

Two-photon exchange in elastic electron-proton scattering : QCD factorization approach

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We estimate the two-photon exchange contribution to elastic electron-proton scattering at large momentum transfer Q^2 . It is shown that the leading two-photon exchange amplitude behaves as $1/Q^4$ relative to the one-photon amplitude, and can be expressed in a model independent way in terms of the leading twist nucleon distribution amplitudes. Using several models for the nucleon distribution amplitudes, we provide estimates for existing data and for ongoing experiments.

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Elastic electron-nucleon scattering in the one-photon (1γ) exchange approximation is a time-honored tool for accessing information on the structure of the nucleon. Precision measurements of the proton electric to magnetic form factor ratio at larger Q^2 using polarization experiments [1, 2, 3] have revealed significant discrepancies in recent years with unpolarized experiments using the Rosenbluth technique [4]. As no experimental flaw in either technique has been found, two-photon (2γ) exchange processes are the most likely culprit to explain this difference. Their study has received a lot of attention lately, see [5] for a recent review (and references therein), and [6] for a recent global analysis of elastic electron-proton (ep) scattering including 2γ corrections. In this work we calculate the leading in Q^2 behavior of the elastic ep scattering amplitude with hard 2γ exchange.

To describe the elastic ep scattering, $l(k) + N(p) \rightarrow l(k') + N(p')$, we adopt the definitions : $P = (p + p')/2$, $K = (k + k')/2$, $q = k - k' = p' - p$, and choose $Q^2 = -q^2$ and $\nu = K \cdot P$ as the independent kinematical invariants. Neglecting the electron mass, it was shown in [7] that the T -matrix for elastic ep scattering can be expressed through 3 independent Lorentz structures as :

$$T_{h, \lambda'_N \lambda_N} = \frac{e^2}{Q^2} \bar{u}(k', h) \gamma_\mu u(k, h) \quad (1)$$

$$\times \bar{u}(p', \lambda'_N) \left(\tilde{G}_M \gamma^\mu - \tilde{F}_2 \frac{P^\mu}{M} + \tilde{F}_3 \frac{\gamma \cdot K P^\mu}{M^2} \right) u(p, \lambda_N),$$

where e is the proton charge, M is the proton mass, $h = \pm 1/2$ is the electron helicity and λ_N (λ'_N) are the helicities of the incoming (outgoing) proton. In Eq. (1), \tilde{G}_M , \tilde{F}_2 , \tilde{F}_3 are complex functions of ν and Q^2 . To separate the 1γ and 2γ exchange contributions, it is furthermore useful to introduce the decompositions : $\tilde{G}_M = G_M + \delta\tilde{G}_M$, and $\tilde{F}_2 = F_2 + \delta\tilde{F}_2$, where G_M (F_2) are the proton magnetic (Pauli) form factors (FFs) respectively, defined from the matrix element of the electromagnetic current, with $G_M(0) = \mu_p = 2.79$ the proton magnetic moment. The amplitudes \tilde{F}_3 , $\delta\tilde{G}_M$ and $\delta\tilde{F}_2$

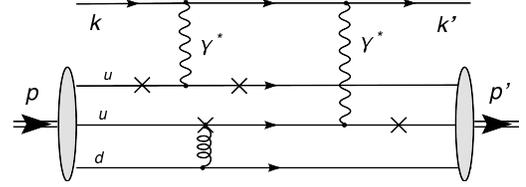


FIG. 1: Typical graph for the elastic ep scattering with two hard photon exchanges. The crosses indicate the other possibilities to attach the gluon. The third quark is conventionally chosen as the d -quark. There are other diagrams where the one photon is connected with u - and d - quarks. We do not show these graphs for simplicity.

originate from processes involving at least 2γ exchange and are of order e^2 (relative to the factor e^2 in Eq. (1)).

The leading perturbative QCD (pQCD) contribution to the 2γ exchange correction to the elastic ep amplitude is given by a convolution integral of the proton distribution amplitudes (DAs) with the hard coefficient function as shown in Fig. 1. In the hard regime, where $Q^2, s \gg M^2$, we calculate the amplitude in the Breit system, where the initial and final proton momenta correspond to two opposite light-like directions :

$$p \simeq Q \frac{\bar{n}}{2}, \quad \text{with } \bar{n} = (1, 0, 0, 1),$$

$$p' \simeq Q \frac{n}{2}, \quad \text{with } n = (1, 0, 0, -1),$$

with $(n \cdot \bar{n}) = 2$. The lepton kinematics are given by :

$$k = \zeta Q \frac{n}{2} - \bar{\zeta} Q \frac{\bar{n}}{2} + k_\perp, \quad k' = -\bar{\zeta} Q \frac{n}{2} + \zeta Q \frac{\bar{n}}{2} + k_\perp,$$

where, at large Q^2 , ζ can be determined from $s \simeq \zeta Q^2$, and $u \simeq -\bar{\zeta} Q^2$, with $\bar{\zeta} \equiv 1 - \zeta$, and $\zeta \geq 1$. Furthermore, the transverse vector in the lepton kinematics is determined from : $k_\perp^2 = -\zeta \bar{\zeta} Q^2$.

Let us now consider the proton matrix element which appears in the graph of Fig. 1. Following the notation from [8], the proton matrix element is described at lead-

ing twist level by three nucleon DAs as :

$$\begin{aligned}
& 4 \left\langle 0 \left| \varepsilon^{ijk} u_\alpha^i(a_1 \lambda n) u_\beta^j(a_2 \lambda n) d_\sigma^k(a_3 \lambda n) \right| p \right\rangle \\
&= V p^+ \left[\left(\frac{1}{2} \bar{n} \cdot \gamma \right) C \right]_{\alpha\beta} [\gamma_5 N^+]_\sigma \\
&+ A p^+ \left[\left(\frac{1}{2} \bar{n} \cdot \gamma \right) \gamma_5 C \right]_{\alpha\beta} [N^+]_\sigma \\
&+ T p^+ \left[\frac{1}{2} i \sigma_{\perp \bar{n}} C \right]_{\alpha\beta} [\gamma^\perp \gamma_5 N^+]_\sigma, \quad (2)
\end{aligned}$$

with light-cone momentum $p^+ = Q$, where C is charge conjugation matrix : $C^{-1} \gamma_\mu C = -\gamma_\mu^T$, and where $X = \{A, V, T\}$ stand for the nucleon DAs which are defined by the light-cone matrix element :

$$X(a_i, \lambda p^+) = \int d[x_i] e^{-i\lambda p^+(\sum x_i a_i)} X(x_i), \quad (3)$$

with

$$d[x_i] \equiv dx_1 dx_2 dx_3 \delta(1 - \sum x_i).$$

In general, the following properties are valid

$$\begin{aligned}
V(x_1, x_2, x_3) &= V(x_2, x_1, x_3), \\
A(x_1, x_2, x_3) &= -A(x_2, x_1, x_3), \\
T(x_1, x_2, x_3) &= \frac{1}{2} [V - A](1, 3, 2) + \frac{1}{2} [V - A](2, 3, 1),
\end{aligned}$$

i.e. we have only two independent functions.

In the large Q^2 limit, the pQCD calculation of Fig. 1 involves 24 diagrams, and leads to hard 2γ corrections to $\delta\tilde{G}_M$, and $\nu/M^2\tilde{F}_3$, which are found as :

$$\begin{aligned}
\delta\tilde{G}_M &= -\frac{\alpha_{em}\alpha_S(\mu^2)}{Q^4} \left(\frac{4\pi}{3!} \right)^2 (2\zeta - 1) \\
&\times \int d[y_i] d[x_i] \frac{4x_2 y_2}{D} \\
&\times \{ Q_u^2 [(V' + A')(V + A) + 4T'T](3, 2, 1) \\
&+ Q_u Q_d [(V' + A')(V + A) + 4T'T](1, 2, 3) \\
&+ Q_u Q_d 2[V'V + A'A](1, 3, 2) \}, \quad (4)
\end{aligned}$$

and

$$\begin{aligned}
\frac{\nu}{M^2} \tilde{F}_3 &= -\frac{\alpha_{em}\alpha_S(\mu^2)}{Q^4} \left(\frac{4\pi}{3!} \right)^2 (2\zeta - 1) \\
&\times \int d[y_i] d[x_i] \frac{2(x_2 \bar{y}_2 + \bar{x}_2 y_2)}{D} \\
&\times \{ Q_u^2 [(V' + A')(V + A) + 4T'T](3, 2, 1) \\
&+ Q_u Q_d [(V' + A')(V + A) + 4T'T](1, 2, 3) \\
&+ Q_u Q_d 2[V'V + A'A](1, 3, 2) \}, \quad (5)
\end{aligned}$$

with quark charges $Q_u = +2/3$, $Q_d = -1/3$, $\alpha_{em} = e^2/(4\pi)$, $\alpha_S(\mu^2)$ is the strong coupling constant evaluated at scale μ^2 , and the denominator factor D is defined as :

$$\begin{aligned}
D &\equiv (y_1 y_2 \bar{y}_2) (x_1 x_2 \bar{x}_2) [x_2 \bar{\zeta} + y_2 \zeta - x_2 y_2 + i\varepsilon] \\
&\times [x_2 \zeta + y_2 \bar{\zeta} - x_2 y_2 + i\varepsilon]. \quad (6)
\end{aligned}$$

The unprimed (primed) quantities in Eqs. (4, 5) refer to the DAs in the initial (final) proton respectively. Eqs. (4, 5) are the central result of the present work. One notices that at large Q^2 , the leading behavior for $\delta\tilde{G}_M$ and $\nu/M^2\tilde{F}_3$ goes as $1/Q^4$. In contrast, the invariant $\delta\tilde{F}_2$ is suppressed in this limit and behaves as $1/Q^6$.

It is interesting to point out that the scaling behavior for the 2γ amplitude obtained in the present calculation differs from the handbag calculation of [9, 10]. Whereas the present calculation gives a model independent leading behavior of $1/Q^4$ for the 2γ amplitude relative to the 1γ amplitude, the Q^2 behavior of the 2γ amplitude within the handbag calculation depends on the specific modeling of the generalized parton distributions. As an example, the modified Regge parameterization considered in [9, 10] leads to a calculation which is of higher twist compared to the leading pQCD calculation considered here.

To evaluate the convolution integrals in Eqs. (4, 5), we need to insert a model for the nucleon twist-3 DAs, V , A , and T . The asymptotic behavior of the DAs and their first conformal moments were given in [8] as :

$$\begin{aligned}
V(x_i) &\simeq 120x_1 x_2 x_3 f_N [1 + r_+(1 - 3x_3)], \\
A(x_i) &\simeq 120x_1 x_2 x_3 f_N r_-(x_2 - x_1), \\
T(x_i) &\simeq 120x_1 x_2 x_3 f_N \left[1 + \frac{1}{2} (r_- - r_+) (1 - 3x_3) \right] \quad (7)
\end{aligned}$$

and depend on three parameters : f_N , r_- and r_+ . In this work, we will provide calculations using two models for the DAs that were discussed in the literature. The corresponding parameters (at $\mu = 1$ GeV) are given in Table I, and are compared with recent lattice QCD calculations (QCDSF [13]), extrapolated to the chiral limit. One notices that the parameters r_- and r_+ in the BLW model for the proton DA are totally compatible with the lattice results, whereas the value of the overall normalization f_N for the lattice DA is about 2/3 smaller than the BLW value. Below, we will provide calculations using the models COZ and BLW, and note that a good estimate using the lattice DA can be directly obtained from our figures by scaling the BLW result by a factor $\approx 2/3$.

	f_N (10^{-3} GeV 2)	r_-	r_+
COZ [11]	5.0 ± 0.5	4.0 ± 1.5	1.1 ± 0.3
BLW [12]	5.0 ± 0.5	1.37	0.35
QCDSF [13]	3.23 $\pm 0.06 \pm 0.09$	1.06 $\pm 0.09 \pm 0.31$	0.33 $\pm 0.03 \pm 0.11$

TABLE I: Parameters entering the proton DA (at $\mu = 1$ GeV) for two models (COZ, BLW) used in this work. For comparison we also show a recent lattice evaluation (QCDSF).

We next calculate the effect of hard 2γ exchange, given through Eqs. (4, 5), on the elastic ep scattering observables. The general formulas for the observables including

the 2γ corrections $\delta\tilde{G}_M$, $\delta\tilde{F}_2$, and $\nu/M^2\tilde{F}_3$ were derived in [7], to which we refer for the corresponding expressions.

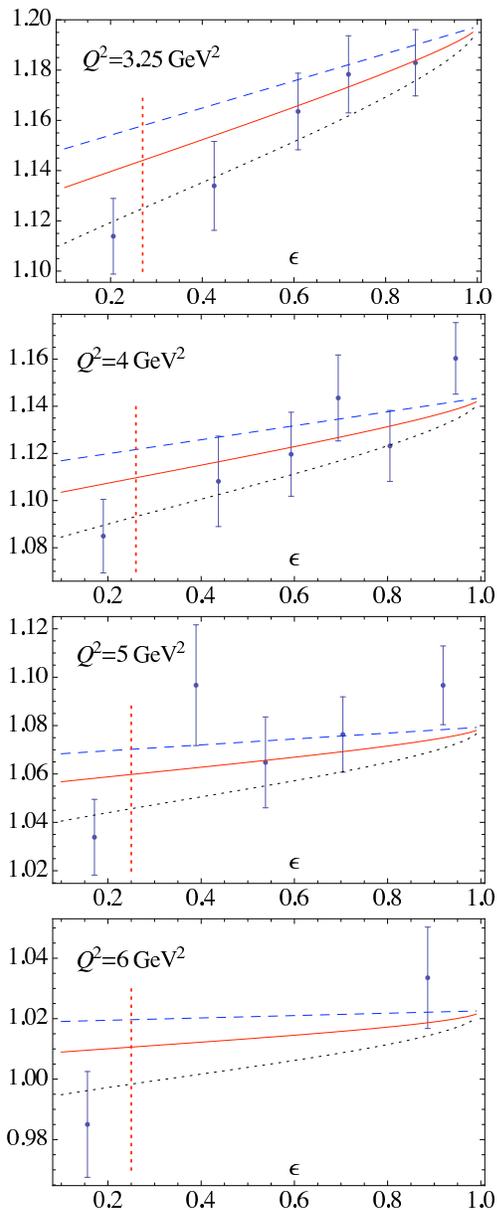


FIG. 2: Rosenbluth plots for elastic ep scattering: σ_R divided by $\mu_p^2/(1 + Q^2/0.71)^4$. Dashed (blue) curves : 1γ exchange, using the G_{Ep}/G_{Mp} ratio from polarization data [1, 2, 3]. Solid red (dotted black) curves show the effect including hard 2γ exchange calculated with the BLW (COZ) model for the proton DAs. The vertical dotted line shows the boundary where the lhs of Eq. (8) is 0.5. The data are from Ref. [4].

In Fig. 2, we calculate the reduced cross section σ_R as a function of the photon polarization parameter ϵ and different values of Q^2 . In the 1γ exchange, $\sigma_R = G_M(Q^2) + \epsilon/\tau G_E(Q^2)$, with $\tau = Q^2/(4M^2)$, and the Rosenbluth plot is linear in ϵ , indicated by the dashed straight lines in Fig. 2. The effect including the hard 2γ exchange is shown for both the COZ and BLW mod-

els of the proton DAs. One sees that including the 2γ exchange changes the slope of the Rosenbluth plot, and that sizeable non-linearities only occur for ϵ close to 1. The inclusion of the hard 2γ exchange is able to well describe the polarization dependence of the unpolarized data, when using the polarization data [1, 2, 3] for the proton FF ratio G_{Ep}/G_{Mp} as input. Quantitatively, the COZ model for the nucleon DA leads to a correction about twice as large as when using the BLW model. The question arises as to the applicability of the hard scattering calculation for the Q^2 values of the data shown in Fig. 2. On the one hand, the argument of the running coupling α_s is defined by the renormalization scale $\mu \sim Q$, which should be sufficiently large to validate a pQCD calculation. On the other hand, \tilde{F}_3 contains a logarithmic singularity when $\epsilon \rightarrow 0$ (i.e. when $\zeta \rightarrow 1$). Therefore, the applicability of the hard description is restricted by the condition :

$$\alpha_s(Q^2) |\ln(1 - 1/\zeta)| \ll 1. \quad (8)$$

In Fig.2 we indicate a boundary by dashed vertical lines, where $\alpha_s(Q^2) |\ln(1 - 1/\zeta)| \leq 0.5$, which corresponds with $\epsilon \gtrsim 0.25$. We like to note here that in contrast to the pQCD treatment of the proton FFs, which requires two hard gluon exchanges, the 2γ correction to elastic ep scattering only requires one hard gluon exchange. One therefore expects the pQCD calculation to set in for Q^2 values in the few GeV^2 range, which is well confirmed by the results shown in Fig. 2.

The real part of the 2γ exchange amplitude can be accessed directly as the deviation from unity of the ratio of e^+/e^- elastic scattering. The precision of past experiments performed at SLAC [14], was not sufficient to see a clear deviation from unity over a large range in ϵ . Presently, several new experiments are planned or are underway at VEPP-3 [15], JLab/CLAS [16], and Olympus@DESY [17] to make precision measurements of the e^+/e^- ratio in elastic scattering off a proton. In Fig. 3, we show the predictions for the $\sigma_{e^+p}/\sigma_{e^-p}$ ratio for different values of Q^2 and ϵ planned by the Olympus@DESY experiment [17]. In order to make a comparison with our pQCD calculations, we only show kinematics for which $Q^2 > 2 \text{ GeV}^2$. We also made an estimate of the theoretical uncertainty of our predictions. For this purpose, we varied the normalization scale μ (entering α_s) and the normalization f_N (in units 10^{-3} GeV^2) for both COZ and BLW models of the proton DAs over the ranges : $Q^2/2 < \mu^2 < Q^2$, and $4.5 < f_N < 5.5$. Furthermore, for the COZ model, we also varied the parameters r_+ and r_- in the range : $0.8 < r_+ < 1.4$, and $2.5 < r_- < 5.5$. The resulting theoretical error estimate is also indicated on Fig. 3. The Olympus@DESY experiment aims at a statistical precision of the e^+/e^- cross section ratio of better than one percent for an average $Q^2 = 2.2 \text{ GeV}^2$. Our calculations predict a deviation from unity for these ratios in the range 2.5% (BLW) to 5% (COZ), allowing for an unambiguous test with the upcoming measurements.

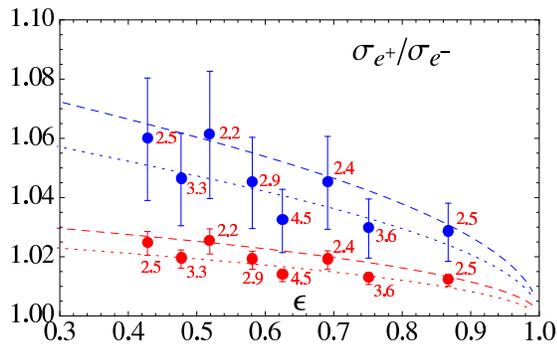


FIG. 3: Predictions for the ratio $\sigma_{e^+}/\sigma_{e^-}$ for different values of Q^2 (shown by numbers in GeV^2) and ϵ planned by the Olympus@DESY [17] experiment (we only show kinematics for which $Q^2 > 2 \text{ GeV}^2$). The upper blue (lower red) points correspond with the COZ (BLW) models. The error-bars show the theoretical uncertainties due to the parameters of the proton DAs and the running coupling scale, as described in the text. For comparison, we also show the theoretical curves for the average values $Q^2 = 2.4 \text{ GeV}^2$ (dashed curves) and $Q^2 = 3.25 \text{ GeV}^2$ (dotted curves) for both the COZ (two upper curves) and BLW (two lower curves) models.

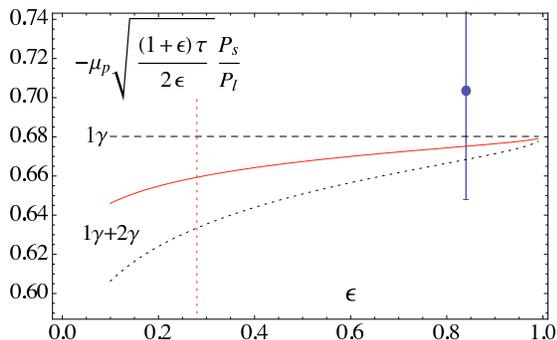


FIG. 4: The ratio P_s/P_l as a function of ϵ for $Q^2 = 2.5 \text{ GeV}^2$. The horizontal curve is the result of the 1γ calculation. The dotted black (solid red) curves correspond to the $1\gamma + 2\gamma$ calculations using the COZ (BLW) models for the proton DAs. The data point is from the JLab/Hall A experiment [1, 2].

In Fig. 4, we show the ratio of the proton recoil polarization components P_s/P_l as a function of ϵ for $Q^2 = 2.5 \text{ GeV}^2$. The 1γ prediction is given by the horizontal line. Our $1\gamma + 2\gamma$ prediction yields a negative correction which increases with decreasing ϵ . Around $\epsilon = 0.3$ it reduces the P_s/P_l ratio by 2 % for the BLW model, and by around 4 % for the COZ model. The JLab/Hall A experiment [1, 2] has measured this ratio at a large ϵ value around 0.85. A new JLab/Hall C experiment [18], which is currently under analysis, has recently measured this ratio at $Q^2 = 2.5 \text{ GeV}^2$ for three ϵ values between 0.15 and 0.8. The expected experimental precision of around 1% for this ratio, will therefore allow to test our predictions, which are in the 2 – 5 % range.

The observables discussed above test the real parts of the 2γ amplitudes. The imaginary parts of Eqs. (4,

5), arising from lepton propagator singularities, can be tested by polarizing the target or recoiling proton perpendicular to the scattering plane.

In summary, we calculated the leading in Q^2 behavior of the 2γ exchange contribution to elastic ep scattering. It was found that the leading 2γ amplitude is given by processes involving one hard gluon exchange, resulting in a $1/Q^4$ behavior of the 2γ amplitude relative to the 1γ amplitude. We expressed the leading 2γ amplitude in terms of the leading twist nucleon DAs. Using two models for the nucleon DAs, we found that, for Q^2 in the few GeV^2 range, these calculations can quantitatively explain the slope of the Rosenbluth plot when using the G_{Ep}/G_{Mp} polarization data as input. Furthermore, we have shown that ongoing and planned elastic ep scattering experiments both for the ϵ dependence of the recoil polarization ratio P_s/P_l as well as for the e^+/e^- ratio, have the precision to test our predictions.

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