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Planar Entry Flow of Low Viscosity Elastic Fluids in Micro-Fabricated Geometries

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ABSTRACT

The non-Newtonian flow of dilute aqueous polyethylene oxide (PEO) solutions through microfabricated planar abrupt contraction-expansions is investigated. The contraction geometries are fabricated from a high-resolution chrome mask and cross-linked PDMS gels using the tools of soft-lithography. The small length scales and high deformation rates in the contraction flow regions lead to significant extensional flow effects even with dilute polymer solutions having time constants on the order of milliseconds. The dimensionless extra pressure drop across the contraction increases by more than 200% and is accompanied by significant upstream vortex growth. Streak photography and video-microscopy using epifluorescent particles shows that the flow ultimately becomes unstable and three-dimensional. The moderate Reynolds numbers ($0.03 \leq Re \leq 44$) associated with these high Deborah number ($0 \leq De \leq 600$) microfluidic flows results in the exploration of new regions of the Re-De parameter space in which both the effects of both elasticity and inertia can be observed. Understanding such interactions will be increasingly important in microfluidic applications involving complex fluids and can best be interpreted in terms of the Elasticity number, El = De/Re.

INTRODUCTION

The small lengthscales and high deformation rates achievable in microfabricated devices enable the generation of strong viscoelastic effects even in dilute aqueous polymer solutions. This is particularly important in the development of lab-on-a-chip and inkjet printing devices, which typically utilize non-Newtonian fluids. In traditional macro-scale experiments, the enhanced elastic effects (vortex growth) associated with the flow of dilute polymer solutions through abrupt contraction-expansions have relied on a high solvent viscosity to attain long relaxation times and maintain low Reynolds (Re) numbers [1]. The dependence of the elasticity number on the inverse of the length-scale of the geometry makes it possible to attain high Deborah (De) and elasticity numbers using semi-dilute aqueous polymer solutions in microfluidic devices; conditions not typically accessible in the equivalent macro-scale experiment. The use of small length-scales to enhance non-linear rheological effects of the fluid has already been demonstrated in a microfluidic flow rectifier [2], which makes use of the additional and irreversible pressure drop of dilute aqueous polymer solutions flowing through a saw-tooth geometry at low Re.

In the present work, we investigate the flow of dilute and semi-dilute polyethylene oxide (PEO) solutions through micro-fabricated planar abrupt contraction-expansions. Experiments are performed over a range of Deborah and Reynolds numbers ($0.27 < De < 1020$, $0.03 < Re < 44$), in a region of Re-De parameter space that has previously been unexplored. We investigate the competing effects of inertia and elasticity on vortex growth and the structure of flow instabilities in the micro-contraction flows. Video-microscopy and streak imaging with 1.1 micron epifluorescent seed particles are used to characterize the steady flow kinematics upstream of the contraction for both polyethylene oxide solutions and a Newtonian fluid.

EXPERIMENTAL

Channel Geometry and Fabrication

Figure 1 contains images of the 16:1:16 planar abrupt contraction-expansion and an 8:1:8

hyperbolic contraction- abrupt expansion, both with upstream channel widths of 400 microns. (Figures 1(a) and (b)). All channels were fabricated in polydimethylsiloxane (PDMS) using soft-lithography and SU-8 molds fabricated using stardard photo-lithographic procedures. The use of a high-resolution chrome mask permitted the fabrication of sharp features at the contraction entrance and near-vertical channel walls along the entire length of the channel as illustrated in Figures 1(c) and (d).

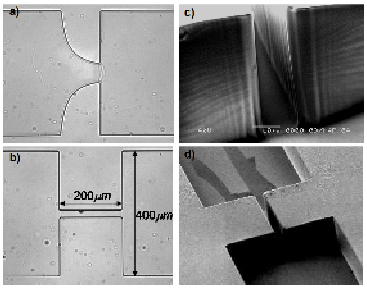

Figure 1. Transmission microscopy images of PDMS abrupt planar contraction-expansions; (a) hyperbolic contraction-abrupt expansion 400:50:400 (Hencky strain=1) and (b) 16:1:16 (400:25:400μm) and SEM images of a 16:1:16 contraction expansion (c) and (d)

Pressure taps were integrated into the device at axial locations 6 mm upstream and downstream of the contraction, in order to measure the differential pressure drop, ΔP_{12} as a function of flowrate for each of the polymer solutions and the Newtonian fluid. Inlet and outlet ports for connection to a syringe pump ($0.1 \le Q \le 12$ml/hr) were located a further 6 mm upstream and downstream of the pressure taps.

Fluid Rheology

Two aqueous solutions containing 0.10% and 0.30% polyethylene oxide (PEO; 2×10^6 g/mol) and a 55% wt. glycerol/water solution containing 0.10% PEO were used in this work. For brevity, we denote these compositions by 0.1% and 0.3%, respectively. The rheological properties of these solutions are given in Table 1. Relaxation times were measured using capillary breakup extensional rheometry [3], and the steady shear viscosities were determined using a controlled stress rheometer (AR2000) with a double gap Couette cell at shear rates, $0.1 \le \dot{\gamma} \le 1000$ s^{-1}. All three solutions were found to be slightly shear-thinning at high deformation rates, exhibiting a power law behaviour.

Table 1: Fluid Rheology

Fluid Property (at 20°C)	0.1% PEO in water	0.1% PEO in 55% wt. glycerol/water	0.3% PEO in water
Relaxation time, λ (ms)	1.5	23	4.4
Zero-shear viscosity, η_0 (mPa.s)	2.3	18.2	8.3
Density, ρ (g/cm^3)	0.996	1.085	0.989
c/c^*	1.34	1.34	4.01

Dimensionless Parameters

Non-Newtonian flow of each of the solutions through the micro-scale geometries is characterized by two dimensionless quantities, Deborah number (De) and Reynolds number (Re) which are defined according to Equations 1-3. The Deborah number is defined in terms of the average shear rate, $\dot{\gamma}$ in the contraction throat:

$$\mathrm{De} = \lambda\dot{\gamma} = \frac{\lambda\bar{v}_\mathrm{c}}{w_\mathrm{c}/2} \qquad (1)$$

where λ is the relaxation time of the fluid, $\bar{v}_\mathrm{c} = Q/(w_\mathrm{c}h)$ is the average velocity, w_c is the contraction width, h is the depth of the channel and Q is the volumetric flowrate. The Reynolds number is

$$\mathrm{Re} = \frac{\rho\bar{v}_\mathrm{c}D_\mathrm{h}}{\eta_0} = \frac{2\rho Q}{(w_\mathrm{c}+h)\eta} \qquad (2)$$

where ρ is the fluid density, η_0 is the zero shear viscosity and the hydraulic diameter is given by $D_\mathrm{h}=2w_\mathrm{c}h/(w_\mathrm{c}+h)$, where h is the height of the channel. Finally, the Elasticity number is

$$\mathrm{El} = \frac{\mathrm{De}}{\mathrm{Re}} = \frac{\lambda\eta}{\rho w_\mathrm{c}D_\mathrm{h}} \qquad (3)$$

The Elasticity number, El is dependent on only fluid properties and the inverse square of the length-scale of the channel and is constant for a given fluid and geometry. In these experiments, the Elasticity numbers are El = 9 and El = 90 for the 0.1% and 0.3% PEO solutions, respectively.

The dimensionless vortex length, $\chi = L_\mathrm{v}/w_1$ is defined according to the convention in previous macro-scale entry flow experiments [1], to quantify the axial distance upstream from the contraction plane, at which the primary flow first detaches from the channel wall. The differential pressure, ΔP_{12} is normalized by the linear slope of the pressure drop/flowrate curve that is observed

in all experiments at low De, such that $\Delta P(\mathrm{Re},\mathrm{De}) = \Delta P_{12}/(mQ)$, where $m=\Delta P_{12}/dQ$ when $Q \rightarrow 0$.

Flow Visualisation

All fluids are seeded with 1μm fluorescent latex particles (Molecular Probes, Ex/Em 520/580nm) at a concentration of 0.02% wt. The evolution in steady state kinematics upstream of the contraction is shown qualitatively with fluorescent streak imaging (Figs. 2 – 3), using a high resolution (2184 x 1472 px) single frame camera and exposure times of 30ms. The transient behaviour in start-up flow is also captured using video rate (29.97 fps) fluorescent microscopy. Flows were visualized for all 3 solutions over the range of operating conditions given in Table 2.

Table 2: Experimental operating space

	0.1%	0.3%	Water
De	0.27 – 373	19.6 – 548	0
Re	0.03 – 44	0.22 – 6	3.7 – 59
El	9	90	0
Shear rate (s^{-1})	200 – 2.5x10^5	4000 – 1.25x10^5	8900 – 1.4x10^5

Figures 2 and 3 illustrate the effect of inertia on the development of elastic flow instabilities in the 16:1 planar contraction for the 0.1% PEO (a-c) and 0.3% PEO solutions (d-f).

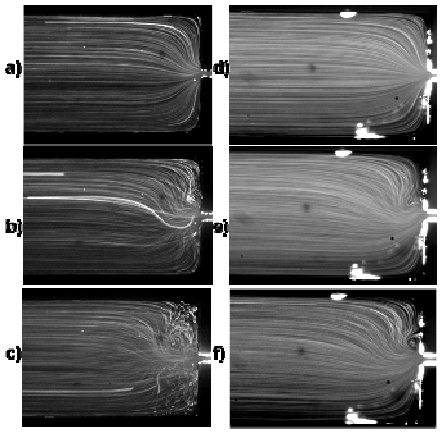

Figure 2. Streak images of flow upstream of a 400:25 μm contraction for 0.1% PEO in water at (a)* Re=*4.7,* De=*40 (b)* Re=*7.8,* De=*67 (c)* Re=*9.4,* De=*80 and 0.3% PEO in water at (d)* Re=*0.44,* De=*39, (f)* Re=*0.66,* De=*59, and (g)* Re=*0.87,* De=*78.

For all adjacent image pairs in Figures 2 and 3, the structures of these instabilities are comparable for the same De although their stability is substantially reduced by the 10-fold increase in Re associated with the 0.1% PEO solution.

Newtonian-like flow behaviour is observed for both solutions for De < 27, after which point elastic instabilities (Figs. 2 (b-c)) develop into an enhanced vortex growth regime for Deborah numbers 40 < De < 107. This regime is unstable for the 0.1% PEO solution, such that vortices are continually forming and collapsing, and the dominant vortex alternates between 'top' and 'bottom' positions. At higher De, diverging flow and substantial elastic vortex growth upstream are observed. Figure 3 represents a 'semi-stable' vortex regime, in which the mean size and location of both top and bottom vortices is constant in time although the local structure is slightly oscillatory in nature. This transient behaviour is observed primarily for the 0.1% PEO solution. Although more stable in time, the 0.3% PEO displays asymmetric vortex growth which is believed to be a result of imperfections in the channel geometry. Flow symmetry for the 0.3% PEO solution is restored at De > 313, which corresponds to approximately the same flowrate at which point the flow structures of the 0.1% PEO solution became symmetric again (De >120).

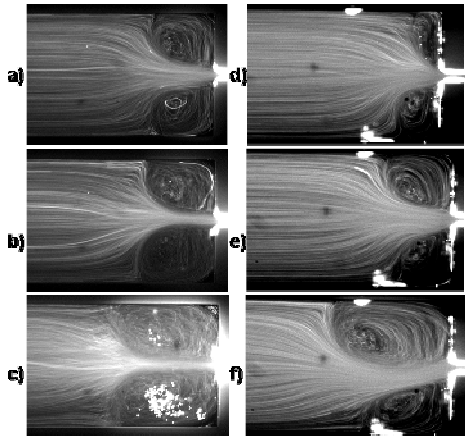

Figure 3. Streak images of flow upstream of a 400:25μm abrupt contraction for 0.1% PEO at (a)* Re=*14,* De=*120, (b)* Re=*19,* De=*160, (c)* Re=*28,* De=*240 and 0.3% PEO at (d)* Re=*1.3,* De=*117, (e)* Re=*18,* De=*156, (f)* Re=*2.6,* De=*235.

Pressure Drop and Vortex Growth

The enhanced pressure drop associated with the increased extensional viscosity of the PEO solutions at high extension rates is illustrated in

Figure 4. Pressure measurements were taken over a range of flow conditions ($0.3 < Re < 11.7$) and $2.7 < De < 215$), such that measurements within the vortex growth regime were obtained for the 0.3% PEO solution only. The onset of a substantial excess pressure drop for the 0.1% PEO solution at $Re \approx 3$, correlates well with the onset of elastic instabilities in Figure 3.

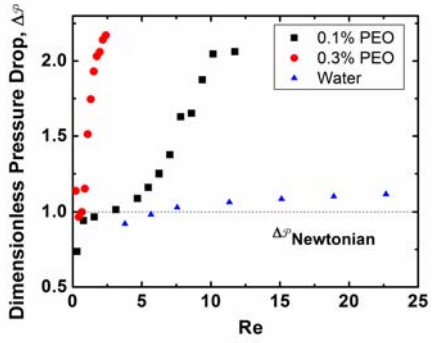

Figure 4. Dimensionless pressure drop as a function of Reynolds number for the 0.1% and 0.3% PEO solutions and for water.

The corresponding lengths of the upstream vortex size for both fluids are also shown in Fig. 5. Regardless of the temporal instability and asymmetry of the top and bottom vortices, their growth is almost linearly dependent on Deborah number, for both solutions. The limited range of pressure measurements ($De < 100$, for 0.1% PEO and $De < 200$ for 0.3% PEO) correspond to only a small section of the vortex growth data shown in Figure 5.

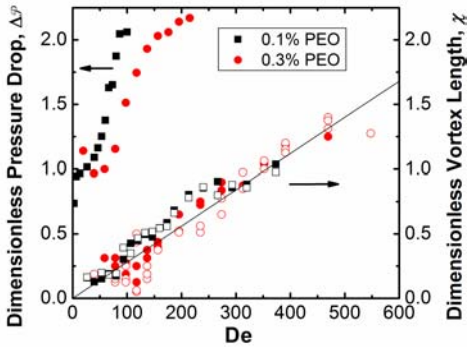

Figure 5. Dimensionless pressure drop and vortex length as a function of Deborah number for 0.1% and 0.3% PEO in water; lower vortex (hollow symbols) and upper vortex (solid symbols)

The linear growth in vortex length for $100 < De < 500$ for both solutions suggest that further additional pressure measurements beyond De=100 and De=200 should show continued increases in a similar fashion. Excess dimensionless pressure drops in flow regimes beyond the unstable vortex regime are yet to be explored.

SUMMARY AND CONCLUSIONS

This work illustrates the evolution of non-Newtonian flow phenomena that arises in micro-fabricated planar abrupt contraction-expansions. The high deformation rates and small length-scales inherent to microfluidic devices make it possible to generate very high Elasticity numbers and Deborah numbers for dilute aqueous polymer solutions. The range of Deborah numbers presented ($0 < De < 1020$) are far in excess of those typically attainable in the equivalent macroscale experiment ($De < 150$). Vortex growth was observed, but appears to be a fairly unstable process, as a result of moderate Reynolds numbers ($0.03 < Re < 44$). A substantial excess pressure drop is observed for both the 0.1% and 0.3% PEO solutions. The linear response of the corresponding pressure drop in a Newtonian fluid at the same Reynolds numbers indicates that the enhanced pressure drop is an effect of strong viscoelasticity rather than being inertial in origin. The results presented here illustrate the interaction of both elasticity and inertia. They correspond to a region of parameter space associated with high Deborah numbers and moderate Reynolds numbers, which has been previously unexplored in contraction flow experiments.

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