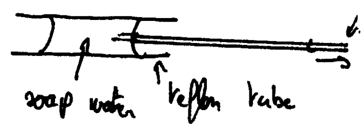
⑤ Interfacial hydrodynamics

demo :



→ capillary "rise"

spreading of a wetting liquid.

Navier & Stokes eq:

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v}.\vec{\nabla})\vec{v} = \rho \vec{g} - \vec{\nabla} P + \mu \nabla^2 \vec{v}$$

body force, pressure

orders of magnitude $\rho \frac{V^2}{L}$ $\mu \frac{V}{L^2}$

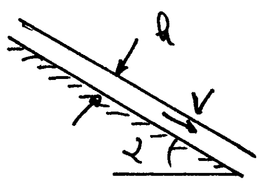
$$Re = \frac{\text{inertia}}{\text{viscous}} = \frac{\rho V^2/L}{\mu V/L^2}.$$

$$\boxed{Re = \frac{\rho V L}{\mu}}$$

$Re \gg 1$ → inertia (Bernoulli ...)

$Re \ll 1$ → viscous (Stokes flow)

examples scaling laws : → N & S difficult to solve → scalings.



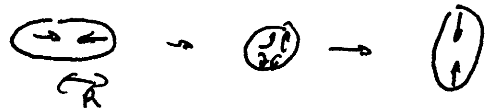
→ if viscous flow. $\left(\frac{\rho V h}{\mu} \ll 1 \right)$

$$\mu \frac{V}{h^2} \sim \rho g \sin\alpha .$$

$$\Rightarrow \boxed{V \sim \frac{\rho g h^2}{\mu} \sin\alpha}$$ → Darcy law.

Oscillations of a droplet:

→ slide: many modes.



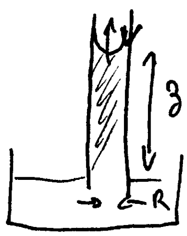
driving: surface tension / inertia / surface tension ...

$$\rho V^2 \sim \frac{\sigma}{R}$$

$$V \sim \frac{R}{\tau_{osc}} \Rightarrow \rho \frac{R^2}{\tau_{osc}^2} \sim \frac{\sigma}{R} \to \boxed{\tau_{osc} \sim \left(\frac{\rho R^3}{\sigma}\right)^{1/2}}$$

5.1 Spontaneous wetting

5.1.1 Capillary rise



$$\mu \frac{V}{R^2} \sim \frac{2\sigma\cos\theta}{Rz} - \rho g .$$

$$\mu \frac{V}{R^2} \sim \frac{\sigma\cos\theta}{Rz}\left(1 - \frac{z}{h_{eq}}\right).$$

if $z \ll h_{eq}$ ($h_{eq} \to \infty$: horizontal tube).

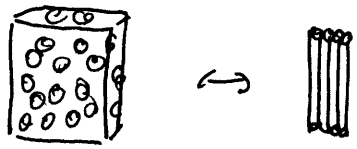
$$V \sim \frac{R\sigma\cos\theta}{\mu z} \to z\frac{dz}{dt} \sim R\frac{\sigma}{\mu}\cos\theta .$$

$$\boxed{z \sim \left(R\frac{\sigma\cos\theta}{\mu}t\right)^{1/2}}$$ Washburn law.

$V^* = \frac{\sigma}{\mu}$ characteristic capillary speed.

↳ may have some inertia at the beginning.

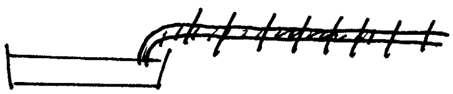
→ <u>imbibition of a porous media:</u>



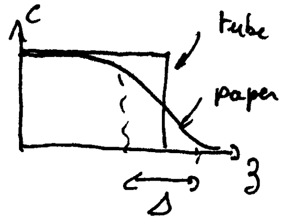
series of capillaries.

$R \sim r_{pores}$

↳ dilution of the front.

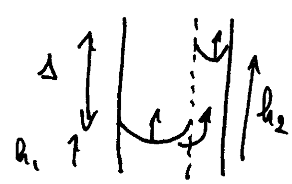


paper band cut in slices during imbibition.



$\Delta(t) \nearrow$

↳ effect of the pores polydispersity:



small use big as reservoir.

<u>Capillary rise in trees?</u>

→ slide: sequoia.

$R \sim 10\mu$ $\quad \mu \sim 10^{-3}\ Pa \cdot s$ $\quad h \sim 50\ m$ $\quad \sigma \sim 70\ mN/m$.

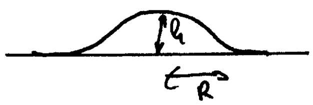
$\tau \sim \dfrac{h^2 \mu}{R\sigma}$ ~ many months!

best situation
g neglected

→ surface tension just holds the sap

driving force: evaporation.

5.1.2 Spreading



Laplace pressure: $\frac{1}{r} = \frac{\ddot{z}}{(1+\dot{z}^2)^{3/2}}$.

↳ scaling for smooth surface

$$\frac{1}{r} \sim \frac{h}{R^2}.$$

$$\Rightarrow \Delta P_L \sim \frac{\sigma h}{R^2} \qquad \nabla P \sim \frac{\sigma h}{R^3}.$$

capillary driven spreading:

$$\mu \frac{V}{h^2} \sim \frac{\sigma h}{R^3}$$

$$\Rightarrow V \sim \frac{\sigma}{\mu}\left(\frac{h}{R}\right)^3$$

$$\Omega \sim R^2 h \quad \rightarrow \quad h \sim \frac{\Omega}{R^2}$$

$$\frac{dR}{dt} \sim \frac{\sigma}{\mu}\left(\frac{\Omega}{R^3}\right)^3 \quad \rightarrow \quad R^9 \frac{dR}{dt} \sim \frac{\sigma}{\mu}\Omega^3.$$

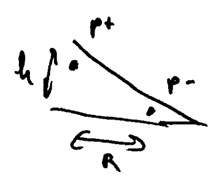
$$R^{10} \sim \Omega^3 V^* t \qquad \boxed{R \sim (\Omega^3 V^* t)^{1/10}}$$

↳ delicate point: integration in the edge ⇒ ln term and cut-off.

→ slide: A.M. Cazabat. → 2nd regime 1/8.

gravity driven:
(if time enough)

$$\mu \frac{V}{h^2} \sim \rho g \frac{h}{R}$$



$$\nabla P \sim \rho g \frac{h}{R}.$$

$$V \sim \frac{\rho g}{\mu}\frac{h^3}{R}$$

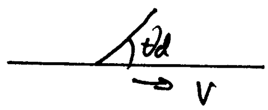
$$V \sim \frac{\rho g}{\mu}\frac{\Omega^3}{R^7}$$

$$R^7 \frac{dR}{dt} \sim \frac{\rho g}{\mu}\Omega^3 \quad \rightarrow \quad R \sim \left(\frac{\rho g}{\mu}\Omega^3 t\right)^{1/8}.$$

cross over: $\rho g \frac{h}{R} \sim \frac{\sigma h}{R^3} \quad \rightarrow \quad R \sim l_c.$

5.2 Forced wetting:

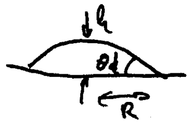
5.2.1 Dynamic contact angle:



capillarity wants $\theta \to 0$

viscosity: liquid ~~follow~~ follow the plate.

→ analogue spreading



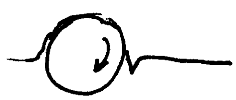
$h/R \sim \theta_d \Rightarrow V \sim \frac{\sigma}{\mu}\theta_d^3$.

$\boxed{\theta_d \sim \left(\frac{\mu V}{\sigma}\right)^{1/3}}$ Tanner's law. $\boxed{Ca = \frac{\mu V}{\sigma}}$ capillary number

→ slide. θ_d from Hoffman.

↳ application → limit to coating process: Ca too big ⇒ $\theta_d \sim 180°$ ⇒ air entrainment.

slide: cusp near a rotating cylinder



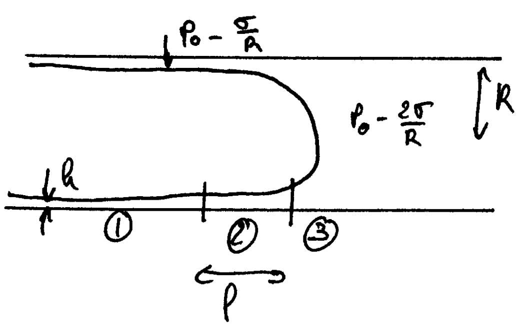
→ film if $\omega \nearrow$.

5.2.2 Coating

demo:

→

$l_f < l_0$ ⇒ coating.



$\begin{cases} \text{viscosity} \Rightarrow \text{film thick.} \\ \text{surface tension} \Rightarrow \text{sucks the film.} \end{cases}$

1. static film
2. dynamic meniscus
3. static meniscus

→ dynamic meniscus: $\mu \frac{V}{h^2} \sim \frac{\sigma}{R \ell}$

connection static & dynamic: same ~~curv~~ curvatures:

$$\frac{1}{R} \sim \frac{h}{\ell^2} \qquad \left(\frac{\ddot{z}}{(1+\dot{z}^2)^{3/2}} \right)$$

$$\Rightarrow \underline{\ell \sim (Rh)^{1/2}.}$$

$$\frac{\mu V}{h^2} \sim \frac{\sigma}{R(Rh)^{1/2}}$$

$$\Rightarrow \boxed{h \sim R \left(\frac{\mu V}{\sigma}\right)^{2/3}} \quad \begin{cases} \text{Landau-Levich} \\ \text{Bretherton} \end{cases}$$

↳ works for $Ca \ll 1$ ($h < R$ for the tube).

<u>same for a plate</u> $\ell_c \leftrightarrow R$:

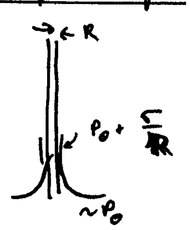
$$\boxed{h \sim \ell_c \, Ca^{2/3}.}$$

effect of gravity? $\rho g \sim \frac{\sigma}{\ell_c \ell}$ → $\ell \sim \ell_c$. ($\Rightarrow Ca \sim 1$)

→ $Ca \gg 1$

$$h \sim \left(\frac{\mu V}{\rho g}\right)^{1/2} \rightarrow \underline{h \sim \ell_c \, Ca^{1/2}}.$$

<u>same for a fiber</u>:



→ $\underline{h \sim R \, Ca^{2/3}.}$