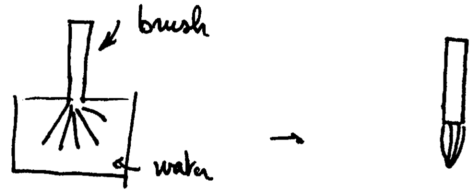
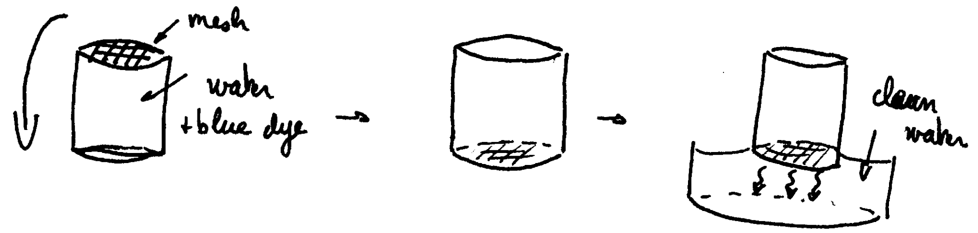
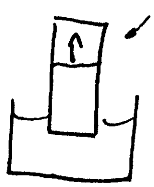
(1) Surface tension

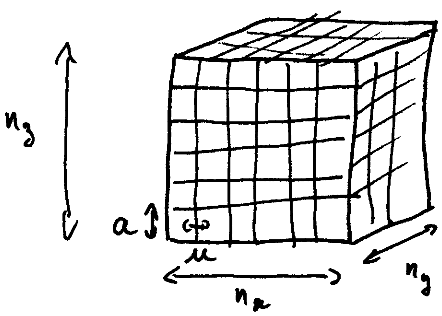
some examples:







1.1 Surface energy → the price of the interface



typical distance between molecules: $a \sim 3\,\text{Å}$ (0.3 nm)

" interaction between 2 molecules: $u \sim kT$ $(\sim 4.10^{-21} J)$
↓
entropy
(if $u \ll kT \Rightarrow$ gas)

Volume: $V = n_x n_y n_z a^3$

surface: $S = 2(n_x n_y + n_x n_z + n_y n_z) a^2$

→ number of pairs of interactions: in x direction $n_x - 1$ planes with $n_y n_z$ interactions
y $n_y - 1$ $n_x n_z$
z $n_z - 1$ $n_x n_y$

⇒ energy of the material:

$$E = -\left[(n_x-1) n_y n_z + (n_y - 1) n_x n_z + (n_z-1) n_x n_y\right] u$$

↓
$E < 0$ if condensed (attractive interactions).

$$\boxed{E = -\frac{3V}{a^3}u + \frac{S}{2a^2}u}$$

$< 0 \Rightarrow$ cohesion $\quad\quad > 0 \rightarrow$ surface Energy : E_s.

$E_s = \sigma S \quad \Rightarrow \quad \sigma \sim \frac{u}{2a^2} \sim \frac{4 \cdot 10^{-21}}{2 \cdot 9 \cdot 10^{-20}} \sim 0.02\ J/m^2 \quad (20\ mJ/m^2)$.

(σ: surface tension; 20 mJ/m²: typical value)

~ if other phase: the same but u_{12}

$\Rightarrow$ E_s ~ contrast between phases.

some examples. (room temperature)

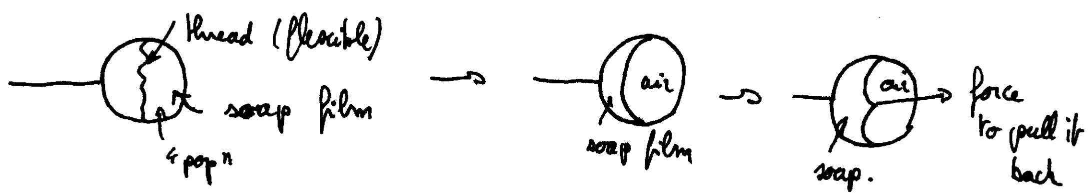
liquid:	Helium (4 °K)	ethanol	olive oil	soap water	water/oil	water	mercury
σ (mJ/m²)	0.1	23	~30	~35	50	72	~500

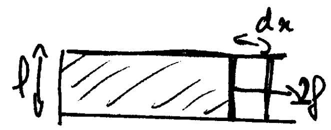
↳ if 2nd phase not precised $\sigma \rightarrow$ liquid/vapor.

→ variation with T : $T \nearrow \Rightarrow \sigma \searrow \quad \sim -0.1\ mJ/m^2/°C$.

Force :

demo:





2 interfaces : air | soap film | air

→ $dW = E_s\, dS = 2\sigma l\, dx$.

$dW = 2f\, dx$
↳ 2 forces

$$\boxed{f = \sigma l}$$ $\quad \sigma$: J/m² or N/m

1.2 The shape of droplets & bubbles:

↳ E_s minimum for V given ⇒ sphere.

enemples in litterature: ↳ slides.

↳ Dr Seuss, curious Georges.

↳ Nasa: movies at 0g

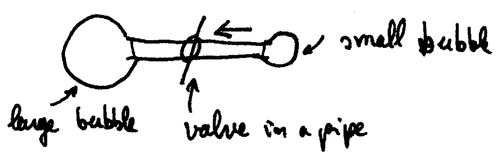
↳ facted droplets (liquid cristals) → not the same σ in all the directions

↳ bubbles in irradiated material (α rays ⇒ helium bubbles)

1.3 Laplace law:

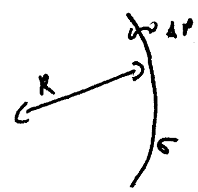
→ To make a soap bubble ⇒ put some pressure inside.

↳ experiment:



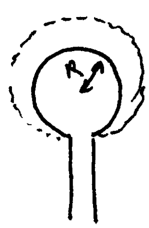
↳ the big eats the small.

↳ non-dimensional analysis:



$$\sigma \sim \frac{f}{L} \Rightarrow \Delta P \sim \frac{f}{L^2} \Rightarrow \boxed{\Delta P \sim \frac{\sigma}{R}}$$

↳ "clean" analysis:

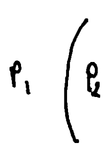


$E_s = 4\pi R^2 \sigma \qquad dE_s = 8\pi\sigma R\,dR.$

$V = \frac{4}{3}\pi R^3 \qquad dV = 4\pi R^2\,dR$

$$dE_s = \Delta P\,dV \Rightarrow \boxed{\Delta P = \frac{2\sigma}{R}}$$

sign:



$P_2 > P_1$.

exemples: soap bubble: $R \sim 5$ cm $\sigma \sim 35$ mN/m

$$\Delta P = 2 \times \frac{2 \times 35.10^{-3}}{5.10^{-2}} \to 2.8 \text{ Pa} \sim 3.10^{-5} \text{ Atm.}$$

(2 interfaces)

champagne bubble: $R \sim 0.1$ mm $\sigma \sim 50$ mN/m

$$\Delta P \simeq \frac{2 \times 50.10^{-3}}{10^{-4}} = 1000 \text{ Pa} \quad (\sim 10^{-2} \text{ Atm})$$

helium bubbles: $R \sim 10$ nm $\sigma \sim 1$ N/m

$$\Delta P = \frac{2 \times 1}{10^{-8}} = 2.10^{8} \text{ Pa} \to 2000 \text{ Atm.}$$

<u>Generalization</u>: any 3D surface: 2 main radii of curvature, R_1 & R_2.

$$\boxed{\Delta P = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

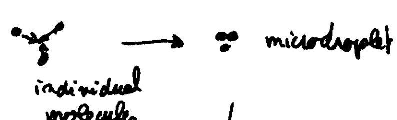
↳ careful with the signs!

ex: open ~~soap~~ bubble $\Delta P = 0$

↳ catenoid

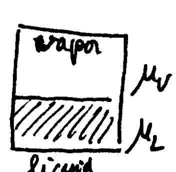
1.4 <u>Nucleation / Condensation</u>

↳ exemple: Condensation from a vapor



problem $\Delta P = \frac{\sigma}{R}$ gets huge => explosion?

some thermodynamics:



vapor pressure with a pressure jump at the interface?

equilibrium => $\mu_L = \mu_V$

dP at the interface => $d\mu_L = d\mu_V$

$$\Rightarrow \quad V_m\, dP\Big)_L = V_m\, dP\Big)_V$$

↓ molecular vol

$$\rightarrow \quad V_m \Delta P = \int_{P_V}^{P_V'} \frac{RT}{P}\, dP \qquad \left(\text{perfect gas: } V_m = \frac{RT}{P}\right)$$

↓ almost non compressible

$$V_m \Delta P = RT \ln\left(\frac{P_V'}{P_V}\right) \qquad \boxed{P_V' = P_V \exp\left(\frac{V_m \Delta P}{RT}\right).}$$ Kelvin's law

$P^* = \frac{RT}{V_m}$ water: $V_m = 18.10^{-6}\ m^3/mol$

$RT = 8 \times 300 \sim 2.4\ 10^3\ J/mol.$

$\Rightarrow P^* \simeq \frac{2.4\ 10^{-3}}{18.10^{-6}} = 1{,}3\ .10^8\ Pa \sim 1300\ Atm.$

for a droplet $\Delta P = \frac{2\sigma}{R}$

$$\Rightarrow \boxed{P_V' = P_V \exp\left(\frac{R^*}{R}\right)}, \qquad R^* = \frac{2\sigma}{P^*}.$$ critical radius.

↓ Vapor pressure for a flat interface

→ water: $R^* = \frac{2.7\ 10^{-2}}{1.3\ 10^8} = 10^{-9}\ m$ $\underline{R^* \simeq 1\ nm.}$

$v \simeq 4\ nm^3$, $v_m = \frac{V_m}{N_a} = \frac{18.10^{-6}}{6.10^{23}} = 3.10^{-29}\ m^3 = 0.03\ nm^3$

↳ $\underline{R^* \leftrightarrow 100 \text{ molecules.}}$

⇒ heterogeneous nucleation.

↳ dust particles, irregularities...

ex in clouds

↙ dust $> R^*$

↳ molecules adsorb → germ $> R^*$.

slide: champagne bubbles (trains from a piece of dust).