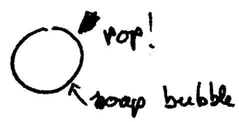
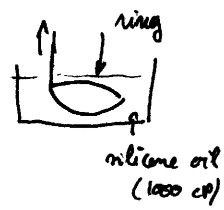
⑥ Instabilities

demo :

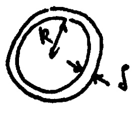




6.1 Pure surface tension driven instabilities

↳ too much surface in the initial configuration.

ex: burst of a bubble

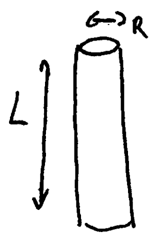


$V = 4\pi R^2 \delta$
$S = 4\pi R^2$ → $S/V = \frac{1}{\delta}$.

↳ same liquid in a droplet:

$O \updownarrow r$ $\quad V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$ → $S/V = \frac{3}{r}$ → $\delta \ll r \Rightarrow$ much better.

6.1.1 Rayleigh instability :

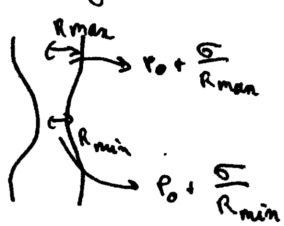


⟶ ideally one single big droplet.

↳ pb: it takes inertia or viscous dissipation to move liquid from far away →

→ slide : jet break up.

Breaking mechanism:



R_{max} & R_{min} ~ R.

→ driving pressure ~ $\frac{\sigma}{R}$

→ slowing process : viscosity or inertia.

→ viscous flow:

$$\mu \frac{V}{R^2} \sim \frac{\sigma}{R^2} \rightarrow V \sim \frac{\sigma}{\mu} \quad (=V^*)$$

$$V \sim \frac{R}{\tau_{vis}} \Rightarrow \boxed{\tau_{visc} \sim \frac{\mu R}{\sigma}}$$

→ inertia:

$$\rho V^2 \sim \frac{\sigma}{R} \rightarrow \rho \frac{R^2}{\tau_{iner}^2} \sim \frac{\sigma}{R}.$$

$$\boxed{\tau_{iner} \sim \left(\rho \frac{R^3}{\sigma}\right)^{1/2}}$$

inertia or viscous!

↳ Ohnesorge number:

$$\boxed{Oh = \frac{\tau_{iner}}{\tau_{visc}} = \frac{\left(\rho \frac{R^3}{\sigma}\right)^{1/2}}{\frac{\mu R}{\sigma}} = \frac{(\rho \sigma R)^{1/2}}{\mu}}$$

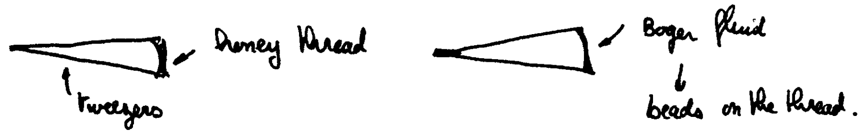
$$\begin{cases} Oh \gg 1 \Rightarrow \text{inertia} \\ Oh \ll 1 \Rightarrow \text{viscous} \end{cases}$$

instability wave lenght $\lambda \sim R\, f_1(Oh)$

final droplet size $r \sim R\, f_2(Oh)$

6.1.2 Capillary break up rheometer

demo:



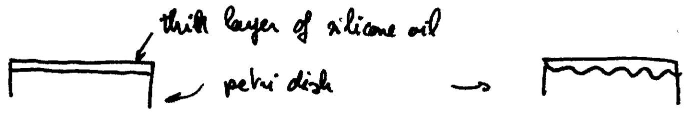
→ measure extensional viscosity (relevant for complex fluids)

$$\mu_{ext} \sim \frac{\sigma}{dR/dt}$$

slides: fishbones J. Bush, electric fishbones.

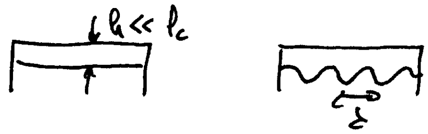
6.2 Applied field

demo:



Rayleigh-Taylor instability

↳ same in the shower roof.



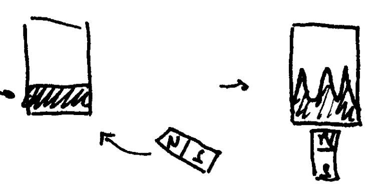
instability from competition surface tension vs gravity

$$\Rightarrow \lambda \sim l_c .$$

↳ h is growing rate: $$\tau_{inst} \sim \frac{\sigma \mu}{\rho^2 g^2 h^3}$$

↳ other fields: electrostatic, magnetic.

demo: ferrofluid



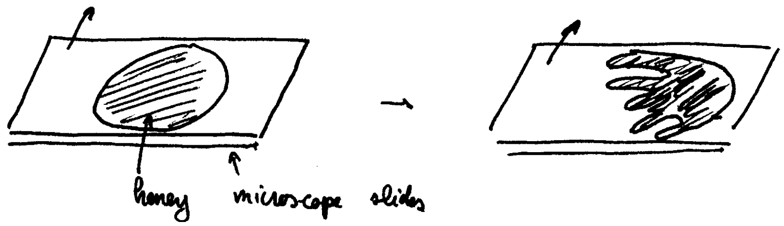
λ ↘ when magnet closer.

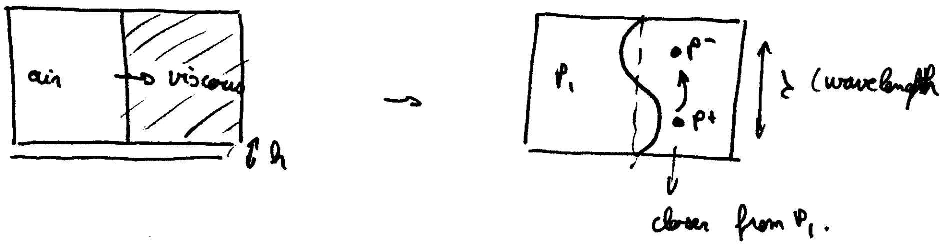
in a thin cell

6.3 Flow induced instabilities:

6.3.1 Viscous fingering (Saffman-Taylor)

demo:





flow from P^+ to P^- ⇒ instability grows.

Wavelength?

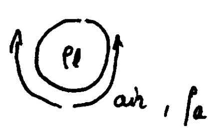
$$\mu \frac{V}{h^2} \sim \frac{\sigma}{\lambda}\frac{1}{\lambda}$$

($\frac{\sigma}{\lambda}$ → typical curvature; $\frac{1}{\lambda}$ → gradient scale)

$$\Rightarrow \boxed{\lambda \sim h\, Ca^{-1/2}}$$

→ at the end more complex, just big fingers.

6.3.2 Size of a rain droplet



→ terminal speed: $\rho_{air} V^2 R^2 \sim \rho R^3 g$ (drag ∼ weight)

$$\Rightarrow \boxed{V \sim \left(\frac{\rho_{liq}}{\rho_{air}} R g\right)^{1/2}}$$

↳ pinches off until $\rho_{air} V^2 \sim \frac{\sigma}{R}$

$$\Rightarrow R_{max} \sim \frac{\sigma}{\frac{\rho_{liq}}{\rho_{air}} R g \rho_{air}} \qquad \boxed{R_{max} \sim l_c}$$

splash (only if time enough)

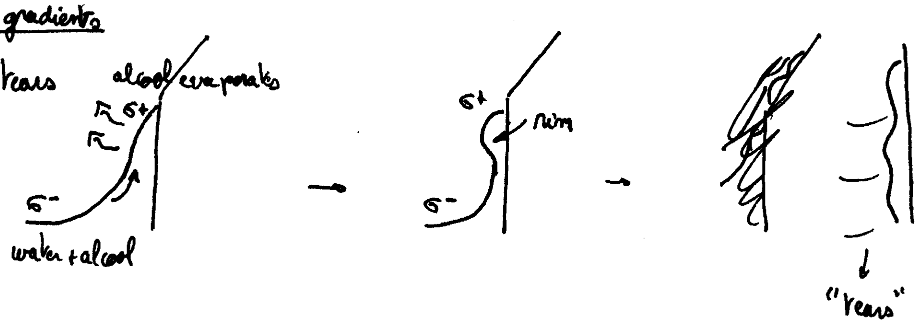
inertia → spreading

surface tension → prevents

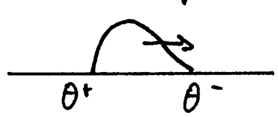
slide: splash solder droplet.

6.4 surface tension gradients

slide wine tears



surfaces with wetting gradients:



slide: Chaudhury.