Homepage of Dr. Eugene Guth, Theoretical and Experimental Physicist

About the Webmaster

Eugene Guth Homepage

Pictures of Eugene Guth & his family

Additional Pictures of Eugene Guth

University of Miami Lecture on History of Physics

Notes on Conversations with Albert Einstein (forthcoming)

The Kinetic Theory of Rubber Elasticity
1934 Seminal Article (forthcoming)

The James and Guth Network Model
of Rubber Elasticity (forthcoming)

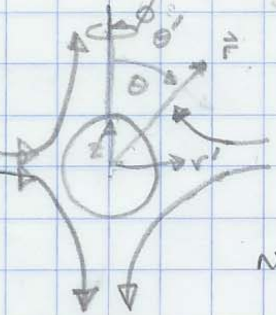
EUGENE GUTH, Ph.D., (1905-1990)
Oak Ridge National Laboratory Physicist
Research Professor of Physics
send webmaster e-mail
(E-mail is best method of contact).
116 Oklahoma Ave.
Oak Ridge, TN
37830-8604
Phone: (865) 483-8309

Professor Eugene Guth

Biographical Data

Born in Budapest, Hungary, Aug. 21, 1905; naturalized 1942 (American), married 1947, wife's maiden name, Roma Claire Lynch, 4 children. Ph.D. (Theoretical Physics), University of Vienna 1928. Research Associate, University of Vienna, 1928-30. Post-doc Research Associate (with Wolfgang Pauli), Austrian-German Science Foundation Federal Institute of Technology (ETH) Zurich, and University of Leipsig, (with Werner Heisenberg) 1930-31. Professor, University of Vienna, 1932-37. Faculty, University of Notre Dame 1937-41; Professor 1941-45; Research Professor 1945-55. Director, Polymer Physics Laboratory, University of Notre Dame, 1941-55; Director, Office of Rubber Research Project, 1943-44; Office of Naval Research Projects (Polymers and Theoretical Physics), 1946-55. Technical Adviser to the Director, Oak Ridge National Laboratory, 1956-71. Visiting Professor of Physics, Rice University, 1971-72. Part-time Prof. of Physics and Astronomy, University of Tennessee, 1968-90. Died July 5, 1990.

If students reflect the mark of the teacher, then students do him great credit. Four have been Physics Department Chairmen: C.J. Mullin (University of Notre Dame), L. V. Holroyd (University of Missouri); L. S. Dart (Claremont College); P. Urban (University of Graz, Austria) who also was editor of *Acta Physica Austriaca* and Director of the world-renowned Schladming Winter School on nuclear and elementary particle physics. Also F. E. Dart (University of Oregon); R. L. Sells (Geneseo State University); D. G. Ivey (University of Toronto); M. L. Wiedenbeck (University of Michigan); W. B. Thompson (University of California at San Diego) formerly first professor of plasma physics at Oxford University, England, have all been students or research associates of Guth, as was J. A. Thie, who wrote two books on reactor physics; R. S. Codrington (section head at Vairan's); J.R. Feldmyer (Director of Franklin Institute Laboratories); J. F.



•cylindrical

$v_{r'} = -\frac{1}{2}\dot{\varepsilon} r'$

$v_{\theta'} = 0$

$v_{z'} = \dot{\varepsilon} z'$

•spherical

$r' = r\sin\theta$

$z' = r\cos\theta$

$\theta' = \phi$

Note: $v_z = -\frac{1}{r}\frac{\partial\psi}{\partial r}$ $\quad v_r = \frac{1}{r}\frac{\partial\psi}{\partial z}$

$\psi \to -\frac{1}{2} r^2 z \dot{\varepsilon}$ $\qquad \psi \to -\frac{1}{2} r^3 \dot{\varepsilon} \sin^2\theta\cos\theta$

spherical - axis symmetrical no ϕ dependence

$$v_r = -\frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta} = \frac{-1}{\cancel{r^2\sin\theta}}\left(-\frac{1}{2} r^{\cancel{3}}\dot{\varepsilon}\right)\left(2\cancel{\sin\theta}\cos^2\theta - \sin^{\cancel{3}\,2}\theta\right) = \boxed{\frac{1}{2}\dot{\varepsilon} r(2\cos^2\theta - \sin^2\theta) = v_r}$$

$$v_\theta = \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r} = \frac{1}{\cancel{r\sin\theta}}\left(-\frac{3}{2} r^{\cancel{2}}\dot{\varepsilon}\sin^2\theta\cos\theta\right) = \boxed{-\frac{3}{2}\dot{\varepsilon} r\sin\theta\cos\theta = v_\theta}$$

$\boxed{v_\phi = 0}$

constant

$E^4\psi = 0$ where $\psi = f(r)\dot{\varepsilon}\sin^2\theta\cos\theta$

$$E^2\psi = \frac{\partial^2\psi}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\psi}{\partial\theta}\right)$$

$$E^2\psi = f'' + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\underbrace{\left(2\cos^2\theta - \sin^2\theta\right)}_{-6\sin\theta\cos\theta} \qquad E^2\psi = \left(\frac{\partial^2\psi}{\partial r^2} - \frac{6}{r^2}\right)$$

$$\Rightarrow E^4\psi = \left(\frac{\partial^2}{\partial r^2} - \frac{6}{r^2}\right)\left(\frac{\partial^2\psi}{\partial r^2} - \frac{6}{r^2}\right) f = 0$$

$$\frac{\partial^4 f}{\partial r^4} - \frac{12}{r^2}\frac{\partial^2 f}{\partial r^2} + \frac{24}{r^3}\frac{\partial f}{\partial r} = 0$$

$f = r^n$

$$n(n-1)(n-2)(n-3)\cancel{r^{n-4}} - 12n(n-1)\cancel{r^{n-4}} + 24n\cancel{r^{n-4}} = 0$$

$$n\left[(n-1)(n-2)(n-3) - 12(n-3)\right]$$

$$n(n-3)\left[n^2 - 3n - 10\right] = 0$$

$$n(n-3)(n-5)(n+2) = 0 \Rightarrow n = -2, 0, 3, 5$$

$$f(r) = \frac{A_1}{r^2} + A_2 + A_3 r^3 + A_4 r^5$$

$$f(r \to \infty) = -\frac{1}{2} r^3 \qquad \Rightarrow A_4 = 0$$
$$\Rightarrow A_3 = -\frac{1}{2}$$

B.Cs @ $r = a$

$$f(a) = 0 = \frac{A_1}{a^2} + A_2 - \frac{1}{2} a^3 \qquad \Rightarrow \quad A_2 = \frac{1}{2} a^3 + \frac{3}{4} a^3 = \frac{5}{4} a^3$$

$$f'(a) = -\frac{2A_1}{a^3} + 3 A_3 a^2 = 0$$

$$A_1 = \frac{3}{2} A_3 a^5 = -\frac{3}{4} a^5 = A_1$$

$$f(r) = -\frac{3}{4}\frac{a^5}{r^2} + \frac{5a^3}{4} - \frac{1}{2} r^3$$

$$\Rightarrow \psi(r,\theta) = \left(-\frac{3}{4}\frac{a^5}{r^2} + \frac{5a^3}{4} - \frac{1}{2} r^3\right)\dot{\varepsilon}\sin^2\theta\cos\theta$$

$$v_r = \dot{\varepsilon}\left(2\cos^2\theta - \sin^2\theta\right)\left(\frac{3}{4}\frac{a^5}{r^4} - \frac{5a^3}{4r^2} + \frac{r}{2}\right)$$

$$v_\theta = \dot{\varepsilon}\sin\theta\cos\theta\left(\frac{3}{2}\frac{a^5}{r^4} - \frac{3}{2} r\right)$$

$$v_\phi = 0$$

$$\frac{\partial v_r}{\partial \theta} = \dot{\varepsilon}\sin\theta\cos\theta\left(-\frac{9}{2}\frac{a^5}{r^4} + \frac{15}{2}\frac{a^3}{r^2} - 3r\right)$$

continuity: $\nabla \cdot \underline{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(v_\theta \sin\theta\right) = 0$

$$\dot{\varepsilon}\left(2\cos^2\theta - \sin^2\theta\right)\left[\frac{3}{2}\frac{a^5}{r^5} - \cancel{\frac{5}{2}\frac{a^3}{r^3}} + 1 - 3\frac{a^5}{r^5} + \cancel{\frac{5}{2}\frac{a^3}{r^3}} + \frac{1}{2}\right]$$

$$\dot{\varepsilon}\left(\cos^2\theta - \sin^2\theta\right)\left(\frac{3}{2}\frac{a^5}{r^5} - \frac{3}{2}\right) + \dot{\varepsilon}\cos^2\theta\left(\frac{3}{2}\frac{a^5}{r^5} - \frac{3}{2}\right) = 0 \ \checkmark$$

continuity satisfied

$$\dot{\gamma}_{rr} = 2\frac{\partial v_r}{\partial r} \qquad \dot{\gamma}_{r\theta} = r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}$$

$$\dot{\gamma}_{\theta\theta} = 2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) \qquad \dot{\gamma}_{\theta\phi} = \cancel{\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}} = 0$$

$$\dot{\gamma}_{\phi\phi} = 2\left(\frac{v_r}{r} + \frac{v_\theta \cot\theta}{r}\right) \qquad \dot{\gamma}_{r\phi} = \cancel{\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)} = 0$$

$$\dot{\gamma}_{r\theta} = \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} + \frac{1}{r}\frac{\partial v_r}{\partial \theta}$$

$$\dot{\gamma}_{r\theta} = \dot{\varepsilon}\sin\theta\cos\theta\left(-\frac{6a^5}{r^5} - \cancel{\frac{3}{2}} - \frac{3a^5}{2r^5} + \cancel{\frac{3}{2}} - \frac{9a^5}{2r^5} + \frac{15a^3}{2r^3} - 3\right)$$

$$\boxed{\dot{\gamma}_{r\theta} = \dot{\varepsilon}\sin\theta\cos\theta\left(-12\frac{a^5}{r^5} + \frac{15}{2}\frac{a^3}{r^3} - 3\right)}$$

$$\boxed{\dot{\gamma}_{rr} = \dot{\varepsilon}(2\cos^2\theta - \sin^2\theta)\left(-6\frac{a^5}{r^5} + 5\frac{a^3}{r^3} + 1\right)}$$

$$\dot{\gamma}_{\theta\theta} = \dot{\varepsilon}(\cos^2\theta - \sin^2\theta)\left(3\frac{a^5}{r^5} - 3\right) + \dot{\varepsilon}(2\cos^2\theta - \sin^2\theta)\left(\frac{3a^5}{2r^5} - \frac{5a^3}{2r^3} + 1\right)$$

$$\boxed{\dot{\gamma}_{\theta\theta} = \dot{\varepsilon}\left[\cos^2\theta\left(\frac{6a^5}{r^5} - \frac{5a^3}{r^3} - 1\right) - \sin^2\theta\left(\frac{9a^5}{2r^5} - \frac{5a^3}{2r^3} - 2\right)\right]}$$

$$\dot{\gamma}_{\phi\phi} = \dot{\varepsilon}(2\cos^2\theta - \sin^2\theta)\left(\frac{3a^5}{2r^5} - \frac{5a^3}{2r^3} + 1\right) + \dot{\varepsilon}\cos^2\theta\left(\frac{3a^5}{r^5} - 3\right)$$

$$\boxed{\dot{\gamma}_{\phi\phi} = \dot{\varepsilon}\left[\cos^2\theta\left(\frac{6a^5}{r^5} - \frac{5a^3}{r^3} - 1\right) - \sin^2\theta\left(\frac{3a^5}{2r^5} - \frac{5a^3}{2r^3} + 1\right)\right]}$$

$$\frac{\partial p}{\partial r} = -\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\tau_{\theta r}\sin\theta) - \frac{(\tau_{\theta\theta} + \tau_{\phi\phi})}{r}\right]$$

$$\frac{\partial p}{\partial \theta} = -\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^3\tau_{r\theta}) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\tau_{\theta\theta}\sin\theta) - \tau_{\phi\phi}\cot\theta\right]$$

$$\frac{\partial p}{\partial r} = -\mu\left[\frac{\partial\dot\gamma_{rr}}{\partial r} + \frac{2\dot\gamma_{rr}}{r} + \frac{1}{r}\frac{\partial\dot\gamma_{\theta r}}{\partial\theta} + \frac{\dot\gamma_{\theta r}\cos\theta}{r\sin\theta} - \left(\frac{\dot\gamma_{\theta\theta}+\dot\gamma_{\phi\phi}}{r}\right)\right]$$

$$\frac{\partial p}{\partial r} = -\mu\Big[\dot\varepsilon(2\cos^2\theta-\sin^2\theta)\left(\cancel{\frac{30a^5}{r^6}} - \cancel{\frac{15a^3}{r^4}} - \cancel{\frac{12a^5}{r^6}} + \cancel{\frac{10a^3}{r^4}} + \frac{2}{r}\right)$$

$$+ \dot\varepsilon(2\cos^2\theta-\sin^2\theta)\left(\cancel{-\frac{12a^5}{r^6}} + \cancel{\frac{15a^3}{2r^4}} - \frac{3}{r}\right)$$

$$\cancel{+\dot\varepsilon\cos^2\theta\left(-\frac{12a^5}{r^6} + \frac{15a^3}{2r^4} - \frac{3}{r}\right)}$$

$$- \dot\varepsilon\left[2\cos^2\theta\left(\cancel{\frac{6a^5}{r^6}} - \frac{5a^3}{r^4} - \frac{1}{r}\right) - \sin^2\theta\left(\cancel{\frac{6a^5}{r^6}} - \frac{5a^3}{r^4} - \frac{1}{r}\right)\right]\Big]$$

$$\frac{\partial p}{\partial r} = -\mu\dot\varepsilon(2\cos^2\theta-\sin^2\theta)\left(\frac{15a^3}{2r^4}\right)$$

$$\Rightarrow \boxed{\frac{\partial p}{\partial r} = -\frac{15a^3\mu\dot\varepsilon}{2r^4}(2\cos^2\theta-\sin^2\theta)} \underbrace{-\rho g\cos\theta}_{\text{gravity}}$$

$$\frac{\partial p}{\partial \theta} = -\mu\left[r\frac{\partial\dot\gamma_{r\theta}}{\partial r} + 3\dot\gamma_{r\theta} + \frac{\partial\dot\gamma_{\theta\theta}}{\partial\theta} + \frac{\cos\theta}{\sin\theta}(\dot\gamma_{\theta\theta} - \dot\gamma_{\phi\phi})\right]$$

$$\frac{\partial p}{\partial \theta} = -\mu\Big[\dot\varepsilon\sin\theta\cos\theta\left(\cancel{\frac{60a^5}{r^5}} - \cancel{\frac{45a^3}{2r^3}} - \cancel{\frac{36a^5}{r^5}} + \cancel{\frac{45a^3}{2r^3}} - \cancel{9}\right)$$

$$-2\dot\varepsilon\cos\theta\sin\theta\left(\frac{21a^5}{\cancel{2}r^5} - \frac{15a^3}{2r^3} - \cancel{3}\right)$$

$$+ \cancel{\dot\varepsilon\frac{\cos^3\theta}{\sin\theta}}(0) - \dot\varepsilon\sin\theta\cos\theta\left(\cancel{\frac{3a^5}{r^5}} - \cancel{3}\right)\Big]$$

$$\boxed{\frac{\partial p}{\partial \theta} = -\frac{15a^3\mu\dot\varepsilon}{r^3}\sin\theta\cos\theta} + \underbrace{\rho g\sin\theta}_{\text{gravity}}$$

$$\boxed{p = \frac{15a^3}{6r^3}\mu\dot\varepsilon(2\cos^2\theta-\sin^2\theta) + p_0} \underbrace{-\rho g r\cos\theta}_{\text{gravity}}$$

$$\Pi_{zz} - p_0 = p + \tau_{zz} - p_0 = \tau_{zz} - \tau_{rr} = -\mu(\dot\gamma_{zz} - \dot\gamma_{rr}) = -\mu\left(\dot\gamma_{rr}\Big|_{\substack{\theta=0\\ r\to\infty}} - \dot\gamma_{rr}\Big|_{\substack{\theta=\frac{\pi}{2}\\ r\to\infty}}\right)$$

$$\tau_{zz} - \tau_{rr} = -\mu\dot\varepsilon\left[(2 - -1)\left(-\frac{6a^5}{r^5}\overset{\to 0}{} + \frac{5a^3}{r^3}\overset{\to 0}{} + 1\right)\right] = -\underbrace{3\mu}_{\eta = 3\mu}\dot\varepsilon$$

$\boxed{\text{pg 4}}$

$$\dot{\gamma}_{r\theta}^{2} = \dot{\varepsilon}^{2}\sin^{2}\theta\cos^{2}\theta\left(-\frac{12a^{5}}{r^{5}} + \frac{15a^{3}}{2r^{3}} - 3\right)^{2}$$

$$\boxed{\dot{\gamma}_{r\theta}^{2} = \dot{\varepsilon}^{2}\sin^{2}\theta\cos^{2}\theta\left(144\frac{a^{10}}{r^{10}} - 300\frac{a^{8}}{r^{8}} + \frac{225a^{6}}{4r^{6}} + 72\frac{a^{5}}{r^{5}} - 45\frac{a^{3}}{r^{3}} + 9\right)}$$

$$\dot{\gamma}_{rr}^{2} = \dot{\varepsilon}^{2}\left(4\cos^{4}\theta - 4\cos^{2}\theta\sin^{2}\theta + \sin^{4}\theta\right)\left(-\frac{6a^{5}}{r^{5}} + \frac{5a^{3}}{r^{3}} + 1\right)^{2}$$

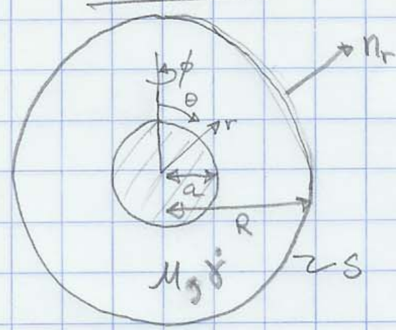
$$\boxed{\dot{\gamma}_{rr}^{2} = \dot{\varepsilon}^{2}\left(4\cos^{4}\theta + \sin^{4}\theta - 4\cos^{2}\theta\sin^{2}\theta\right)\left(\frac{36a^{10}}{r^{10}} - \frac{60a^{8}}{r^{8}} + \frac{25a^{6}}{r^{6}} - \frac{12a^{5}}{r^{5}} + \frac{10a^{3}}{r^{3}} + 1\right)}$$

$$\dot{\gamma}_{\theta\theta}^{2} = \dot{\varepsilon}^{2}\left[\cos^{4}\theta\left(\frac{6a^{5}}{r^{5}} - \frac{5a^{3}}{r^{3}} - 1\right)^{2} - 2\cos^{2}\theta\sin^{2}\theta\left(\frac{6a^{5}}{r^{5}} - \frac{5a^{3}}{r^{3}} - 1\right)\left(\frac{9a^{5}}{2r^{5}} - \frac{5a^{3}}{2r^{3}} - 2\right) + \sin^{4}\theta\left(\frac{9a^{5}}{2r^{5}} - \frac{5a^{3}}{2r^{3}} - 2\right)^{2}\right]$$

$$\boxed{\begin{aligned}\dot{\gamma}_{\theta\theta}^{2} = \dot{\varepsilon}^{2}\Bigg[&\cos^{4}\theta\left(\frac{12a^{10}}{r^{10}} - 60\frac{a^{8}}{r^{8}} + 25\frac{a^{6}}{r^{6}} - 12\frac{a^{5}}{r^{5}} + 10\frac{a^{3}}{r^{3}} + 1\right)\\ &- 2\cos^{2}\theta\sin^{2}\theta\left(\frac{54a^{10}}{2r^{10}} - \frac{70a^{8}}{2r^{8}} + \frac{25a^{6}}{2r^{6}} - \frac{33a^{5}}{2r^{5}} + \frac{25a^{3}}{2r^{3}} + 2\right)\\ &+ \sin^{4}\theta\left(\frac{81a^{10}}{4r^{10}} - \frac{90a^{8}}{4r^{8}} + \frac{25a^{6}}{4r^{6}} - 18\frac{a^{5}}{r^{5}} + 10\frac{a^{3}}{r^{3}} + 4\right)\Bigg]\end{aligned}}$$

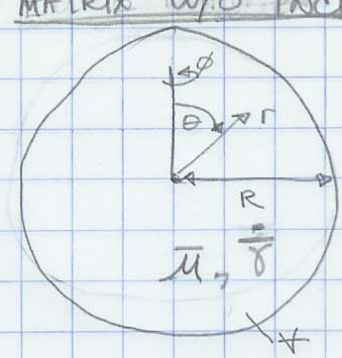
$$\boxed{\begin{aligned}\dot{\gamma}_{\phi\phi}^{2} = \dot{\varepsilon}^{2}\Bigg[&\cos^{4}\theta\left(\frac{12a^{10}}{r^{10}} - 60\frac{a^{8}}{r^{8}} + 25\frac{a^{6}}{r^{6}} - 12\frac{a^{5}}{r^{5}} + 10\frac{a^{3}}{r^{3}} + 1\right)\\ &- 2\cos^{2}\theta\sin^{2}\theta\left(9\frac{a^{10}}{r^{10}} - \frac{45a^{8}}{2r^{8}} + \frac{25a^{6}}{2r^{6}} + \frac{9a^{5}}{2r^{5}} - \frac{5}{2}\frac{a^{3}}{r^{3}} - 1\right)\\ &+ \sin^{4}\theta\left(\frac{9a^{10}}{4r^{10}} - \frac{30a^{8}}{4r^{8}} + \frac{25a^{6}}{4r^{6}} + \frac{3a^{5}}{r^{5}} - \frac{5a^{3}}{r^{3}} + 1\right)\Bigg]\end{aligned}}$$

INCLUSION



vs.

MATRIX W/O INCLUSION



Note: $R >> a$

Energy Applied at boundary surface = Energy dissipated within Volume

$$\frac{dE}{dt} = \int_S \frac{1}{2}(\underline{\sigma} \cdot \underline{n}_r) \cdot \underline{v}\, dS = \int_\forall \frac{1}{2}\bar{\mu}(\dot{\underline{\bar{\gamma}}} : \dot{\underline{\bar{\gamma}}})\, d\forall$$

assume $\dot{\bar{\gamma}} = k\dot{\gamma}$

$\Rightarrow$ constant

Recall

$$A_{sphere} = \int_S dA_S = \int_0^{2\pi}\int_0^{\pi} R^2 \sin\phi \, d\theta \, d\phi$$

$$\forall_{sphere} = \int_\forall d\forall = \int_0^R \int_0^{2\pi}\int_0^{\pi} r^2 \sin\phi \, dr \, d\theta \, d\phi$$

with inclusion

$$\dot{\gamma}_{rr} = \dot{\varepsilon}\left(2\cos^2\theta - \sin^2\theta\right)\left(-\frac{6a^5}{r^5} + \frac{5a^3}{r^3} + 1\right)$$

$$\dot{\gamma}_{\theta\theta} = \dot{\varepsilon}\left[\cos^2\theta\left(\frac{6a^5}{r^5} - \frac{5a^3}{r^3} - 1\right) - \sin^2\theta\left(\frac{9}{2}\frac{a^5}{r^5} - \frac{5}{2}\frac{a^3}{r^3} - 2\right)\right]$$

$$\dot{\gamma}_{\phi\phi} = \dot{\varepsilon}\left[\cos^2\theta\left(\frac{6a^5}{r^5} - \frac{5a^3}{r^3} - 1\right) - \sin^2\theta\left(\frac{3a^5}{2r^5} - \frac{5}{2}\frac{a^3}{r^3} + 1\right)\right]$$

$$\dot{\gamma}_{r\theta} = \dot{\varepsilon}\sin\theta\cos\theta\left(-12\frac{a^5}{r^5} + \frac{15a^3}{2r^3} - 3\right)$$

$$v_r = \dot{\varepsilon}(2\cos^2\theta - \sin^2\theta)\left(\frac{3}{4}\frac{a^5}{r^4} - \frac{5a^3}{4r^2} + \frac{r}{2}\right)$$

$$v_\theta = \dot{\varepsilon}\left(\sin\theta\cos\theta\left(-\frac{9}{2}\frac{a^5}{r^4} + \frac{15a^3}{2r^2} - 3r\right)\right)$$

Without inclusion

$$\dot{\gamma}_{rr} = \dot{\varepsilon}\left(2\cos^2\theta - \sin^2\theta\right)$$

$$\dot{\gamma}_{\theta\theta} = -\dot{\varepsilon}\left(\cos^2\theta - 2\sin^2\theta\right)$$

$$\dot{\gamma}_{\phi\phi} = -\dot{\varepsilon}$$

$$\dot{\gamma}_{r\theta} = -3\dot{\varepsilon}\sin\theta\cos\theta$$

$$v_r = \frac{\dot{\varepsilon} r}{2}\left(2\cos^2\theta - \sin^2\theta\right)$$

$$v_\theta = -3r\dot{\varepsilon}\sin\theta\cos\theta$$

$$k^2(\dot\gamma:\dot\gamma) = \dot\gamma_{rr}^2 + \dot\gamma_{\theta\theta}^2 + \dot\gamma_{\phi\phi}^2 + 2\dot\gamma_{r\theta}^2$$

$$= k^2\dot\varepsilon^2\big(4\cos^4\theta - 4\sin^2\theta\cos^2\theta + \sin^4\theta$$
$$+ \cos^4\theta - 4\sin^2\theta\cos^2\theta + 4\sin^4\theta$$
$$+ 1 \quad + 18\sin^2\theta\cos^2\theta\big)$$

$$\dot\gamma:\dot\gamma = k^2\dot\varepsilon^2\left(1 + 5\cos^4\theta + 10\sin^2\theta\cos^2\theta + 5\sin^4\theta\right)$$

$$= k^2\dot\varepsilon^2\Big(1 + 5\underbrace{(\cos^2\theta + \sin^2\theta)^2}_{=1}\Big)$$

$$\dot\gamma:\dot\gamma = 6k^2\dot\varepsilon^2$$

$$\Rightarrow \int_V \frac{1}{2}\bar\mu(\dot\gamma:\dot\gamma)\,dV = 3\bar\mu\dot\varepsilon^2\int_0^R\int_0^{2\pi}\int_0^\pi r^2\sin\phi\,dr\,d\theta\,d\phi$$

$$= 2\pi\bar\mu\dot\varepsilon^2R^3\left[(-\cos\pi) - -(\cos 0)\right]$$
$$\qquad 1 \quad + \quad 1$$

$$= 4\pi\bar\mu\dot\varepsilon^2R^3k^2$$

Constant

$$\boxed{\left.\frac{dE}{dt}\right|_{\bar\mu} = 4\pi k^2\bar\mu\dot\varepsilon^2R^3}$$

$$\tau_{rr} = +\mu(\dot\gamma_{rr}) = \mu\dot\varepsilon(2\cos^2\theta - \sin^2\theta)\left(-\frac{6a^5}{r^5} + \frac{5a^3}{r^3} + 1\right)$$

$$v_r = \dot\varepsilon(2\cos^2\theta - \sin^2\theta)\left(\frac{3}{4}\frac{a^5}{r^4} - \frac{5a^3}{4r^2} + \frac{r}{2}\right)$$

$$\tau_{rr}v_r\Big|_{r=R} \cong \mu\dot\varepsilon^2(4\cos^4\theta - 4\sin^2\theta\cos^2\theta + \sin^4\theta)\left(-\frac{9a^5}{4R^5} + \frac{5a^3}{4R^2} + \frac{R}{2}\right)$$

$$\frac{dE}{dt} = \mu\dot\varepsilon^2\left(-\frac{9}{4}\frac{a^5}{R^2} + \frac{5}{4}a^3 + \frac{R^3}{2}\right)\int_0^{2\pi}\left[\underbrace{4\cos^4\theta}_{4\left(\frac{3\pi}{4}\right)} - \underbrace{4\sin^2\theta\cos^2\theta}_{4\left(\frac{\pi}{4}\right)} + \underbrace{\sin^4\theta}_{\frac{3}{4}\pi}\right]d\theta$$

$$\boxed{\left.\frac{dE}{dt}\right|_{\mu} = \frac{11}{8}\pi\mu\dot\varepsilon^2\left(-\frac{9}{2}\frac{a^5}{R^2} + \frac{5}{2}a^3 + R^3\right)} = 4\pi k^2\bar\mu\dot\varepsilon^2R^3$$

$$\Rightarrow \bar\mu = \frac{11}{32k^2}\mu\left(\cdots + \frac{5}{2}\frac{a^3}{R^3} + 1\right) \Rightarrow \boxed{\bar\mu \sim \mu\left(1 + \frac{5}{2}\phi\right)}$$

$$k^2 = 11/32 \quad \Rightarrow \quad a \to 0 \quad \bar\mu = \mu$$

$$\frac{dE}{dt} = \int_0^R \int_0^\pi \int_0^{2\pi} \sin\phi \, \frac{1}{2} \mu \dot{\varepsilon}^2 \Big[\cos^4\theta \left(-72\frac{a^5}{r^3} + 60\frac{a^3}{r} + 6r^2 \right)$$

$$+ 2\cos^2\theta \sin^2\theta \left(108\frac{a^5}{r^3} - 75\frac{a^3}{r} + 6r^2 \right)$$

$$+ \sin^4\theta \left(-27\frac{a^5}{r^3} + 15\frac{a^3}{r} + 6r^2 \right) \Big] \, dr \, d\phi \, d\theta$$

$-\cos\pi$

$$\frac{dE}{dt} = \mu \dot{\varepsilon}^2 \int_0^{2\pi}\int_0 \Big[\cos^4\theta \left(-72\frac{a^5}{r^3} + 60\frac{a^3}{r} + 6r^2 \right)$$

$$+ 2\cos^2\theta \sin^2\theta \left(108\frac{a^5}{r^3} - 75\frac{a^3}{r} + 6r^2 \right)$$

$$+ \sin^4\theta \left(-27\frac{a^5}{r^3} + 15\frac{a^3}{r} + 6r^2 \right) \Big] \, dr \, d\theta$$

Recall

$$\int \sin^n u \cos^m u \, du = -\frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} u \cos^m u \, du$$

$$= \frac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m} \int \sin^n u \cos^{m-2} u \, du$$

$$\Rightarrow \int_0^{2\pi} \cos^4\theta \, d\theta = \underbrace{-\frac{\cos^3\theta \sin\theta}{4}\Big|_0^{2\pi}}_{\to 0} + \frac{3}{4}\int_0^{2\pi} \cos^2\theta \, d\theta$$

$$= \frac{3}{4}\left(\frac{1}{2}\theta + \cancel{\frac{1}{4}\sin 2\theta} \right)\Big|_0^{2\pi} = \frac{3}{4}\pi$$

$$\Rightarrow \int_0^{2\pi} \sin^2\theta \cos^2\theta \, d\theta = \cancel{-\frac{\sin\theta \cos^3\theta}{4}} + \frac{1}{4}\int_0^{2\pi} \cos^2\theta \, d\theta$$

$$= \frac{1}{4}\left(\frac{1}{2}\theta + \cancel{\frac{1}{4}\sin 2\theta} \right)\Big|_0^{2\pi} = \frac{\pi}{4}$$

Recall

$$\Rightarrow \int \sin^n u \, du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u \, du$$

$$\Rightarrow \int_0^{2\pi} \sin^4\theta \, d\theta = \cancel{-\frac{1}{4}\sin^3\theta \cos\theta}\Big|_0^{2\pi} + \frac{3}{4}\int_0^{2\pi} \sin^2\theta \, d\theta$$

$$= \frac{3}{4}\left(\frac{1}{2}\theta - \cancel{\frac{1}{4}\sin 2\theta} \right)\Big|_0^{2\pi} = \frac{3}{4}\pi$$

→ Now apply the same technique to a solid

Fluid

$\sigma = \eta \dot{\gamma}$

Solid

$\sigma = E\varepsilon$

$$\Delta E = \int_{dS} \frac{1}{2}(\sigma \cdot n) \cdot \underset{\uparrow \text{displacement}}{x} \, dS = \int_{V} \frac{E}{2}(\varepsilon : \varepsilon) \, dV$$

→ This method not only works for E and η but for K: THERMALLY CONDUCTIVITY

ϵ: DIELECTRIC CONSTANT

WORKS ONLY FOR SMALL CONCENTRATIONS less than 2 vol%

(Ref. Batchelor)

Utilize a power series to capture behavior at higher concentrations

P: property

$$P^* = P\left[1 + \underbrace{\alpha_1 \phi}_{\text{independent action}} + \underbrace{\alpha_2 \phi^2}_{\text{mutual interaction of pairs}} + \underbrace{\alpha_3 \phi^3}_{\text{of triplets}} + \ldots\right]$$

"$\alpha_1, \alpha_2, \ldots$ cannot, in general, be obtained in an elementary manner; in most cases a somewhat intricate analysis is necessary for their determination"

$\alpha = f$(particle shape)

Carbon black system

→ $\phi \leq 10$ vol% discrete particles w/in matrix

$10 \leq \phi \leq 30$ vol% network forms

$\phi > 30$ vol% carbon black diluted by rubber

What does $\alpha_2 = ?$ (THANKS TREVOR!)

14.1 – Guth, Gold, Simha 1936
⇒ accounts for a pair of spheres (similar to method of images)

12.6 – Simha 1940
⇒ accounts for brownian motion

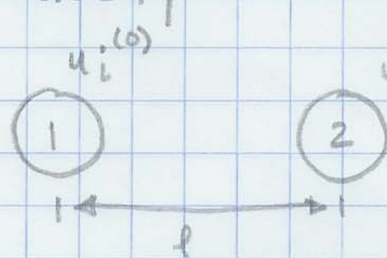
7.35 – Vand, 1948

6.2 – 7.6 Batchelor
← accounts for longrange Hydrodynamical Interactions

varies from 5 to 15 depending on assumptions or empirical derivation

(See Handbook of Surface + Colloid Chemistry (Birdi) for more thorough review)

Method of Images
- single reflection
- 2 spheres in translational fluid velocity



$$u_i^{(1)} = f\left(u_i^{(0)}, 1 + \frac{\alpha_1}{\ell} + \frac{\alpha_2}{\ell^2} + \frac{\alpha_3}{\ell^3} + \ldots\right)$$

- must satisfy B.C.s (no slip), Continuity + N.S.

Equates energy dissipated in this flow

$$\frac{dE}{dt} = 2\eta\left[a_{ii}^{(0)}\right]^2 \forall \left(1 + \frac{1}{2}c - \frac{1}{2}c^2\right)$$

to energy dissipated in a comparison flow where the axes of this flow do not coincide with those of the primary flow

$$\frac{dE}{dt} = 2\bar{\eta}\left[a_{ii}^{(0)}\right]^2 \forall \left[1 - 2c - 9.6c^2\right]$$

$$\bar{\eta} = \eta\,\frac{\left(1 + \frac{1}{2}c - \frac{1}{2}c^2\right)}{1 - 2c - 9.6c^2}$$

$$\Rightarrow \quad 1 + \frac{5}{2}c + 14.1c^2$$

$$\begin{array}{r|l} 1 - 2c - 9.6c^2 & 1 + 0.5c - 0.5c^2 \\ & -1 + 2c + 9.6c^2 \\ \hline & 2.5c + 9.1c^2 \\ & -2.5c + 5c^2 + 24c^3 \\ \hline & 14.1c^2 \end{array}$$

$$\boxed{\bar{\eta} = \eta\left(1 + \frac{5}{2}c + 14.1c^2\right)}$$

The analog for rod-like filler particles

$$\bar{E} = E\left[1 + 0.67fc + 1.62f^2c^2\right]$$

$$f = \frac{\text{length}}{\text{breadth}}$$

for $f \cong 1$ $\quad \bar{E} = E\left[1 + 2.5c + 14.1c^2\right]$

? I could not get this to work out suggestions ?