

## Viscoelastic Flows in Abrupt Contraction-Expansions

### IV. Pressure Drop Definition

In our experiments we measure the total pressure drop across the axisymmetric abrupt contraction-expansion geometry. We then systematically correct this to report an ‘extra pressure drop’ associated with the contraction-expansion alone.

Table 1 shows the axial distance of each possible pressure transducer location from the contraction plane. For clarity, it is useful to define several *differential pressures* by the location of the pressure transducers used. For instance, the pressure drop across the contraction-expansion measured between pressure transducers 2 and 4 is denoted as  $\Delta P_{24} = (P_2 - P_4)$ . Several different pressure transducers locations were created sufficiently far upstream of the contraction plane to ensure that the pressure transducers were not affected by the presence of the elastically-driven growth of the upstream (and downstream) vortices. In the subsequent measurements only the upstream pressure transducer at location  $P_2$  was used.

	Pressure Transducer <b>i</b>	Distance From Contraction Plane $z_i$ [cm]
<b>Upstream:</b>	$P_1$	-22.86
	$P_2$	-7.62
	$P_3$	-2.54
<b>Downstream:</b>	$P_4$	+8.26

**Table 1:** Location of the flush mounted pressure transducers with respect to the contraction plane (located at  $z = 0$ ).

As the flow rate is increased, the pressure transducers measure a combination of the pressure drop arising from fully-developed rectilinear flow in the straight pipe and the extra pressure drop caused by the presence of the orifice plate  $\Delta P(Q) = \Delta P_{\text{straight pipe}} + \Delta P_{\text{ext.}}$ . We are only interested, however, in the extra pressure drop which we denote as  $\Delta P'_{24}$  where the prime indicates that the contribution to the total pressure drop resulting from the Poiseuille flow in the pipe connecting the pressure transducers and in the contraction-expansion has been removed in order to isolate the extra pressure drop across the contraction-expansion. In other words, we linearly decompose the pressure

measurements according to

$$\Delta P'_{24} = \Delta P_{24} - \frac{8QL_c\eta}{\pi R_2^4} - \frac{8QL\eta}{\pi R_1^4} \quad (1)$$

where  $L \equiv z_4 - (z_2 + L_c)$  is the length of straight tubing of radius  $R_1$  between the pressure transducers and the contraction-expansion plate,  $Q$  is the volume flow rate of the fluid and  $\eta$  is the fluid viscosity. Lastly, we define a dimensionless pressure drop

$$\mathcal{P} = \frac{\Delta P'_{24}(Q, De)}{\Delta P'_{24}(Q, De = 0)} \quad (2)$$

$\Delta P'_{24}(Q, De = 0)$  is the pressure drop across a given contraction-expansion orifice of the Newtonian oligomeric polystyrene oil at a given flow rate and  $\Delta P'_{24}(Q, De)$  is the pressure drop across the same contraction-expansion ratio of the viscoelastic 0.025% PS/PS polymer solution at the same flow rate. This dimensionless pressure drop differs from the Couette correction often used to present contraction flow pressure drop data by a constant factor which we determine below.

In 1891, R. A. Sampson first solved the pressure-driven flow of a Newtonian fluid at low Reynolds number through an infinitesimally thin circular hole in an unbounded rigid plane wall using oblate spheroidal coordinates [1]. Of particular interest is Sampson's result for pressure drop across the orifice which may be simply expressed

$$\Delta P_S = \frac{3Q\eta_s}{R_2^3} \quad (3)$$

where  $Q$  is the volume flow rate of the fluid and  $R_2$  is the radius of the hole in the orifice plane. For large contraction ratios  $\beta = R_1 / R_2 \gg 1$ , Sampson's solution should approximate the flow near the plane of the contraction reasonably well. In our paper we show that we can replace the measured pressure drop of the oligomeric polystyrene  $\Delta P'_{24}(Q, De = 0)$  by Sampson's solution ( $\Delta P_S$ ) for  $\beta \geq 4$  [2]. However, because our orifice plate is not infinitely thin, and has finite aspect ratio ( $L_c / R_2 \neq 0$ ) there is an additional contribution to the pressure drop. This has been considered analytically by Dagan *et al.* [3]. They show that their numerical calculations can be accurately approximated by

linearly combining the pressure drop associated with Sampson flow and the pressure drop of the assumed Poiseuille flow through the orifice itself to give:

$$\Delta P_D = \frac{Q\eta}{R_2^3} \left( 3 + \frac{8L_c}{\pi R_2} \right) \quad (4)$$

In our experiments, because of the finite size of the contraction ( $0.5 \leq L_c / R_2 \leq 2$ ) the additional pressure drop arising from the latter term in Equation 2 is not negligible. In fact, it is of the same order as the Sampson pressure drop.

An alternative approach commonly used in presenting contraction flow results is to report a *Couette correction* [4] in which the extra pressure drop arising from the orifice is scaled with the wall shear stress in the downstream tube  $C = \Delta P'_{\text{ext}} / 2 \tau_w$ , where  $\tau_w = 4 \eta_s \langle v_2 \rangle / R_2 = 4 \eta_s Q / \pi R_2^3$ . For  $\beta \gg 1$ , the Couette correction for a Newtonian fluid entering an abrupt contraction computed using the Sampson solution thus predicts

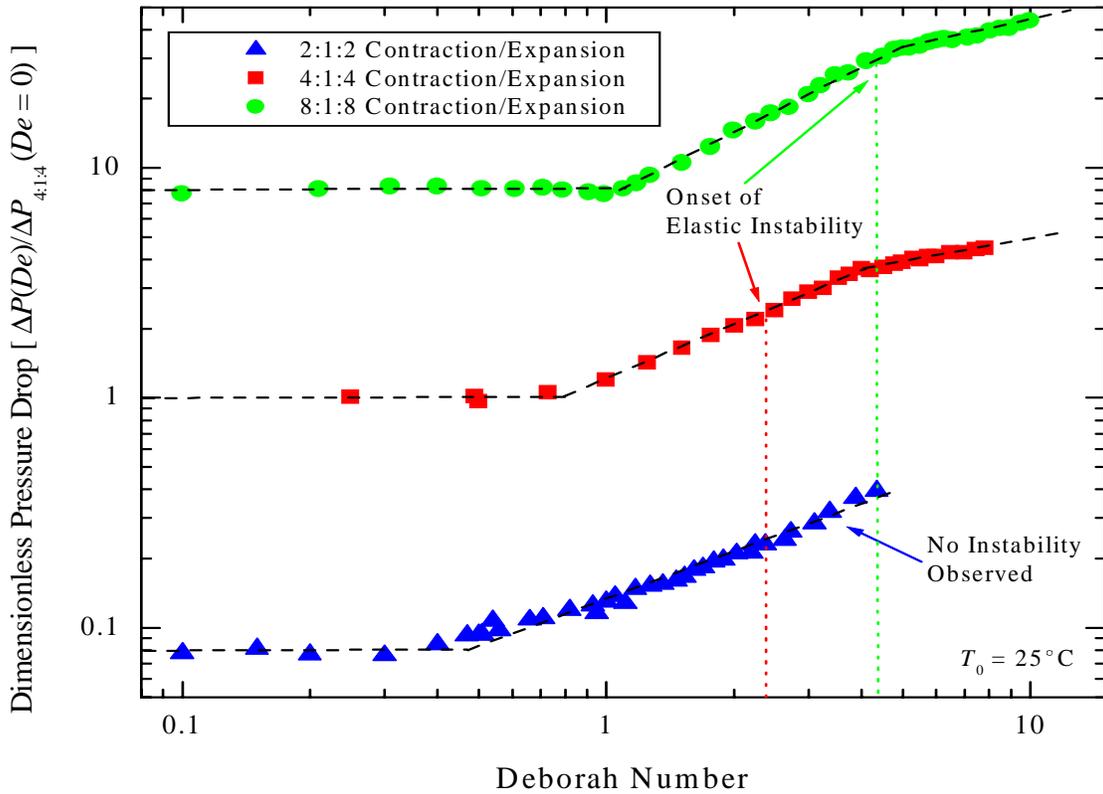
$$C = \frac{1}{2} \frac{\Delta P_s}{2 \tau_w} = \frac{3\pi}{16} = 0.589 \quad (5)$$

which is in good agreement with simulations and measurements for  $\beta \geq 4$  [5].

## Results and Discussion

Figure 1 shows a comparison of the dimensionless pressure drop between contraction-expansion ratios of  $\beta = R_1 / R_2 = 2, 4$  and 8. For convenience, all pressure drops in this figure have been normalized with the Newtonian pressure drop associated with the 4:1:4 axisymmetric contraction-expansion. In all cases, at low Deborah numbers the pressure drop of the PS/PS solution across the contraction-expansion are equal to the pressure drop observed for a Newtonian fluid with the same zero-shear-rate viscosity ( $\mathcal{P} = 1$ ). As the Deborah number is increased, the dimensionless pressure drop across the contraction-expansion grows linearly with Deborah number until at a critical Deborah number is reached at which point the dimensionless pressure drop begins to saturate and the value of the dimensionless pressure drop increases at a greatly reduced rate.

Figure 2 shows a comparison between a 4:1:4 axisymmetric contraction-expansion with a

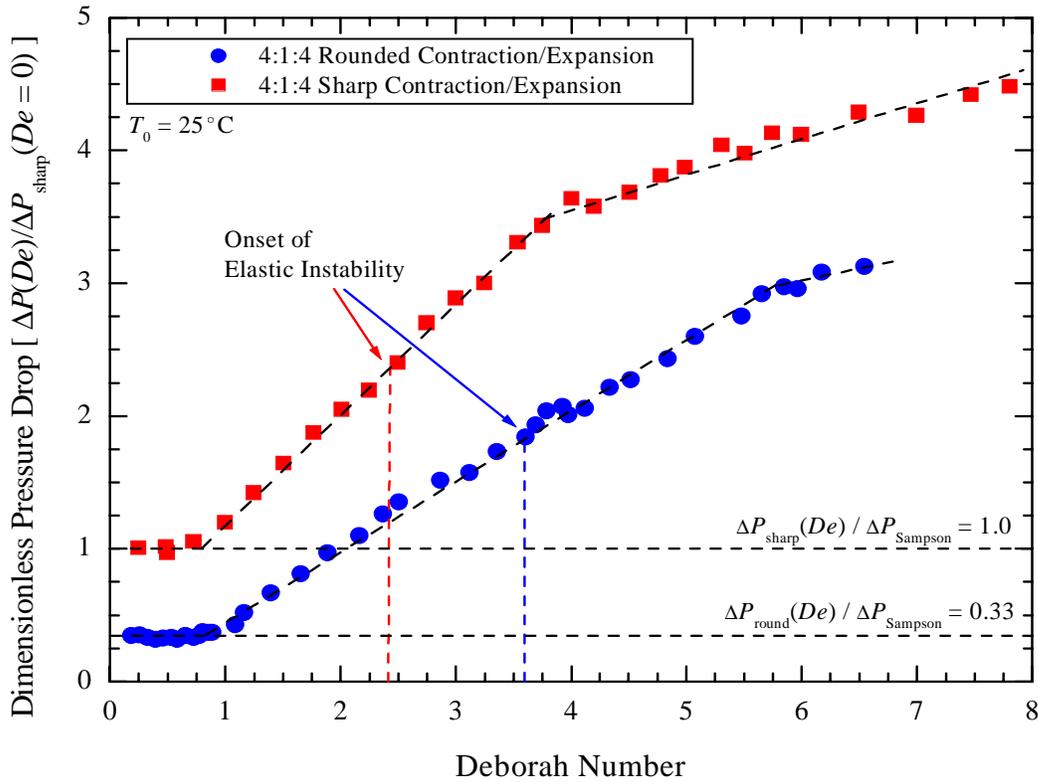


**Figure 1:** Comparison of dimensionless pressure drops for  $\beta = 2, 4$  &  $8$ .

sharp entrance lip ( $R_c \cong 0$ ) and with a entrance lip rounded with a radius of curvature equal to  $R_c = 0.5 R_2$ . One observes that rounding the entrance lip does not change the pressure growth dynamics, but simply delays the transition into each pressure growth regime.

### Conclusions

We have shown how we systematically decompose our data into a dimensionless extra pressure drop  $\mathcal{P} ( De, \beta )$ . Our experimental data is typically scaled with the *measured*  $\Delta P'_{24}(Q, De = 0)$ . For comparing pressure drops associated with different contraction-expansions we choose to scale our data with the Sampson solution of Equation 1. **We would be very interested in hearing from other groups performing numerical simulations of the validity of the linear decomposition of Equation 1.**



**Figure 2:** Comparison of dimensionless pressure drops for  $\rho = 0$  &  $0.5R_2$ .

For a copy of any or all of these data sets please contact Jonathan Rothstein by email at [jproth@mit.edu](mailto:jproth@mit.edu).

## References

- [1] R.A. Sampson. Phil. Trans. Roy. Soc., **182** (1891) 449.
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