the problem's dynamics. This suggests that pricing decisions induce losses of a qualitatively different nature than inventory decisions, requiring potentially more complex contractual terms to alleviate.

Our work is also related to a large body of finance and economics literature that addresses agency issues inherent when holding debt. Jensen and Meckling

(1976) and Myers (1977) were among the first to challenge the classical Modigliani-Miller insight that a firm's decisions are independent of its capital structure, by arguing how the equity holders of a leveraged firm could extract value from the debt holders by increasing the risk in the firm's cash flows after the debt is in place. A large volume of subsequent literature in corporate finance has been devoted to examining how the design of the firm's capital structure can recognize and control the resulting efficiency losses, known as the *agency costs of debt*. Within this literature, several recent papers quantified these costs using dynamic models of the firm, typically within a real options framework; see, e.g., Leland and Toft (1996), Leland (1998), Childs et al. (2005), and Manso (2008) for a more in-depth review. The majority of these papers document costs of 0.5%–1.5% of firm value. Closer to our work, Décamps and Faure-Grimaud (2002) study a model where the equity holders can liquidate the firm at a fixed set of "scrapping times," and numerically document larger agency costs. Décamps and Djembissi (2007) consider a firm with the ability to dynamically switch operations to a poor activity, with a higher volatility and lower expected returns, and numerically document costs of more than 7%. No papers in this stream model a firm that has *dynamic pricing* ability, which is the main emphasis of our work. Furthermore, to the best of our knowledge, the only paper in this literature that discusses some form of time evolution for the agency issues is Décamps and Faure-Grimaud (2002). This paper numerically documents that the distortions in the firm's operating policy could increase or decrease over time, and the agency costs of debt (measured at the initial time) could increase or decrease with the introduction of additional decision points. Of key difference is that decision epochs in Décamps and Faure-Grimaud (2002) are separated by arbitrary time intervals, so that the firm's profits/revenues across decision points are nonstationary. By contrast, we show analytically and numerically that for a firm faced with stationary willingness-to-pay distributions, the pricing distortions always compound over time, giving rise to actions and revenues that increasingly deviate from the system-optimal ones in expectation.

Also related are papers showing how debt can be strategically used as a precommitment tool to increase firm value. Brander and Lewis (1986) show how firms engaged in Cournot competition can use debt to precommit to larger production quantities, improving equilibrium outcomes. Note though that Faure-Grimaud (2000) argues how using an optimal renegotiation-proof contract can reverse the positive effect of debt documented by Brander and Lewis (1986). Chemla and Faure-Grimaud (2001) consider a monopolist selling to strategic consumers with private valuations for the good, and show how debt can reverse the negative effect of adverse selection, and allow the firm to charge higher prices. Some of our results are aligned with these findings: In Online Appendix C, we also show how a more leveraged firm can charge prices that are closer to optimal, and improve its value. However, the main focus of our work is different: We discuss the dynamic evolution of pricing distortions and losses, and quantify the effectiveness of contractual mechanisms for alleviating the resulting inefficiencies.

Our pricing policy results are validated by the empirical work of Genesove and Mayer (1997), who find homeowners with larger debt loads (i.e., higher loan-to-value (LTV) ratio) listing their homes at higher prices (as mentioned earlier). The effects are of significant magnitude, and are found to be more pronounced for investors than for individual owners. For the latter category, the authors rationalize the behavior using the model in Stein (1995), which relies on the liquidity constraints introduced by a required down payment. For investors, Genesove and Mayer (1997) suggest an explanation relying on the limited liability effect. Our work formalizes this intuition providing rigorous theoretical backing.

Viewing the decision maker's payoff as a bonus corresponding to the residual revenues in excess of a quota also relates our paper to a growing body of work focusing on salesforce compensation. Basu et al. (1985) were the first to rationalize the existence of revenue quotas, by adopting the principal-agent framework of Holmstrom (1979) to argue that the optimal compensation package for a sales agent exerting unobservable effort may depend nonlinearly on the generated revenues. See also the related studies of Oyer (1998). Sales quotas have also been considered in the operations literature, with a focus on coordinating operating decisions with an agent's compensation. For instance, Chen (2000) considers a dynamic model in which a manufacturer compensates a sales agent through an annual salary and a per-unit bonus when *quantity* sales exceed a target. The paper shows how the well-known "sales hockey stick" (SHS) effect can arise, whereby the agent's optimal choice of delaying effort generates increasing sales toward the end of the horizon. See also Sohoni et al. (2010). While we also document distortions in the agent's policy, by contrast with the SHS effect observed in this line of work, which could be viewed as a form of *positive* spiraling, we show that pricing distortions compound *negatively* over time, and we quantify how this generates a downward spiral in efficiency.

2. Problem Formulation

We present the base model we use to assess the effects of debt on pricing decisions and efficiency. We introduce the dynamics, followed by the seller's problem, and the metrics we use to quantify the effects.

Consider a decision maker (DM) in charge of selling a given number of units of a single product in discrete time over a horizon of T periods. We denote by $Y \in \mathbb{N}$ the number of units available at the start of the selling horizon. In each period $t \in \{1, \ldots, T\}$, the DM sets a posted price p_t selected from an interval of feasible prices $\mathcal{P} = [\underline{p}, \bar{p}] \subseteq \mathbb{R}$. Subsequently, a single customer arrives with a willingness-to-pay (WTP) $\mathcal{W}_t$. We assume $\mathcal{W}_1, \ldots, \mathcal{W}_T$ to be i.i.d., and let $\lambda: \mathcal{P} \to [0,1]$ denote the corresponding *demand function* that gives the probability that the WTP exceeds a posted price, $\lambda(p_t) = \mathbb{P}(\mathcal{W}_t \geq p_t)$.[5]

The DM selects the posted prices according to a *pricing policy* $\mathbf{p} = (p_1, \ldots, p_T)$. In particular, the set of admissible pricing policies for the DM, denoted by $\mathcal{Q}$, consists of all nonanticipating policies for which the price p_t is $\mathcal{F}_t$-measurable, where $\mathcal{F}_t = \sigma(\mathcal{W}_1, \ldots, \mathcal{W}_{t-1})$ denotes the information set available to the DM at the beginning of period $t = 1, \ldots, T$.

Let $\mathcal{Y}_t$ denote the number of remaining units at the start of period t, which evolves according to:

$$\mathcal{Y}_1 = Y, \quad \mathcal{Y}_{t+1} = (\mathcal{Y}_t - \mathbf{1}\{\mathcal{W}_t \geq p_t\})^+, \quad t = 1, \ldots, T-1.$$

Consistent with typical dynamic pricing models, we assume that unmet demand is lost without any penalty, and unsold units at the end of the horizon are discarded without any salvage value. Consequently, the total revenues generated throughout the horizon are given by

$$\mathcal{R}(\mathbf{p}) := \sum_{t=1}^{T} p_t(\mathcal{Y}_t - \mathcal{Y}_{t+1}).$$

In this setting, a natural measure of efficiency is the total expected revenue $\mathbb{E}[\mathcal{R}(\mathbf{p})]$, which is the usual focus in dynamic pricing and revenue management models (Talluri and van Ryzin 2005).

To introduce debt, we assume that the DM is faced with a *debt* repayment B that is due at the end of the horizon. The generated revenues $\mathcal{R}(\mathbf{p})$ are used to first pay off the debt, and the DM only collects the residual revenues. The DM is shielded by limited liability, so that if revenues are insufficient to pay off the debt, he collects zero. That is, the DM's *payoff* at the end of the horizon equals

$$(\mathcal{R}(\mathbf{p}) - B)^+.$$

We shall say that the DM *pays off* or *covers* the debt in the event that $\mathcal{R}(\mathbf{p}) \geq B$.

Decision Maker's Policy. The DM selects his pricing policy, denoted by $\mathbf{p}^\dagger$, to maximize his expected payoff, i.e.,

$$\mathbf{p}^\dagger \in \arg\max_{\mathbf{p} \in \mathcal{Q}} \mathbb{E}[(\mathcal{R}(\mathbf{p}) - B)^+].$$

We denote the expected revenue generated under such a policy by

$$J^\dagger := \mathbb{E}[\mathcal{R}(\mathbf{p}^\dagger)].$$

Note that the self-interested DM, by optimizing over an objective that is different from the expected revenue, follows a policy that might incur efficiency losses compared to what would be possible.

Optimal Policy. An optimal policy maximizes expected revenue. Let $\mathbf{p}^\star$ denote such a policy, i.e.,

$$\mathbf{p}^\star \in \arg\max_{\mathbf{p} \in \mathcal{Q}} \mathbb{E}[\mathcal{R}(\mathbf{p})],$$

which we shall also refer to as a *revenue-maximizing* policy. In accordance with our previous notation, let $J^\star$ be the expected revenue under $\mathbf{p}^\star$ or the *optimal revenue*, i.e.,

$$J^\star := \mathbb{E}[\mathcal{R}(\mathbf{p}^\star)].$$

By definition, we have that $J^\star \geq J^\dagger$. Note that for $B = 0$, the DM's policy is also a revenue-maximizing policy, and consequently the inequality holds with equality. For $B > 0$, the policies generally differ, and a loss in expected revenues is likely when the DM's policy is followed.

Policy Comparison and Efficiency Loss. We aim to understand the pricing distortions and resulting efficiency loss induced by debt. We characterize the former by comparing the DM's prices with the revenue-maximizing ones. We quantify the latter as the loss in expected revenue due the DM's policy, relative to the optimal revenue. Formally, the *efficiency loss* in our setting is given by

$$L := \frac{J^\star - J^\dagger}{J^\star}.$$

Note that L is the standard way of defining and measuring normalized efficiency losses in the academic literature, and has been used extensively in economics, finance, and operations management (see, e.g., Perakis and Roels 2007). However, L remains an eminently static measure, aggregating losses across time and across all future (uncertain) states of the world. To quantify the *dynamic* progression of revenue losses over time as the customers' uncertain WTP is being realized, we introduce efficiency loss metrics that capture time and path dependence. In particular, let $\mathcal{J}_t^\dagger$ ($\mathcal{J}_t^\star$) be the *conditional expected total revenue* under the DM's (optimal) policy at time t, given that $\mathcal{F}_t$ has realized, i.e.,

$$\mathcal{J}_t^\dagger := \mathbb{E}[\mathcal{R}(\mathbf{p}^\dagger) \mid \mathcal{F}_t] \quad \text{and}$$
$$\mathcal{J}_t^\star := \mathbb{E}[\mathcal{R}(\mathbf{p}^\star) \mid \mathcal{F}_t], \quad t = 1, \ldots, T.$$

Similarly, let $\mathcal{L}_t$ be the resulting *conditional efficiency loss*, dependent on the information at time t,

$$\mathcal{L}_t := \frac{\mathcal{J}_t^\star - \mathcal{J}_t^\dagger}{\mathcal{J}_t^\star}, \quad t = 1, \ldots, T. \tag{1}$$

Note that the conditional expected revenues $\mathcal{J}_t^\dagger$, $\mathcal{J}_t^\star$ and efficiency losses $\mathcal{L}_t$ are all $\mathcal{F}_t$-measurable random variables. We emphasize here that $\mathcal{L}_t$ depends on both policies $\mathbf{p}^\star$ and $\mathbf{p}^\dagger$. It compares the total expected revenues generated by either policy conditional on a given history of WTP values up to $t-1$. Note that the inventory and debt levels under $\mathbf{p}^\star$ and $\mathbf{p}^\dagger$ can be different for some history realizations. In particular, $\mathcal{L}_t$ corresponds to an updated measurement of the efficiency loss at time t, conducted under additional information. To facilitate the comparison, we also define the associated *expected efficiency loss* at time t,

$$L_t := \mathbb{E}[\mathcal{L}_t], \quad t = 1, \ldots, T.$$

This generalized definition of efficiency loss is the exact counterpart of L extended to a future time, when measurements are conducted under additional information. Note that, since no additional information is available at $t=1$ ($\mathcal{F}_1 = \emptyset$), $\mathcal{J}_1^\star = J^\star$, $\mathcal{J}_1^\dagger = J^\dagger$, and $L_1 = L$, so that our definitions are consistent.

Model Discussion. One way to interpret our model is that it corresponds to a setting where a debt contract is already in place between an inventory-selling firm and some debt holders. In particular, the debt holders are entitled to a debt repayment B; their expected payoff, which equals $\mathbb{E}[\min(B, \mathcal{R}(\mathbf{p})]$, is commonly referred to as *debt value* in the corporate finance literature (see p. 75 of Tirole 2006). The expected payoff for the firm's equity holders, which equals $\mathbb{E}[(\mathcal{R}(\mathbf{p}) - B)^+]$, is commonly referred to as *equity value*. Thus, the DM in our model acts in the interest of the firm's equity holders, which is the fiduciary duty of corporate managers (see p. 56 of Tirole 2006).[6] The sum of the expected payoffs to the debt and equity holders, which equals $\mathbb{E}[\mathcal{R}(\mathbf{p})]$, is referred to as *firm value*, and constitutes the de facto way of measuring the efficiency of contracts in economics and finance (in the operations literature, it corresponds to the "supply chain profit/payoff"). The efficiency losses L are commonly referred to as *agency costs of debt* in the corporate finance literature.

Our model adopts the established paradigm in corporate finance that the equity holders/the DM have *zero* liability, so that they suffer no losses or penalty when revenues fail to cover the debt (see, e.g., p. 75 or p. 115 of Tirole 2006). Zero-liability loans are commonly referred to as *nonrecourse* in practice (White and Kitchen 2016, p. 2). In Online Appendix A, we consider a setting of nonzero liability: If unable to pay off the debt, the DM/equity holders incur losses (due to, e.g., recourse by the lender or bankruptcy and reputational costs when the firm goes into default) that are proportional to the shortfall, so that the DM's payoff becomes $\mathbb{E}[(\mathcal{R}(\mathbf{p}) - B)^+] - k\mathbb{E}[(B - \mathcal{R}(\mathbf{p}))^+]$, for some $k \in [0, 1]$.

That generated revenues are used to first repay the debt is consistent with the literature (Myers 1977), as well as with practice, where lockbox arrangements or escrow/impound accounts are routinely set up in conjunction with use-of-proceeds covenants to allow collecting debt payments from operational cashflow (see p. 2 of White and Kitchen 2016 and section 7:6.1 of Hilson 2013).

Here, we assume for simplicity that there is no discounting, or equivalently that the risk-free interest rate is zero (see, e.g., p. 115 of Tirole 2006). Furthermore, we model discrete decision epochs to simplify the analysis. We emphasize, however, that these choices do not qualitatively affect the insights obtained.

Notation. We use script font notation to denote random variables (e.g., $\mathcal{W}_t, \mathcal{Y}_t, \ldots$), and use comparisons (such as "increasing/decreasing," "greater/smaller," etc.) in their nonstrict sense. Furthermore, we say that $f: [a,b] \to \mathbb{R}$ is *piecewise convex*, (respectively, *piecewise increasing*) if there exists a finite set of points $a_0 = a < a_1 < a_2 < \cdots < a_m = b$ such that the restriction of f on $(a_i, a_{i+1}]$ is convex, (respectively, increasing), for any $i \in \{0, \cdots, m-1\}$.

3. Dynamic Program and Properties of the Value Function

In our analysis, we restrict attention to Markov policies, a choice that is without loss of generality given our problem's structure. A sufficient state representation at the start of any period t is given by the remaining inventory units $\mathcal{Y}_t$ and the *outstanding debt*, which is equal to the initial debt B less the revenues generated throughout periods $1, \ldots, t-1$. Formally, the outstanding debt at the start of period t, denoted by $\mathcal{B}_t$, evolves as

$$\mathcal{B}_1 = B, \quad \mathcal{B}_{t+1} = \mathcal{B}_t - p_t \mathbf{1}\{\mathcal{W}_t \geq p_t, \mathcal{Y}_t > 0\}, \quad t = 1, \ldots, T-1.$$

Positive values of $\mathcal{B}_t$ correspond to the additional revenues that the DM needs to generate in periods $t, \ldots, T$ to pay off the debt. Negative values of $\mathcal{B}_t$ indicate that the DM has already generated enough revenues to pay off the debt, in which case $-\mathcal{B}_t$ corresponds to a payoff that the DM has already secured. In other words, when the DM faces a positive (negative) outstanding debt at some period, this means that some amount (no amount) of the additional revenues he will generate in the remaining periods must be withheld to service the debt.

We use $V_t(b, y)$ to denote the DM's optimal expected payoff at the start of period t when his outstanding debt is b and he has y remaining inventory units; we also refer to V_t as the value function. Clearly, the value function satisfies the following recursion:

$$V_t(b, y) = \max_{p \in \mathcal{P}} \{\lambda(p) V_{t+1}(b - p, y - 1) + (1 - \lambda(p)) V_{t+1}(b, y)\}, \quad y \geq 1,\ t = 1, \ldots, T, \quad (2)$$

$$V_t(b,0) = (-b)^+, \quad t = 1,\ldots,T+1, \\ V_{T+1}(b,y) = (-b)^+. \tag{3}$$

Note that the debt payment causes the terminal payoff function V_{T+1} to be convex in the outstanding debt b, a feature that differentiates our model from classical models in the operations management literature addressing maximization of concave profits (or minimization of convex costs). When $B = 0$, one obtains the classical dynamic pricing recursion for a seller with no debt who maximizes expected revenues (Gallego and van Ryzin 1994). Thus, the case $B = 0$ serves as our benchmark, from which distortions will be measured.

In this setting, the DM's price in period t, denoted by $p_t^\dagger(b,y)$, satisfies

$$p_t^\dagger(b,y) \in \arg\max_{p\in\mathcal{P}} \{\lambda(p)[V_{t+1}(b-p,y-1) - V_{t+1}(b,y)]\}, \quad y \geq 1.$$

Per our discussion above, for negative outstanding debt at the start of period t, the DM collects all revenue generated in the remaining periods $t,\ldots,T$. Consequently, the DM's price would coincide in such cases with the revenue-maximizing price, denoted by $p_t^\star(y)$. That is, $p_t^\dagger(b,y) = p_t^\star(y)$ for $b \leq 0$. If for some t, $b \geq \min\{y, T-t+1\}\bar{p}$, where $\bar{p} = \sup_{p\in\mathcal{P}} p$, the DM's payoff is equal to zero almost surely, in which case we take $p_t^\dagger(b) = \bar{p}$ without loss of generality.

Next, we provide a set of structural properties for the value function.

Lemma 3.1 (Properties of the DM's Value Function). *We have that*

(i) *$V_t(b,y)$ is convex, decreasing in the outstanding debt b and decreasing in t.*

(ii) *$b + V_t(b,y)$ is increasing in b.*

(iii) *at time t, the probability of paying off the debt is given by $-(\partial V_t/\partial b)(b,y)$, for $b > 0$.*

Part (i) confirms that the DM's expected payoff decreases as the debt burden increases, or as less time is available to pay it off. It also shows that the recursion preserves the convexity of the terminal value function V_{T+1} in b, i.e., $V_t(\cdot,y)$ is convex for all t and y. The intuition behind this result is that an increasing debt, because it becomes unlikely to be paid off, marginally reduces the DM's expected payoff by a decelerating amount. The convexity of V_t also makes the characterization of the DM's prices analytically challenging. However, we derive some insightful structural properties that facilitate our subsequent analysis.

Part (ii) shows that the DM's payoff decreases "slowly" with the debt b. In particular, the marginal decrease is always less than 1. When interpreting b as revenue that the DM needs to "return," this result suggests that the DM is expected to return only a fraction of an additional unit of debt, a testament to the DM's limited liability.

Part (iii) shows that the probability of covering the debt is precisely given by the marginal decrease rate of the DM's expected payoff at time t with respect to the outstanding debt, a result that follows from an application of the Envelope Theorem.

We observe that there are no obvious structural properties of the value function with regard to the dependence on the inventory level. Indeed, the presence of debt fundamentally changes the nature of the dynamic pricing problem studied in Gallego and van Ryzin (1994). While it is well known that units have decreasing marginal returns in the absence of debt, the value function $V_t(b,y)$ will generally *not* be concave in y for $b > 0$. To see why, note that an additional unit could determine whether the DM achieves a positive payoff, and hence $V_t(b,y)$ might have increasing marginal returns in y in some region of (b,y), as the following example illustrates.

Example 3.1 (Increasing Marginal Returns in Inventory). We illustrate that the value function might have decreasing (increasing) marginal returns in inventory for small (high) values of b. Suppose that $T = 2$, $Y = 2$ and $\lambda(p) = \alpha - \beta p$, for some $\alpha \in (0,1]$, $\beta > 0$. For $b \leq 0$, the DM's policy is aligned with the revenue-maximizing policy and we have that $V_{T-1}(b,2) - V_{T-1}(b,1) < V_{T-1}(b,1) - V_{T-1}(b,0)$.[7] By continuity, the inequality continues to hold for small enough positive values of b. However, for $b \in (\alpha/\beta, 2(\alpha/\beta))$, we have that $V_{T-1}(b,2) - V_{T-1}(b,1) > V_{T-1}(b,1) - V_{T-1}(b,0)$.[8]

The above discussion highlights the complexity of the interplay between the debt faced by the DM and the inventory level at his disposal. In Section 4, we analyze a problem with two periods, fully characterizing the distortions and the associated dynamics. We then address the multiperiod case in Section 5 by generalizing the theoretical results for the asset selling problem, and establishing the robustness of the insights for instances with an arbitrary number of units.

4. The Two-Period Case

Consider the case in which the DM has two periods to go, i.e., $T = 2$. Despite being the minimal instance in which dynamics can be analyzed, this setting already provides a rich enough model to understand the key distortions introduced by debt, their time-evolution, and their impact on the efficiency loss. We make the following assumption about the WTP, which is intended primarily to facilitate the analysis, and is relaxed in some of the subsequent sections.

Assumption 4.1. *For some $\alpha \in (0,1]$ and $\beta > 0$, the WTP is uniformly distributed on $\mathcal{P} = [0,\alpha/\beta]$ such that $\lambda(p) = \alpha - \beta p$.*

Our first result characterizes the DM's pricing policy.

Proposition 4.1 (Pricing Under Debt). *Let $T=2$ and suppose that Assumption 4.1 holds. For all $b>0$,*

(i) *the DM's price is strictly higher than the revenue-maximizing price, i.e., $p_t^\dagger(b,y)>p_t^\star(y)$ for all $y\geq 1$ and t;*
(ii) *$p_T^\dagger(b,y)$ is increasing in b, for all $y\geq 1$;*
(iii) *$p_{T-1}^\dagger(b,1)$ is increasing in b, and $p_{T-1}^\dagger(b,y)$ is piecewise increasing in b for all $y\geq 2$.*

Our result derives the distortions that debt induces on the pricing policy: The DM prices strictly higher than the revenue-maximizing price, even for arbitrarily small debt levels, and tends to increase prices further when facing an increasing debt. To understand the intuition behind the DM's preference for prices higher than $\mathbf{p}^\star$, despite them naturally reducing expected revenues, note that this increase also leads to an increase in the revenue *variability*. More precisely, pricing higher decreases the probability of a sale, but increases revenue if a sale does indeed occur; consequently, the probability mass in the revenue distribution gets more dispersed, as both the mass at the zero revenue point and the mass in the right tail of the distribution increase. Because the DM only achieves a positive payoff when the total revenue covers the debt, without being penalized when it falls short, he consequently attends to the right tail of the revenue distribution, and prefers a higher variability in outcomes; in turn, this makes higher prices preferable, despite the overall expected revenue loss.

Our insights are well aligned with the agency theory of debt developed in the corporate finance literature (Jensen and Meckling 1976, Myers 1977), which suggests that the manager of a leveraged firm would have the incentive to take excessively risky actions instead of firm-optimal ones, due to the limited liability inherent in the payoff structure. In our setting, we show that such excess risk taking is manifested through higher prices, which increase the revenue distribution's dispersion.

It is worth comparing our results with those in Levin et al. (2008), who consider dynamic pricing with a risk-averse objective that mixes expected revenues with the probability of meeting a revenue target. Similar to Levin et al. (2008), we find that prices are not distorted once the target is met: If the debt is covered by the revenues generated to date, so that $b\leq 0$, the DM follows the revenue-maximizing policy, $p_t^\dagger(b,y)=p_t^\star(y)$. However, whereas Levin et al. (2008) find that the optimal risk-averse policy may involve either markups (when revenues are far below the target) or *discounts* (immediately below the target) relative to the revenue-maximizing policy, we find consistent risk-seeking behavior and prices always above the revenue-maximizing ones, even when arbitrarily close to the "target."

Our analysis also elicits an interesting interaction between inventory and outstanding debt, which may cause the DM's price to not be *monotonically* increasing with the outstanding debt in some cases. In particular, $p_{T-1}^\dagger(b,y)$ is only *piecewise* increasing in b for $y\geq 2$. To understand this, note that with two periods to go and sufficient inventory, the DM could use either a strategy aimed at covering the debt through a single-unit sale or an alternative strategy based on two-unit sales. This interplay between inventory units and debt introduces the possibility of the DM's price being discontinuous with the debt amount. Specifically, in the proof of Proposition 4.1, we show that there exists a $\hat{b}\in(0,\alpha/\beta)$ such that the DM's pricing policy in period $T-1$ for $y\geq 2$ always has three regimes:

$$p_{T-1}^\dagger(b,y)=\begin{cases} q_\ell(b) & b\in[0,\hat{b}],\\ q_m(b) & b\in(\hat{b},\alpha/\beta],\\ q_h(b) & b\in(\alpha/\beta,2\alpha/\beta],\end{cases}$$

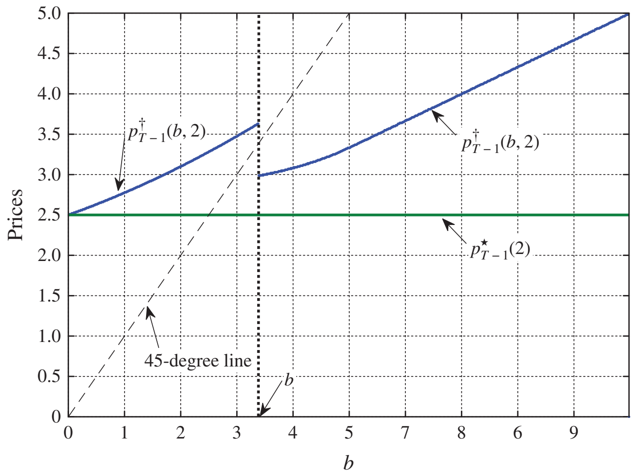
where $q_\ell(\cdot)$, $q_m(\cdot)$, $q_h(\cdot)$ are increasing, $q_\ell(\hat{b})>q_m(\hat{b})>p_{T-1}^\star(2)$, and $q_m(\alpha/\beta)=q_h(\alpha/\beta)$. Specifically, the DM's price has one discontinuity point at $b=\hat{b}$ and

$$p_{T-1}^\dagger(b,y)\begin{cases} >b & b\in(0,\hat{b}],\\ <b & b\in(\hat{b},2\alpha/\beta].\end{cases}$$

Thus, the DM plans to cover a low debt ($b\leq\hat{b}$) by selling a single unit priced sufficiently high, but plans to cover a high debt ($b>\hat{b}$) by selling two units, each priced lower than the debt. Note that this discontinuity cannot occur at time T, when the DM must rely on a single unit to pay off any outstanding debt. The driver of this phenomenon is the discrete nature of inventory and demand in conjunction with the combinatorial possibilities associated with selling the units at hand to maximize payoff under debt. In particular, revisiting the DM's problem in (2), the function $\lambda(p)[V_{t+1}(b-p,y-1)-V_{t+1}(b,y)]$ can be multimodal for more than one unit at hand and multiple periods to go. Each mode can be associated with the type of strategies described above.

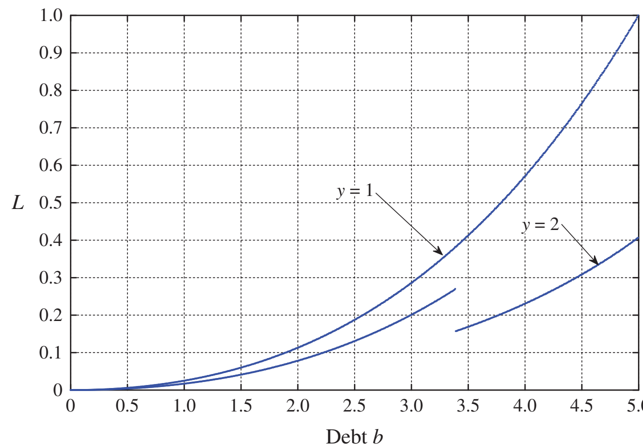
We illustrate the DM's price at $T-1$ as a function of the outstanding debt in Figure 1, for the demand function $\lambda(p)=1-0.2\times p$ and $y=2$. Note that the revenue-maximizing price is $p_{T-1}^\star(2)=2.5$, and the associated optimal revenue is $J^\star=2.5$. We first observe that even a debt of half the optimal revenue ($b=1.25$) leads to a 20% increase in price, a highly nontrivial distortion. The 45-degree line is also shown to highlight when the DM prices below or above the debt; this allows us to discern the two strategies discussed above. In particular, for any debt level below $\hat{b}\approx 3.3$, the DM relies on a strategy that covers the debt through the sale of a single unit. Above $\hat{b}$, the regime switch takes place and

Figure 1. (Color online) DM's Pricing Policy Structure



Notes. The demand function is linear given by $\lambda(p) = 1 - 0.2 \times p$. The horizon is $T = 2$ and the DM has $y = 2$ units in inventory.

Figure 2. (Color online) Efficiency Loss Under $\lambda(p) = 1 - 0.2p$ for $T = 2$, as a Function of the Outstanding Debt b and the Inventory Level y



the DM significantly decreases his price and now relies on two units to cover the debt.

Given that the DM's policy significantly deviates from the revenue-maximizing one, efficiency losses emerge, which we quantify in our next result.

Lemma 4.1 (Efficiency Loss)**.** *Let $T = 2$, $p^\star := \alpha/(2\beta)$, and suppose that Assumption 4.1 holds. Then, under any linear demand model, the efficiency loss L is piecewise increasing and piecewise convex in b, and is bounded from below as follows*:

$$L \geq \begin{cases} 0.093(b/p^\star)^2 & b \in [0, \hat{b}], \\ 0.115 + 0.199(b - \hat{b})/p^\star & b \in (\hat{b}, \alpha/\beta], \\ 0.407 + 0.222(b - \alpha/\beta)/p^\star & b \in (\alpha/\beta, 2\alpha/\beta]. \end{cases}$$

Furthermore, L is piecewise decreasing and piecewise convex in α, and piecewise increasing and piecewise convex in β.

To understand the result, note first that $p^\star = p^\star_{T-1}(2) = p^\star_T(1)$ exactly corresponds to the revenue-maximizing price charged under ample inventory, i.e., in a setting where stock-outs never occur. Thus, $b/p^\star$ can be interpreted as a normalized measure of *leverage*. The lemma shows that efficiency losses always exist under debt, and that they grow with leverage, in a piecewise and convex fashion. In particular, under low debt ($b \leq \hat{b}$), the losses exhibit a convex growth with a quadratic rate. At intermediate debt ($b \in (\hat{b}, \alpha/\beta]$), the losses always exceed 11.5%, and again grow in a convex fashion, at a rate of roughly 20% per unit of leverage. At high debt (i.e., $b > \alpha/\beta$), losses always exceed 40.7% and grow convexly, at a rate of at least 22.2% per unit of leverage. These large values suggest that losses induced by pricing distortions can be quite substantial, and can grow quickly with leverage. The magnitude is also consistent with that documented in several corporate finance papers that consider borrowers endowed with the ability to alter the expected value and the volatility of their cashflow, as is the case in our model (see Décamps and Djembissi 2007 and our discussion in Section 1.3).

The latter part of the lemma suggests that losses decrease and efficiency increases under a larger WTP, i.e., larger α. This is intuitive, since larger WTP implies that the price increases inherent in the DM's policy have less impact on the probability of selling, and thus on the expected revenue losses; furthermore, such revenue losses matter less *relative* to the optimal revenues, which are also higher under larger α. By a similar mechanism, efficiency losses are exacerbated by a greater price sensitivity, i.e., a larger β, which facilitates the DM's ability to engage in riskier behavior, and makes (even small) price changes have more drastic consequences on the generated revenue.

Figure 2 plots the efficiency loss L for our earlier example as a function of the debt, for different inventory levels. Interestingly, L is piecewise (but not globally) increasing and convex, exhibiting a downward jump when the debt exceeds $\hat{b}$. As before, the root cause for this discontinuity is that the DM relies on two sales for covering a debt $b > \hat{b}$. Consequently, he charges a lower price in period $T-1$, which is *closer* to the revenue-maximizing one, and thus reduces efficiency losses (see Figure 1).

4.1. Dynamics and Downward Efficiency Spiral

We now turn to the dynamic evolution of the pricing distortions and efficiency losses we documented. Our first result characterizes the price evolution under the DM's policy.

Proposition 4.2 (Price Evolution Under Debt). *Let $T = 2$ and suppose that Assumption 4.1 holds. Then, under the DM's policy,*

(i) *under ample inventory, the price increases in expectation over time by an amount that is increasing in the debt, i.e.,* $\mathbb{E}[p_T^{\dagger}(\mathcal{B}_T, \mathcal{Y}_T)] - p_{T-1}^{\dagger}(b, y)$ *is positive and increasing in* b *for* $y \geq 2$;

(ii) *conditional on no sale and under limited inventory, the price decreases over time by an amount that is decreasing in the debt, i.e.,* $p_{T-1}^{\dagger}(b, y) - p_T^{\dagger}(b, y)$ *is positive and decreasing in* b *for* $y = 1$;

(iii) *conditional on no sale and under ample inventory, the price increases over time by an amount that is increasing in the debt, i.e.,* $p_T^{\dagger}(b, y) - p_{T-1}^{\dagger}(b, y)$ *is positive and increasing in* b *for* $y \geq 2$.

Part (i) states that, under ample inventory, the DM tends to mark up prices over time, in expectation. This result is in stark contrast with the classical insights in the dynamic pricing literature, where it is known that, under ample inventory, the revenue-maximizing policy maintains constant prices over time (Gallego and van Ryzin 1994), i.e., $p_{T-1}^{\star}(y) = p_T^{\star}(\mathcal{Y}_T)$ almost surely for $y \geq 2$. Furthermore, our result shows that increased debt leads to higher expected markups, thus accentuating the distortion.

Parts (ii) and (iii) of the result focus on the case when no sale occurs. Under these circumstances, the classical insights in dynamic pricing suggest that a revenue-maximizing policy would always mark down prices. Part (ii) shows that, under limited inventory, the DM's policy would also mark down prices, albeit always at a slower rate than the revenue-maximizing policy, i.e., $p_{T-1}^{\dagger}(b, y) - p_T^{\dagger}(b, y) \leq p_{T-1}^{\star}(y) - p_T^{\star}(y)$. Part (iii), however, shows that, under ample inventory, the DM would actually mark *up* the price, highlighting another substantial departure from the revenue-maximizing policy. As before, increased debt accentuates the effects, leading to lower markdowns or higher markups.

Our results also lead to the following corollary for the case when no sale occurs.

Corollary 4.1. *Let* $T = 2$ *and suppose that Assumption 4.1 holds. Then, conditional on no sale, the difference between the DM's and the revenue-maximizing price increases over time in absolute and relative terms, i.e.,*

$$p_t^{\dagger}(b, y) - p_t^{\star}(y) \quad \text{and} \quad \frac{p_t^{\dagger}(b, y) - p_t^{\star}(y)}{p_t^{\star}(y)}$$

are both increasing in t.

By combining the results of Propositions 4.1, 4.2, and Corollary 4.1, we conclude that debt induces the DM to not only consistently price higher but also to mark down prices at a slower pace or even to mark up compared to what would be (revenue-) optimal. These upward pricing distortions that are compounding over time raise the concern that the efficiency under the DM's policy would also deteriorate over time, an issue to which we turn our attention next.

The evolution of generated revenues and the resulting efficiency losses over time is made subtle by the following two opposing forces. On the one hand, when no sales occur, the DM's pricing policy increasingly deviates from the revenue-maximizing one, per Corollary 4.1. This magnified deviation drives higher efficiency losses. On the other hand, when a sale occurs, the higher (than revenue-optimal) prices posted by the DM would generate more revenues in comparison, driving efficiency losses down. Note that all these future scenaria are accounted for by the efficiency loss L, which aggregates all possible sample paths. As time progresses, however, the conditional efficiency loss $\mathcal{L}_t$ defined in (1) could drift either down or up, depending on the more or less favorable WTP realizations and associated sales. As such, the expected efficiency losses L_t, which aggregate these possible realizations of $\mathcal{L}_t$ at time t, capture the evolution of performance over time. Recalling that $L = L_1$, our next result provides a characterization for this dynamic evolution.

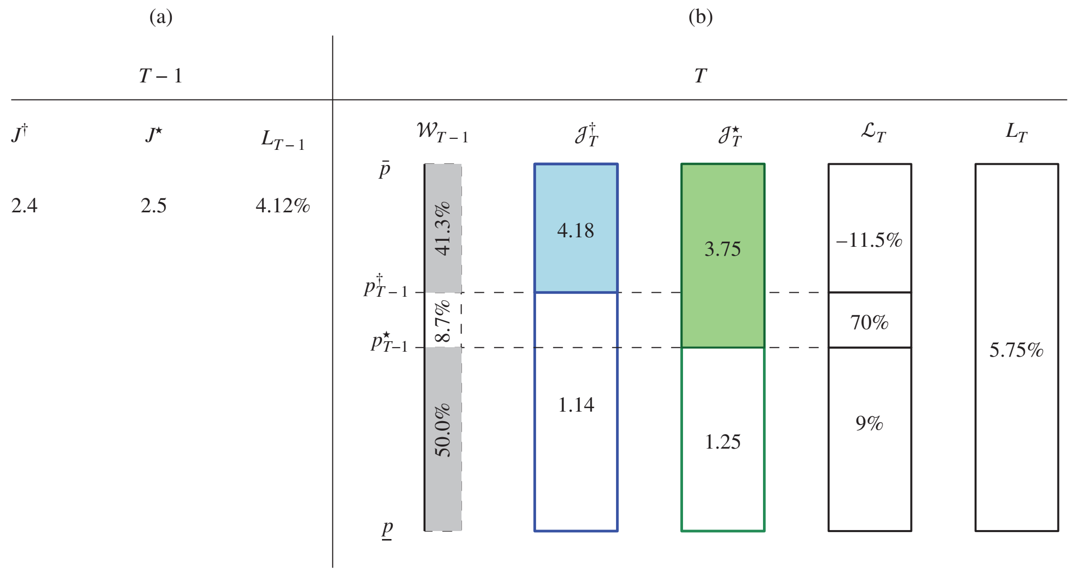
Proposition 4.3 (Downward Efficiency Spiral)**.** *Let* $T = 2$ *and suppose that Assumption 4.1 holds. Then,*

$$L_{T-1} \leq L_T.$$

The result establishes that, in expectation, the pricing distortions inherent in the DM's policy under debt have a compounding effect, leading to a downward spiral in efficiency.

To illustrate this effect, we revisit our earlier example for $\lambda(p) = 1 - 0.2 \times p$, and consider the case in which $B = 1.5$. Figure 3(a) reports the expected revenue at time $T-1$ under the DM's policy ($J^{\dagger} = 2.4$) and the revenue-maximizing policy ($J^{\star} = 2.5$). The resulting efficiency loss is $L_{T-1} = 4.12\%$. Figure 3(b) illustrates how all these quantities could evolve from $T-1$ to T, depending on the realization of the WTP $\mathcal{W}_{T-1}$. In particular, the conditional expected revenue under the DM's policy, $\mathcal{J}_T^{\dagger}$, takes values 4.18 or 1.14, depending on whether the DM sells a unit ($\mathcal{W}_{T-1} \geq p_{T-1}^{\dagger}$, shaded region in the second column) or not ($\mathcal{W}_{T-1} < p_{T-1}^{\dagger}$, nonshaded region in the second column). Note that, since the WTP is uniformly distributed, the size of each region is also proportional to the probability of the associated event. Similarly, the conditional optimal revenue, $\mathcal{J}_T^{\star}$, takes values 3.75 or 1.25, depending on whether $\mathcal{W}_{T-1} \geq p_{T-1}^{\star}$ (shaded region in the third column) or not (nonshaded region in the third column). This gives rise to three possible values for the conditional efficiency loss $\mathcal{L}_T$, depending on the sales under the two different policies, i.e., −11.5% when a sale occurs under both policies, 70% when a sale occurs only under the revenue-maximizing policy, and 9% when no sale occurs under

Figure 3. (Color online) Dynamics of Efficiency Losses



Notes. The efficiency loss L_{T-1} and the conditional and expected loss $\mathcal{L}_T$ and L_T, respectively, for the demand function $\lambda(p) = 1 - 0.2 \times p$. The horizon is $T = 2$, and the DM starts with $Y = 2$ units in inventory and a debt of $B = 1.5$.

either policy. Weighing these three events by their corresponding probabilities yields an expected efficiency loss $L_T = 5.75\%$, which is larger than $L_{T-1} = 4.12\%$.

Figure 3(b) also highlights that the revenue distribution under the DM's policy gets considerably more dispersed compared to the optimal one: the "downside" worsens from 1.25 to 1.14, and the "upside" improves from 3.75 to 4.18.

To the best of our knowledge, these results constitute the first quantification of the *dynamic evolution* of efficiency losses induced by debt. We showed that debt introduces pricing distortions that compound over time, leading to a downward spiral in efficiency. This finding is particularly relevant because it suggests that any mechanism aimed toward reducing this inefficiency must necessarily entail some form of dynamic monitoring of the DM's actions and performance throughout the planning horizon. In Section 6, we discuss some mechanisms and their effectiveness, after first confirming the robustness of our results, and extending them to a multiperiod setting.

5. The Multiperiod Case

We now analyze problems with an arbitrary number of periods, and general demand functions. In Section 5.1, we extend our analytical results to a setting where a single item is sold; in Section 5.2, we explore the case with an arbitrary number of units.

5.1. One Unit Case (Asset Selling)

Consider the special instance of our general model described in Section 2 where the DM is endowed with a single unit of inventory, i.e., $Y = 1$. We make no restrictions on the number of periods, and no longer assume that the demand function is linear. This special case corresponds to a variant of the well-known asset selling problem where the seller posts prices instead of receiving offers, which we extend here by including debt. For an extensive comparison of the two classical models of posted prices and received offers in the absence of debt, see Arnold and Lippman (2001).

In this model, it is sufficient to keep track of whether a sale occurred. However, since we are interested in comparative statics with respect to the debt, we still retain it as part of the state. For $t \le T$, let $V_t(b)$ be the DM's optimal expected payoff at t when the outstanding debt is b and no sale occurred up to t, and let $p_t^\dagger(b)$ be the price he posts. In case a sale occurs, the DM transitions to the terminal period $T+1$, and generates a payoff of $(p_t^\dagger(b) - b)^+$. Otherwise, the DM transitions to the next period $t+1$, without generating any payoff. The Bellman recursion in (2)–(3) can thus be rewritten as follows:

$$V_t(b) = \max_{p \in \mathcal{P}} \{\lambda(p) V_{T+1}(b-p) + (1-\lambda(p)) V_{t+1}(b)\}, \quad t = 1, \ldots, T, \; V_{T+1}(b) = (-b)^+. \tag{4}$$

Accordingly, the DM's price at time t satisfies

$$p_t^\dagger(b) \in \arg\max_{p \in \mathcal{P}} \{\lambda(p) V_{T+1}(b-p) + (1-\lambda(p)) V_{t+1}(b)\}, \quad t = 1, \ldots, T. \tag{5}$$

As before, the DM's price coincides with the revenue-maximizing price in the absence of debt, i.e., $p_t^\dagger(0) = p_t^\star$. For $b \ge \bar{p} = \sup_{p \in \mathcal{P}} p$, the DM's payoff is equal to zero almost surely and so is the maximand in (5), in which case we take $p_t^\dagger(b) = \bar{p}$ for all $t = 1, \ldots, T$ without loss of generality. For $b < \bar{p}$, the DM can generate a positive payoff, provided that he posts a high enough price. In particular, for all $p < b$, $V_{T+1}(b - p) = 0$ and the maximand in (5) is equal to $(1 - \lambda(p))V_{t+1}(b)$, an increasing function in p. Thus, it is never optimal for the DM to post a price lower than the debt.

Lemma 5.1. *The DM's price is always larger than the debt, i.e.,* $p_t^\dagger(b) \geq b$.

Using this property and substituting $V_{T+1}(b - p) = p - b$ for $p \geq b$, we can rewrite (5) equivalently as

$$p_t^\dagger(b) \in \underset{p \in \mathcal{P}, p \geq b}{\arg\max} \{\lambda(p)[p - (b + V_{t+1}(b))]\}, \quad t = 1, \ldots, T, \text{ if } b < \bar{p}.$$

For analytical tractability, we make the following assumptions on the demand function.

Assumption 5.1. *The demand function* λ *is differentiable on* $\mathcal{P}$ *and log-concave.*

This requirement is equivalent to the hazard rate of the WTP distribution being increasing, and is satisfied by many demand functions encountered in the literature, including linear, exponential, normal, generalized linear ($\lambda(p) = (a - bp)^n$, for $a \in [0, 1]$, $b \geq 0$, $n \geq 1$), logit ($\lambda(p) = e^{-bp}/(1 + e^{-bp})$ for $b \geq 0$) or Weibull-distributed WTP, etc.[9]

Our next result further characterizes the DM's pricing policy, and the way it is impacted by debt.

Proposition 5.1 (Asset Selling Under Debt). *Under Assumption 5.1 and for all* $t = 1, \ldots, T$ *and* $b > 0$,

(i) *the DM's price is given by*

$$p_t^\dagger(b) = \begin{cases} \pi(b + V_{t+1}(b)) & b < \bar{p}, \\ \bar{p} & b \geq \bar{p}, \end{cases}$$

where $\pi(x) = \arg\max_{p \in \mathcal{P}} \lambda(p)(p - x)$; *also,* $\pi(x) \geq x$ *and* $0 \leq \pi'(x) \leq 1$;

(ii) *the DM's price is always strictly higher than the revenue-maximizing price, i.e.,* $p_t^\dagger(b) > p_t^\star$;

(iii) *the DM's price is increasing in* b.

This lemma reinforces our earlier conclusions that pricing distortions under debt come in the form of higher prices. More precisely, parts (ii) and (iii) show that the DM always posts prices that are higher than the revenue-maximizing ones, and that the prices increase with the debt. As such, the results are direct counterparts of Proposition 4.1 for the case of multiple periods, and under a more general demand model.

Part (i) of the result provides a more detailed structure of the policy, which is possible when a single unit is being sold. In this case, note that the price is always continuously increasing in the debt level b; this is unlike the case when multiple units are sold, when $p_t^\dagger$ may exhibit a sharp drop as b increases and the DM switches from a strategy relying on a single sale to one relying on multiple sales to cover the debt. Furthermore, it can be seen that $\partial p_t^\dagger / \partial b \leq 1$, implying that extra units of debt would cause subunitary marginal price increases.

We now turn to the dynamics of the DM's pricing policy.

Proposition 5.2 (Price Evolution Under Debt). *Suppose that Assumption 5.1 holds and that* λ *is convex and* $-\lambda'$ *is log-convex. Then, for all* $t = 2, \ldots, T$ *and conditional on no sale to date,*

(i) *the DM marks prices down over time by an amount that is decreasing in the debt, i.e.,* $p_{t-1}^\dagger(b) - p_t^\dagger(b)$ *is positive and decreasing in* b;

(ii) *the DM's policy applies smaller markdowns than the revenue-maximizing policy, in absolute and relative terms, i.e.,* $p_{t-1}^\dagger(b) - p_t^\dagger(b) \leq p_{t-1}^\star - p_t^\star$ *and* $(p_{t-1}^\dagger(b) - p_t^\dagger(b))/p_{t-1}^\dagger(b) \leq (p_{t-1}^\star - p_t^\star)/p_{t-1}^\star$;

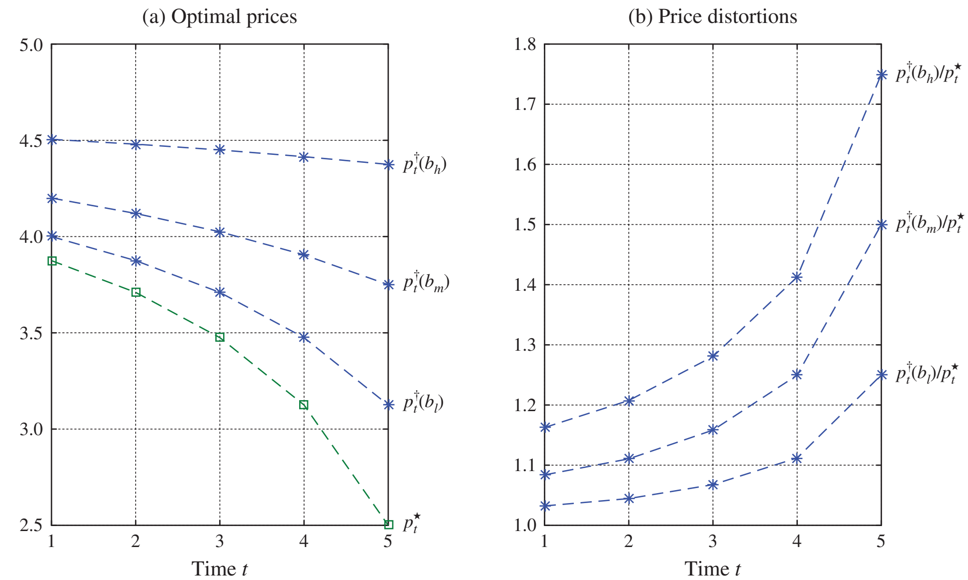
(iii) *the difference between the DM's price and the revenue-maximizing price increases over time, in absolute and relative terms, i.e.,* $p_t^\dagger(b) - p_t^\star$ *and* $(p_t^\dagger(b) - p_t^\star)/p_t^\star$ *are both increasing in* t.

It is well known that the revenue-maximizing policy for the asset selling problem would prescribe a sequence of prices that are decreasing over time (Arnold and Lippman 2001). Proposition 5.2 shows that a debt-facing DM would also apply price markdowns, albeit smaller in magnitude, and decreasing with the debt level. This is in line with Proposition 4.2 obtained for the case of two periods. Furthermore, extending the intuition of Corollary 4.1, part (iii) of the result shows that the pricing distortions consistently increase over time, in an upward spiraling fashion. Note that several of the demand functions satisfying Assumption 5.1, such as linear, exponential, and a Weibull-distributed WTP with shape parameter $k \leq 1$ also satisfy the additional requirement of the lemma.

Figure 4 illustrates the findings of Propositions 5.1 and 5.2 by depicting the prices posted by the DM's policy and the revenue-maximizing policy throughout time for different debt values, under a linear demand function. Specifically, Figure 4(a) shows that under higher debt values, the DM posts higher prices that decrease at a slower rate over time; Figure 4(b) illustrates the compounding distortions. With a debt of $b = p_T^\star$ and five periods to go, the posted price is about 10% higher than the revenue-maximizing price. With two periods to go, the price is 25% higher.

The results in Propositions 5.1 and 5.2 are validated by empirical evidence documented in the literature on

Figure 4. (Color online) (a) Price Evolution for the Revenue-Maximizing Policy and the DM's Optimal Policy Under $\lambda(p) = 1 - 0.2 \times p$ for $T = 5$, and for Different Levels of Debt: "Low" ($b_l = p_T^\star/2$), "Medium" ($b_m = p_T^\star$), and "High" ($b_h = 1.5 \times p_T^\star$); (b) Evolution of Price Distortions Over Time



real estate markets. In particular, Genesove and Mayer (1997) show that homeowners with larger debt loads set higher asking prices for their houses, have them listed for a longer expected time on the market, and receive a higher price upon an eventual sale than owners with smaller debt loads. The documented effects are significant: Compared to an LTV of 80%, an LTV of 100% leads to a list price that is 4% higher, keeping the unit 15% longer on the market, and leading to a sale price that is also 4% higher.

We conclude our analysis of the asset selling problem by studying the efficiency losses and their dynamic evolution. This allows us to generalize our result of compounding efficiency losses in Proposition 4.3 to an arbitrary number of periods. To formally present our result, let $J_t^\dagger$ be the expected revenues under the DM's policy at the beginning of period t, conditional on no sale in periods $1, \ldots, t-1$, i.e.,

$$J_t^\dagger := \mathbb{E}[\mathcal{R}(\mathbf{p}^\dagger) \mid \mathcal{W}_1 < p_1^\dagger(B), \ldots, \mathcal{W}_{t-1} < p_{t-1}^\dagger(B)], \quad t = 1, \ldots, T.$$

Proposition 5.3 (Downward Efficiency Spiral). *Suppose that Assumption 5.1 holds and that $J_t^\dagger/J_{t+1}^\dagger$ is increasing in b for any $t = 1, \ldots, T-1$. Then*

$$L_1 \le L_2 \le \cdots \le L_T.$$

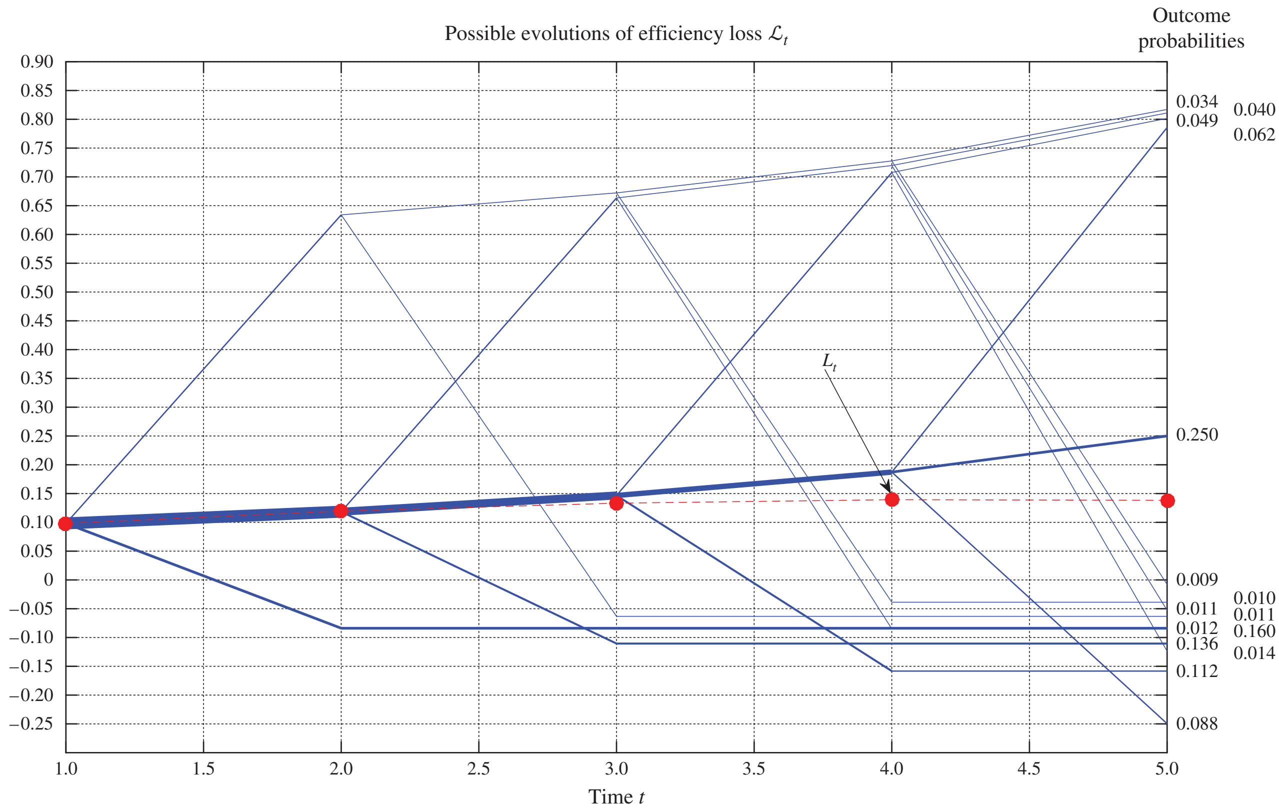
Before commenting on the result, we first note that the monotonicity requirement on $J_t^\dagger/J_{t+1}^\dagger$ is satisfied by any linear or exponential demand function (see Proposition F.1 in Online Appendix F). However, this requirement has a natural interpretation, and we expect it to hold more broadly. To understand this, note that $J_t^\dagger$ ($J_{t+1}^\dagger$) can be interpreted as the revenues that the DM is expected to generate when facing debt b and having $T - t + 1$ ($T - t$) periods to sell the asset. Clearly, $J_t^\dagger$ and $J_{t+1}^\dagger$ are decreasing in b, owing to the increasing pricing distortions that higher debt levels induce. However, when debt levels increase, it is also natural to expect $J_{t+1}^\dagger$ to exhibit a higher relative decrease rate than $J_t^\dagger$, due to the fact that pricing distortions compound across time (or, put differently, a DM with fewer periods to go would distort prices "more"). Thus, it is quite natural to expect $J_t^\dagger/J_{t+1}^\dagger$ to be increasing in b.

As noted in Section 4.1, the evolution of the efficiency loss L_t is made subtle by the higher prices charged by the DM: it decreases or increases depending on whether a sale occurs, respectively. Figure 5 illustrates all possible values of $\mathcal{L}_t$ and the various sample paths that may occur for a specific problem instance. While the figure confirms that the expected efficiency loss L_t is increasing over time, it is also worth observing that the "variability" of $\mathcal{L}_t$ also increases over time, with a wide range of possible outcomes in the last period $t = 5$. Note that $\mathcal{L}_t$ can be negative (whenever the DM successfully sells the unit, which occurs at a higher price than the revenue-maximizing one) but can also be as high as 80% (when the DM does not sell in the first four periods, but would have sold had he applied the revenue-maximizing policy).

5.2. Multiunit Case

Recall from our discussion in Section 4 that a DM endowed with multiple inventory units can follow several strategies for covering the debt, relying on the

Figure 5. (Color online) Efficiency Loss Evolution Under $\lambda(p) = 1 - 0.2 \times p$, for $T = 5$ and $b = p_T^\star$



Notes. Each path in the tree depicts a possible evolution of $\mathcal{L}_t$. In nodes with three edges, the upper edge denotes the event $\{p_t^\star \le \mathcal{W}_t < p_t^\dagger\}$ (when only the revenue-maximizing policy achieves a sale), the middle edge corresponds to the event $\{\mathcal{W}_t < p_t^\star\}$ (when no policy achieves a sale), and the lower node to the event $\{\mathcal{W}_t \ge p_t^\dagger\}$ (when both policies achieve a sale). Conditional on the revenue-maximizing policy having achieved a sale, nodes have two edges, i.e., for the events $\{\mathcal{W}_t < p_t^\dagger\}$ (upper) and $\{\mathcal{W}_t \ge p_t^\dagger\}$ (lower). The width of each edge is proportional to the probability of the corresponding event. The dots on the dashed line depict L_t. The probabilities of all possible outcomes for $\mathcal{L}_5$ are shown on the right.

sale of different numbers of units. This combinatorial feature introduces multimodality in the DM's objective function, which in turn generates discontinuous pricing policies, making a general multiunit, multiperiod setting not amenable for analysis. Consequently, we explore this setting numerically. We find that all our findings persist. For details, we refer the interested reader to Online Appendix B.

6. Managerial and Debt Contract Design Implications

Three main insights that our analysis yields are that (i) under debt, a DM would always tend to charge higher prices than the revenue-maximizing ones; (ii) these price distortions would tend to increase with the debt load; and (iii) these effects would compound over time, generating an upward spiral in prices, and a downward spiral in efficiency. These insights can be used to inform managers and lenders about specific contract terms that could alleviate the inefficiencies we studied.

Early Repayment Option. Offering an early repayment option could be a counterbalancing force for the DM's tendency to charge higher prices. These options, which are routinely included in some forms of lending such as trade credit, typically stipulate a discount for any early repayments, thus encouraging borrowers to repay some portion of the principal to save on interest. In our model, this could induce the DM to charge lower prices in earlier periods, to achieve sales and take advantage of the earlier (reduced) repayment.

Debt Relief. Because a larger debt tends to exacerbate distortions, reducing the debt burden in case of distress could avert further losses. Such reductions in principal and/or interest are informally referred to as "bondholder haircuts" and routinely occur in practice as part of a "debt workout," i.e., a debt restructuring agreement between the creditor(s) and the borrower conducted outside the court system (for more details, see, e.g., Arent Fox 2008 and World Bank Report 2011).

Debt Amortization. To avert the compounding of pricing distortions and efficiency loss, another natural recommendation would be to require debt repayments throughout the entire horizon according to an amortization schedule. This is consistent with real-estate lending practice, where "construction loans that finance multiple units or phases must be structured to ensure that repayment appropriately follows unit sales" (OCC 2013, p. 28). An equivalent mechanism to debt amortization could be the inclusion of financial covenants that require the borrower's revenues to exceed certain thresholds (see Iancu et al. 2017 and chap. 7 of Hilson 2013). The threat of missing a repayment or tripping a covenant could induce the DM to charge lower prices to improve his chances of making sufficient early sales.

We formally model and analyze these three contracting mechanisms in Section 6.1, assessing their potency in alleviating efficiency losses when benchmarked against our plain contract with a single terminal debt repayment. Our findings can be summarized as follows:

$$\begin{aligned}\text{Plain contract} = \text{Early repayment option} \le \text{Debt relief}\\ < \text{Debt amortization} < \text{Optimal.}\end{aligned}$$

A practical implication of these results is that debt contracts would benefit most from including provisions for dynamically monitoring a borrower's sales performance through, e.g., an amortization schedule or financial covenants. Providing debt relief could be another, albeit less effective, means for improving efficiency. Early repayment options, however, appear to be ineffective in this context. Interestingly, although a judicious debt amortization *always* improves performance in our model, it cannot completely restore optimality. In this sense, the result provides a cautious note, as it suggests that to fully alleviate efficiency losses due to pricing distortions, one may require more complex contractual specifications, going beyond simple repayment schedules or financial covenants such as those considered in Iancu et al. (2017). For a more extensive discussion of these findings and the intuition driving them, see Section 6.1.

In conclusion, note that the contracts we considered above were all based on terms commonly encountered in lending agreements. Our work suggests that a natural fix for the price distortions could also be to enforce controls that prevent slow markdowns. However, such controls could be difficult to enforce in practice, due to the inherent contract complexity in specifying contingent pricing policies. Furthermore, prices are often nonverifiable, e.g., when privately communicated to clients, and are also considered operating decisions routinely taken during the course of business, which lenders usually refrain from constraining (DeAngelo et al. 2002).

6.1. Comparing Debt Contract Designs

We formalize the discussion above and now enrich the model with debt contracts that include other terms in addition to the terminal debt repayment. To introduce some notation, we use κ to denote all the terms of a specific contract. Let $J(\kappa)$ be the expected revenues generated under the DM's pricing policy induced by contract κ, and $D(\kappa)$ be the corresponding expected debt repaid, i.e., the debt value. In the context of our model from Section 4, where we analyzed a *plain* contract with a single terminal debt repayment, we have $\kappa = \{B\}$, and $J(\{B\}) = J^{\dagger}$.

An optimal, i.e., efficiency-maximizing, contract κ would seek to maximize $J(\kappa)$. To ensure a meaningful comparison and to be consistent with corporate finance theory, we consider contracts that lead to the same debt value, which we denote with d.[10] The expected revenues generated under the optimal contract are then determined by solving the following optimization problem:

$$\underset{\kappa}{\text{maximize}} \;\; J(\kappa) \tag{6a}$$

$$\text{subject to} \;\; D(\kappa) = d. \tag{6b}$$

Returning again to the class of plain contracts, note that the formulation above would essentially require the debt repayment B to be set so that the debt value equals d. Consequently, there would usually be a single plain contract to consider.

We now analyze the efficiency of optimal contracts within the three classes introduced in this section. As benchmarks, we consider the optimal revenues $J^{\star}$ and the revenues $J^{\dagger}$ under the plain contract. We base the analysis on the model from Section 4 with $T = 2$, $Y = 2$, $\lambda(p) = \alpha - \beta p$, where we take $\alpha = 1$ for simplicity (all our results hold for general α).

To study contracts with an early repayment option, we assume that the DM can make a debt payment x (of his choice) at time $T-1$. In so doing, his terminal debt repayment would be reduced from B to $(B - x/\gamma)^{+}$, for some discount parameter $\gamma \in (0,1]$. Thus, $\kappa = \{B, \gamma\}$ for this class of contracts. Let J^{E} denote the revenues under the optimal contract with an early repayment option. It can be readily seen that for $\gamma = 1$ we recover the plain contract, so that $J^{\dagger} = J(\{B, 1\}) \le J^{E}$.

To capture debt relief, we allow debt holders to dynamically choose whether and by how much to reduce the DM's outstanding debt at the end of period $T-1$, to maximize the expected debt payment made at the end of period T. Thus, $\kappa = \{B, r\}$ for this class, where r is an indicator variable of whether debt relief is allowed. Let J^{R} denote the revenues under the optimal contract with debt relief. Again, $J^{\dagger} = J(\{B, 0\}) \le J^{R}$.

To explore contracts with debt amortization, we assume that instead of facing a single required debt repayment B at the end of period T, the DM faces two required debt repayments, i.e., θB at the end of period $T-1$, and $(1-\theta)B$ at the end of period T, for some parameter $\theta \in [0,1]$.[11] We assume that, upon failing to make the payment θB at $T-1$, the DM loses decision control, and the subsequent price in period T is set to maximize expected revenues. This is reasonable in the context of debt agreements, where missing a loan installment or breaching a covenant can trigger an event of default, in which case a firm's equity holders and debt holders would share decision rights as the firm is undergoing restructuring under Chapter 7 bankruptcy protection (Tirole 2006, Hilson 2013). Thus, $\kappa = \{B, \theta\}$ in this case. Let J^A denote the revenues under the optimal contract with debt amortization. Note that $J^\dagger = J(\{B, 0\}) \leq J^A$. The following result compares the three contracts described above.

Proposition 6.1 (Contract Comparison)**.** *Under the setup described above,*

$$J^\dagger = J^E \leq J^R < J^A < J^\star.$$

Interestingly, according to Proposition 6.1, early repayment discounts *reduce* efficiency, so that the optimal contract in that class actually offers no discount (and becomes a plain contract). To gain some intuition for this result, note that although a discount generally induces the DM to lower the price in period $T-1$, it also results in lower payments to debt holders (due to the discount), which may also occur more often. In fact, the discount may even fail to induce lower prices in period $T-1$, by causing the DM to switch from a less risky strategy relying on two sales to a riskier strategy relying on one to cover the "discounted" debt. In both cases, debt holders would anticipate the lower payments, and increase the debt repayment B, creating a negative feedback loop that ultimately hurts efficiency. This finding is well aligned with practice, where commercial real estate lenders often *discourage* early repayments by including prepayment penalties intended to recover some of the lost interest (White and Kitchen 2016).[12]

By contrast, no negative feedback loop arises when relying on debt relief. The prospect of debt relief lowers the effective debt burden for the DM in period $T-1$, causing him to reduce his price. In turn, this actually *reduces* the need for a debt relief in the first place, and does not decrease the debt value. However, debt relief is not always effective: When the debt burden is low ($B < p_T^\star$), debt holders never find it optimal to trim the debt even when the DM fails to make a sale at $T-1$, rendering the entire mechanism superfluous. This is aligned with findings in corporate finance documenting that the presence of renegotiable debt could actually induce the firm to take up *more* debt and thus engage in *more* risk shifting, thereby increasing inefficiencies (see Flor 2011, Gorton and Kahn 2000).

Finally, debt amortization emerges as the most effective mechanism: A judicious selection of a repayment schedule *always* improves performance, and surpasses debt relief. This confirms that the compounding effects are truly first-order, and are best mitigated by *dynamically* monitoring the borrower's sales performance. An alternative potential explanation is that such mechanisms are qualitatively different from the former two. Early payment discounts and debt relief act as "carrots," which debt holders may have to compensate with an increased debt repayment B. By contrast, by requiring a payment at $T-1$ and threatening to transfer decision control at T, an amortization schedule acts as a "stick," which eliminates the need for a compensating mechanism. However, even the optimal amortization schedule cannot completely restore efficiency:[13] although the revenue-maximizing policy is always followed in period T, the DM continues to charge higher prices than the revenue-maximizing ones at $T-1$ (see the proof of Proposition 6.1). In this sense, the result provides a cautious note, as it suggests that to fully alleviate efficiency losses due to pricing distortions, one may require more complex contractual specifications, going beyond simple repayment schedules or financial covenants such as those considered in Iancu et al. (2017).

7. Limitations and Future Directions

We conclude by highlighting certain limitations of our study, and fruitful directions for future research. To start, our model treated the debt repayment as exogenous. In Online Appendix C, we take a first step toward investigating whether and how endogenizing debt decisions could nuance our insights. Specifically, we consider an extension to our model where before the start of the selling horizon, a cash-constrained DM must decide how to finance the inventory purchase through debt and equity. By endogenizing the DM's and the lender's decisions, we confirm that the main qualitative results derived in Section 4 persist in equilibrium. Confirming our results in an analytical model that endogenizes additional debt contract terms (interest rate, maturity, amortization schedule, financial or borrowing-base covenants Iancu et al. 2017) could be an interesting direction for future research.

We also assumed a nonrecourse loan, i.e., a borrower with zero liability. In Online Appendix A, we consider an alternative specification of our asset selling model in Section 5.1 where a borrower in default faces losses proportional to the shortfall, so that his payoff becomes $\mathbb{E}[(\mathcal{R}(\mathbf{p}) - B)^+] - k\mathbb{E}[(B - \mathcal{R}(\mathbf{p}))^+]$ for some $k \in [0,1]$. We confirm that our main insights are robust: The DM's policy posts prices higher than the revenue-maximizing ones and applies slower markdowns, leading to efficiency losses. As expected, distortions decrease as k increases, and vanish for $k = 1$. However, a new regime emerges in the DM's pricing policy: When debt is sufficiently high, the DM abruptly switches to the revenue-maximizing policy, and continues to follow it for the remaining planning horizon. Qualitatively, the DM acts *as if* he were unable to repay the debt, and thus relies on a strategy that minimizes his losses or equivalently maximizes revenues.[14] Examining the equilibrium lending contracts in such a model (with k as a design parameter) would be an interesting direction for future research. Along the same lines, it would also be relevant to consider settings where the DM could become risk-averse ($k > 1$), such as in privately held companies or when managers have alternative considerations due to, e.g., taxes, risk-aversion, or compensation schemes penalizing losses. Because one could expect prices to be higher or lower than revenue-maximizing ones (Levin et al. 2008), quantifying the efficiency losses could be quite interesting (and challenging).

Our discussion in Section 6 of contract terms that could alleviate inefficiencies could also be enriched. For instance, when the DM fails to achieve sales, we could consider a renegotiation of the loan maturity. Because a DM faced with a longer selling horizon would be less prone to distorting prices, this could alleviate the efficiency losses, in a similar manner to debt relief. An alternative useful lever for deterring the DM from charging higher prices could be a minimum quantity sales requirement. It is known in the marketing literature that salespersons faced with a compensation incentive tied to the total number of units sold would tend to provide customers with discounts, leading to prices that are lower than optimal (see, e.g., Oyer 1998). A quantity sales requirement could thus provide a compensating effect to the revenue requirement induced by debt. This is well aligned with some of the common wisdom in real-estate lending, where lenders financing condominium developments often require a minimum number of units being (pre)sold before releasing their mortgage claim as part of the title conveyance process (OCC 2013, p. 9). Understanding the relative effectiveness of these mechanisms would be an interesting direction for future research.

Finally, the DM may have access to the generated revenues throughout the selling horizon, and could use them for investment (e.g., ordering new items in retail, or repairing/upgrading units in real estate) or for consumption. To capture this, one could consider a more general utility model for the DM, and allow additional inter-temporal decisions involving the revenues. Although such decisions would introduce more opportunities for risk taking, and thus exacerbate the efficiency losses, they could also incentivize the borrower to reduce prices to generate additional revenues earlier, and thus potentially reduce pricing distortions. Understanding how these effects trade off against each other would be a very interesting extension for future research.

Endnotes

[1] Financial covenants are common contingencies in debt contracts, requiring borrowers to maintain certain financial ratios, e.g., a maximum debt-to-equity ratio or a minimum cashflow-to-debt ratio (see Iancu et al. 2017 for a discussion).

[2] Defending the practice, one of the liquidators' senior executives stated: "We have to be economical on our discounts[...] we have commitments to a lot of people–banks, creditors–who are expecting a certain amount of return" (Chang 2009).

[3] An immediate reason is the contract complexity in specifying contingent pricing policies. Prices are also often nonverifiable, e.g., when privately communicated to clients, making such controls hard to enforce in a court of law. Last, such controls would be tantamount to directly dictating operating decisions routinely taken during the course of business, which lenders usually refrain from doing (DeAngelo et al. 2002).

[4] "A prudent development and construction loan policy includes requirements for principal curtailments to ensure periodic remargining if sales [...] fall short of projections" (OCC 2013, p. 28).

[5] To facilitate exposition and avoid degeneracies, we assume that $\lambda(p) > 0 \,\forall\, p \in \operatorname{int}(\mathcal{P})$. To capture demand functions with unbounded support (e.g., exponential), we allow $\bar{p} = \infty$, in which case we have $\mathcal{P} = [\underline{p}, \infty)$.

[6] Our model implicitly assumes that the manager in charge of the firm's operating policy (e.g., pricing) inherits the same fiduciary duty: This assumption is consistent with typical models in corporate finance.

[7] This result follows from Gallego and van Ryzin (1994).

[8] Note that for $b \in (\alpha/\beta, 2(\alpha/\beta))$ we have that $V_{T-1}(b,1) \le (\alpha/\beta - b)^+ = 0$ and hence $V_{T-1}(b,1) - V_{T-1}(b,0) = 0$. In conjunction with (2)–(3), this also implies that $V_{T-1}(b,2) = \max_{0\le p\le \alpha/\beta} \lambda(p) V_T(b-p,1) \ge \lambda(b+\epsilon) \cdot V_T(\epsilon,1) > 0$, for small enough $\epsilon > 0$.

[9] This is exactly the condition discussed in Arnold and Lippman (2001) for an infinite horizon model, which guarantees unimodality of the profit function and natural comparative statics for the pricing decisions.

[10] Requiring a fixed debt value can be thought of as the equilibrium outcome when the lending market is perfectly competitive, in which case lenders determine contract terms so as to recover the time-value of money (Tirole 2006 p. 115). A fixed debt value is also necessary to meaningfully compare contracts: Otherwise, a contract with no terms $\kappa = \{\}$ will trivially emerge as optimal and recover optimal revenues.

[11] This setting could be used to model either loan installments or financial covenants, since both mechanisms are equivalent to a minimum revenue threshold that the borrower must generate by the intermediate time.

[12] "To compensate for early loan repayments, CMBS lenders and most life insurance companies usually require early termination amounts or enforce 'make whole'/'yield maintenance' provisions which attempt to make up the lost revenue [...] expected to make if the loan was repaid on the original term" (White and Kitchen 2016, p. 3).

[13] This also strengthens our assumption concerning the assignment of decision rights in period T upon missing the debt repayment in period $T-1$, since we show that efficiency losses persist even if revenue-maximizing decisions are always followed after a violation.

[14] In fact, since revenue-maximizing prices are lower than the debt here, following this policy actually yields a "self-fulfilling prophecy," and *guarantees* bankruptcy.

References

Archibald T, Thomas L, Betts J, Johnston R (2002) Should start-up companies be cautious? Inventory policies which maximize survival probabilities. *Management Sci.* 48(9):1161–1174.

Arent Fox (2008) Real estate debt restructuring ("work-outs") and foreclosures: Selected tax consequences. Presented by Robert Falb and Robert Honigman, October 15 and 17, Washington, DC. Accessed October 25, 2016, http://tinyurl.com/zaxc4lq.

Arnold M, Lippman S (2001) The analytics of search with posted prices. *Econom. Theory* 17(2):447–466.

Babich V, Sobel M (2004) Pre-IPO operational and financial decisions. *Management Sci.* 50(7):935–948.

Basu AK, Lal R, Srinivasan V, Staelin R (1985) Salesforce compensation plans: An agency theoretic perspective. *Marketing Sci.* 4(4):267–291.

Besbes O, Maglaras C (2012) Dynamic pricing with financial milestones: Feedback-form policies. *Management Sci.* 58(9): 1715–1731.

Board of Governors of the Federal Reserve System (2016) Mortgage debt outstanding. Accessed May 3, http://tinyurl.com/hc4b9qc.

Boyabatlı O, Toktay L (2011) Stochastic capacity investment and flexible versus dedicated technology choice in imperfect capital markets. *Management Sci.* 20(6):2163–2179.

Brander JA, Lewis TR (1986) Oligopoly and financial structure: The limited liability effect. *Amer. Econom. Rev.* 76(5):956–970.

Buzacott A, Zhang Q (2004) Inventory management with asset-based financing. *Management Sci.* 50(9):1274–1292.

Chang A (2009) Price is not right for many shoppers at Circuit City closeout sale. *Los Angeles Times* (January 20), http://tinyurl.com/kta796.

Chemla G, Faure-Grimaud A (2001) Dynamic adverse selection and debt. *Eur. Econom. Rev.* 45(9):1773–1792.

Chen F (2000) Sales-force incentives and inventory management. *Manufacturing Service Oper. Management* 2(2):186–202.

Childs PD, Mauer DC, Ott SH (2005) Interactions of corporate financing and investment decisions: The effects of agency conflicts. *J. Financial Econom.* 76(3):667–690.

Chod J (2017) Inventory, risk shifting, and trade credit. *Management Sci.* 63(10):3207–3225.

Cooper WL, Homem-de Mello T, Kleywegt AJ (2006) Models of the spiral-down effect in revenue management. *Oper. Res.* 54(5): 968–987.

Craig NC, Raman A (2016) Improving store liquidation. *Manufacturing Service Oper. Management* 18(1):89–103.

Dada M, Hu Q (2008) Financing newsvendor inventory. *Oper. Res. Lett.* 36(5):569–573.

DeAngelo H, DeAngelo L, Wruck K (2002) Asset liquidity, debt covenants, and managerial discretion in financial distress: The collapse of L.A. Gear. *J. Financial Econom.* 64(1):3–34.

Décamps J-P, Djembissi B (2007) Switching to a poor business activity: Optimal capital structure, agency costs and covenant rules. *Ann. Finance* 3(3):389–409.

Décamps J-P, Faure-Grimaud A (2002) Excessive continuation and dynamic agency costs of debt. *Eur. Econom. Rev.* 46(9):1623–1644.

Faure-Grimaud A (2000) Product market competition and optimal debt contracts: The limited liability effect revisited. *Eur. Econom. Rev.* 44(10):1823–1840.

Flor C (2011) Asset substitution and debt renegotiation. *J. Bus. Financial Accounting* 38(7–8):915–944.

Foley CF, Raman A, Craig NC (2012) Inventory-based lending industry note. Harvard Business School Background Note 612-057, January 2012 (revised June 2015).

Gallego G, van Ryzin G (1994) Optimal dynamic pricing of inventories with stochastic demand over finite horizons. *Management Sci.* 40(8):999–1020.

Genesove D, Mayer C (1997) Equity and time to sale in the real estate market. *Amer. Econom. Rev.* 87(3):255–269.

Gong X, Chao X, Simchi-Levi D (2014) Dynamic inventory control with limited capital and short-term financing. *Naval Res. Logist.* 61(3):184–201.

Gorton G, Kahn J (2000) The design of bank loan contracts. *Rev. Financial Stud.* 13(2):331–364.

Hilson J (2013) *Asset-Based Lending: A Practical Guide to Secured Financing* (Practicing Law Institute, New York).

Holmstrom B (1979) Moral hazard and observability. *Bell J. Econom.* 10(1):74–91.

Iancu DA, Trichakis N, Tsoukalas G (2017) Is operating flexibility harmful under debt? *Management Sci.* 63(6):1730–1761.

Jensen MC, Meckling WH (1976) Theory of the firm: Managerial behavior, agency costs, and capital structure. *J. Financial Econom.* 3(4):305–360.

Leland H (1998) Agency costs, risk management, and capital structure. *J. Finance* 53(4):1213–1243.

Leland H, Toft K (1996) Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *J. Finance* 51(3):987–1019.

Levin Y, McGill J, Nediak M (2008) Risk in revenue management and dynamic pricing. *Oper. Res.* 56(2):326–343.

Li L, Shubik M, Sobel M (2013) Control of dividends, capital subscriptions, and physical inventories. *Management Sci.* 59(5): 1107–1124.

Manso G (2008) Notes and comments on investment reversibility and agency cost of debt. *Econometrica* 76(2):437–442.

Myers S (1977) Determinants of corporate borrowing. *J. Financial Econom.* 5(2):147–145.

Office of the Comptroller of the Currency (OCC) (2013) *Commercial Real Estate Lending: Comptroller's Handbook* (OCC, U.S. Department of the Treasury, Washington, DC).

Oyer P (1998) Fiscal year ends and nonlinear incentive contracts: The effect on business seasonality. *Quart. J. Econom.* 113(1):149–185.

Pang Z, Berman O, Hu M (2015) Up then down: Bid-price trends in revenue management. *Production Oper. Management* 24(7): 1135–1147.

Perakis G, Roels G (2007) The price of anarchy in supply chains: Quantifying the efficiency of price-only contracts. *Management Sci.* 53(8):1249–1268.

Phillips R (2005) *Pricing and Revenue Optimization* (Stanford Business Books, Stanford, CA).

Porteus EL (1972) Equivalent formulations of the stochastic cash balance problem. *Management Sci.* 19(3):250–253.

Possani E, Thomas LC, Archibald TW (2003) Loans, ordering and shortage costs in start-ups: A dynamic stochastic decision approach. *J. Oper. Res. Soc.* 54(5):539–548.

Sakraida L (2011) 19 out of 25 items increased in price during the Borders liquidation sale. *DealNews* (July 25), http://tinyurl.com/3k3de97.

Sohoni MG, Bassamboo A, Chopra S, Mohan U, Sendil N (2010) Threshold incentives over multiple periods and the sales hockey stick phenomenon. *Naval Res. Logist.* 57(6):503–518.

Stein J (1995) Prices and trading volume in the housing market: A model with down-payment effects. *Quart. J. Econom.* 110(2): 379–406.

Swinney R, Cachon G, Netessine S (2011) Capacity investment timing by start-ups and established firms in new markets. *Management Sci.* 57(4):763–777.

Talluri KT, van Ryzin GJ (2005) *The Theory and Practice of Revenue Management* (Springer Science and Business Media, Inc., New York).

Tirole J (2006) *The Theory of Corporate Finance* (Princeton University Press, Princeton, NJ).

White B, Kitchen D (2016) Recourse vs. nonrecourse: Commercial real estate financing—Which one is right for you? White paper, Wells Fargo. Accessed October 21, http://tinyurl.com/jp4qoyo.

White MC (2016) Why prices at going-out-of-business sales can be rip-offs. *Time* (March 9), http://tinyurl.com/h9otufv.

World Bank Report (2011) Out-of-court debt restructuring. World Bank Study. Accessed October 20, 2016, http://tinyurl.com/h4soxd5.

Xu X, Birge J (2004) Joint production and financing decisions: Modeling and analysis. Working paper, University of Chicago, Chicago.