COLUMBIA
BUSINESS
SCHOOL

On the Choice-Based LP Model for Network Revenue Management

Garrett J. van Ryzin

Paul M. Montrone Professor of Private Enterprise
Columbia University
Graduate School of Business

gjv1@columbia.edu

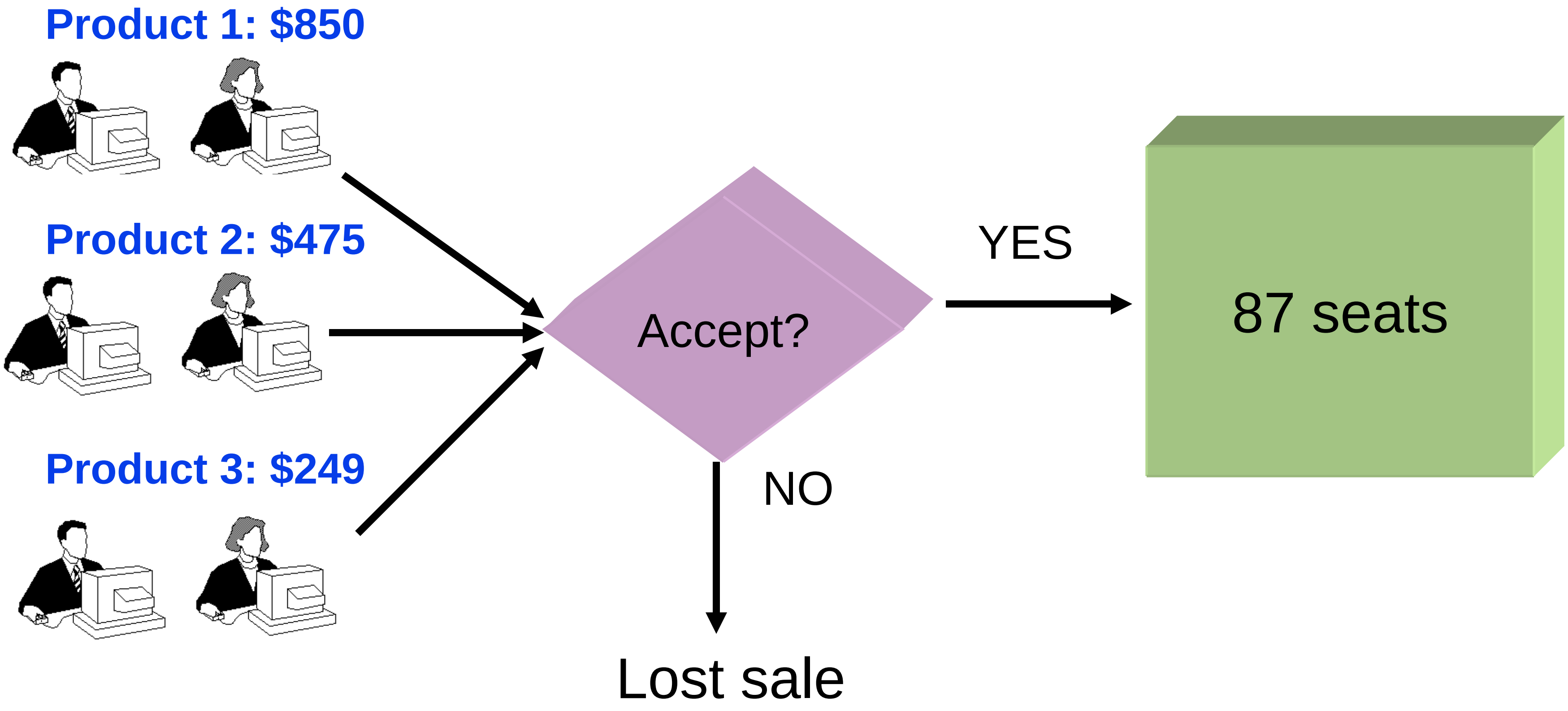
(This work is joint with Qian Liu, Columbia Univ.)

Outline

I. Why choice-based RM is important
II. The choice-based LP of Gallego et al. (2004)
III. Further results on the choice-based LP
    - Asymptotic optimality
    - Efficient sets
    - A DP-based decomposition algorithm
IV. Summary & conclusions

Traditional airline/hotel RM models

Given requests for different products…



Product 1: $850

Product 2: $475

Product 3: $249

Accept?

YES

87 seats

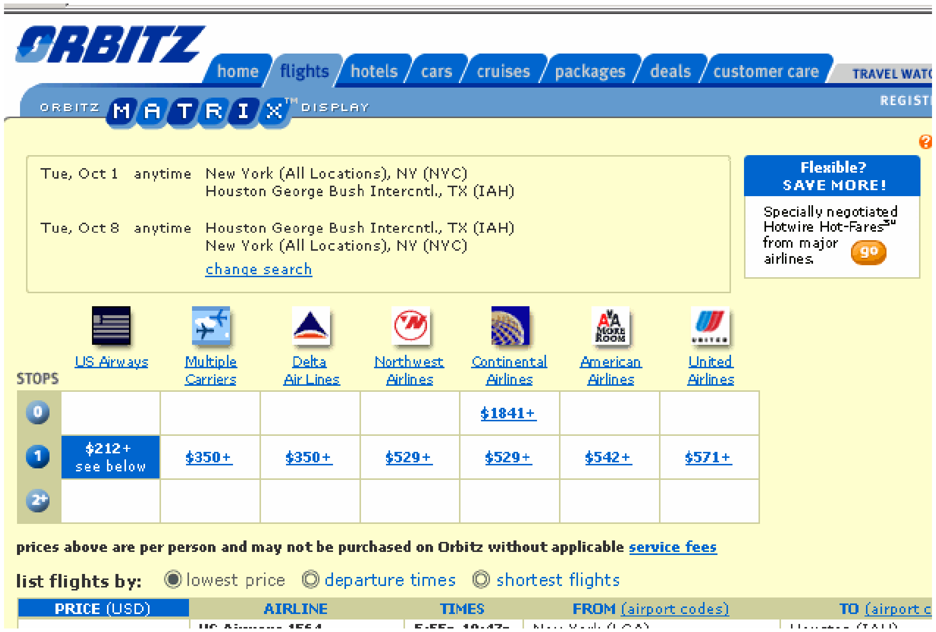
NO

Lost sale

… decide which ones to accept or reject

But demand for products is really the <u>outcome</u> of a consumer choice decision …

Which airline, route, dep. time and fare class to buy?



Tue, Oct 1 anytime New York (All Locations), NY (NYC)
Houston George Bush Intercntl., TX (IAH)

Tue, Oct 8 anytime Houston George Bush Intercntl., TX (IAH)
New York (All Locations), NY (NYC)

change search

Flexible? SAVE MORE!

Specially negotiated Hotwire Hot-Fares℠ from major airlines. go

STOPS	US Airways	Multiple Carriers	Delta Air Lines	Northwest Airlines	Continental Airlines	American Airlines	United Airlines
0					$1841+		
1	$212+ see below	$350+	$350+	$529+	$529+	$542+	$571+
2+							

prices above are per person and may not be purchased on Orbitz without applicable service fees

list flights by: ● lowest price ○ departure times ○ shortest flights

PRICE (USD)	AIRLINE	TIMES	FROM (airport codes)	TO (airport c

To buy now or wait?

Why is choice behavior important?

Choice among fare products: Buy-up and buy-down effects

Flight XY123

Y $320 Unrestricted

Q $189 SA stay required
Nonrefundable

Choice among multiple departure times between same OD

6:50 AM

Open: Y, M, B, Q

9:15 AM

Open: Y, M

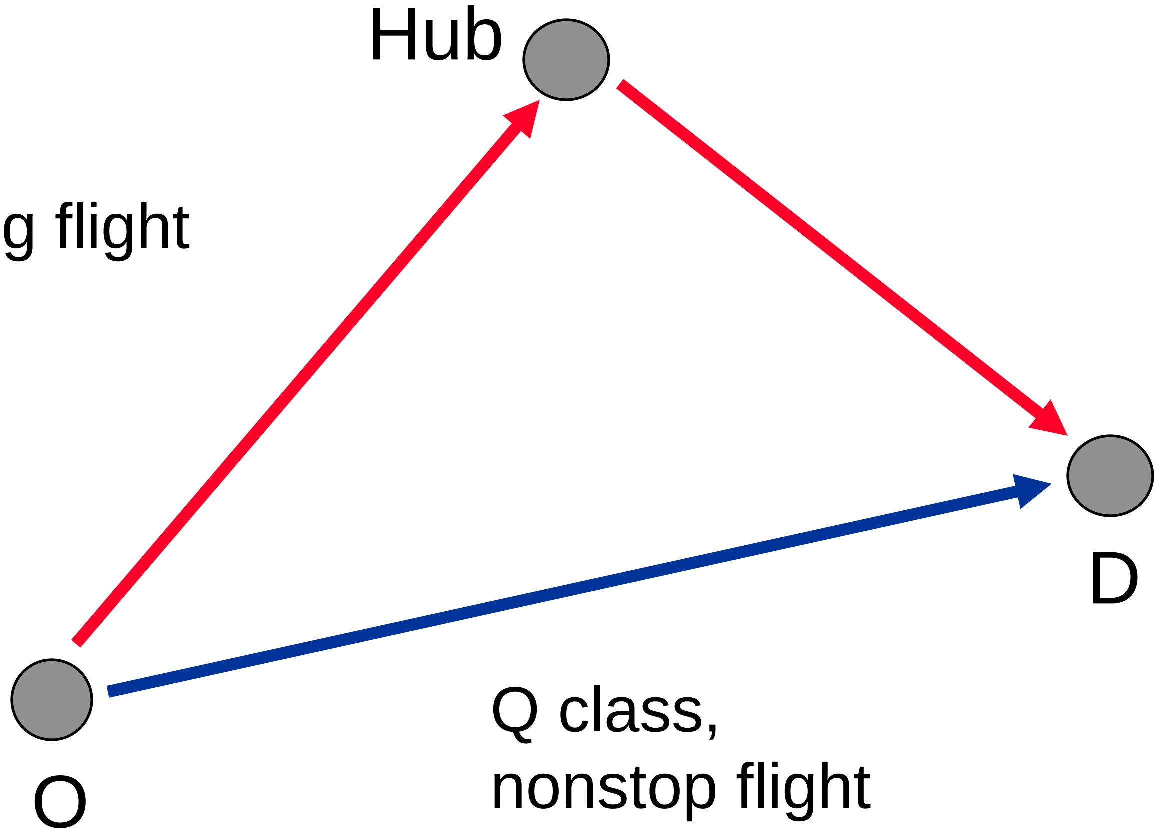
Closed: Q,B

1:10 PM

Open: Y, M, B

Closed: Q

Choice among different routings



Q class,
connecting flight
$189

Q class,
nonstop flight
$279

Undifferentiated, low fare structures

jetBlue
AIRWAYS

○	Web Fare Regular Fare	$94.00 USD $99.00 USD	Wed, 17 Dec 2003 Flight 45	8:20 am 11:05 am	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)
○	Web Fare Regular Fare	$94.00 USD $99.00 USD	Wed, 17 Dec 2003 Flight 41	9:20 am 12:05 pm	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)
○	Web Fare Regular Fare	$94.00 USD $99.00 USD	Wed, 17 Dec 2003 Flight 59	10:45 am 1:25 pm	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)
○	Web Fare Regular Fare	$94.00 USD $99.00 USD	Wed, 17 Dec 2003 Flight 69	1:00 pm 3:40 pm	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)
○	Web Fare Regular Fare	$94.00 USD $99.00 USD	Wed, 17 Dec 2003 Flight 31	4:00 pm 6:45 pm	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)
○	Web Fare Regular Fare	$94.00 USD $99.00 USD	Wed, 17 Dec 2003 Flight 359	5:20 pm 8:05 pm	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)
○	Web Fare Regular Fare	$94.00 USD $99.00 USD	Wed, 17 Dec 2003 Flight 43	6:55 pm 9:50 pm	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)

○	Web Fare Regular Fare	$224.00 USD $229.00 USD	Sun, 21 Dec 2003 Flight 45	8:20 am 11:05 am	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)
○	Web Fare Regular Fare	$224.00 USD $229.00 USD	Sun, 21 Dec 2003 Flight 41	9:20 am 12:05 pm	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)
○	Web Fare Regular Fare	$224.00 USD $229.00 USD	Sun, 21 Dec 2003 Flight 59	10:45 am 1:25 pm	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)
○	Web Fare Regular Fare	$224.00 USD $229.00 USD	Sun, 21 Dec 2003 Flight 69	1:00 pm 3:40 pm	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)
○	Web Fare Regular Fare	$194.00 USD $199.00 USD	Sun, 21 Dec 2003 Flight 31	4:00 pm 6:45 pm	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)
○	Web Fare Regular Fare	$194.00 USD $199.00 USD	Sun, 21 Dec 2003 Flight 359	5:20 pm 8:05 pm	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)
○	Web Fare Regular Fare	$194.00 USD $199.00 USD	Sun, 21 Dec 2003 Flight 43	6:55 pm 9:50 pm	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)
○	Web Fare Regular Fare	$154.00 USD $159.00 USD	Sun, 21 Dec 2003 Flight 79	9:15 pm 11:59 pm	Depart Arrive	New York, NY (JFK) Orlando, FL (MCO)

17 Dec. 2003
$94 all departure

21 Dec. 2003
$154, $194 and $224
depending on departure

And competition is fundamentally about choice…

- Our high fare vs. their low fare
- Our 9:15AM departure vs. their 6:45AM departure
- Our connecting flight vs. their direct flight

Recent literature on choice-based RM

- Belobaba (1989)
  - “Buy-up factors” for EMSR-a
- Bremell et al. (1990) *Trans. Sci.*
  - Fare class allocation with dependent demand
- Phillips, R. (1994) State-contingent RM
- S.E. Anderssen of SAS (1998), *Int. Tran. OR*
  - MNL model used to predict buy-up/diversion
  - Used in simple buy-up heuristic
  - Cf. Algers & Besser (2001), *Int. J. of Serv. Tech.*
- Belobaba & Hopperstad (Boeing/MIT)
  - PODS (passenger origin-destination simulator)
  - Passenger choice behavior modeled
  - Extensive simulation studies of traditional forecasting & control methods
- Talluri & van Ryzin (*Mgmt. Sci.* 2004)
  - General single-leg problem with consumer choice model
  - Structural properties: “efficient sets”, nesting order
- Cooper & Zhang (2003)
  - Parallel flights with choice among flights within each demand class
- Van Ryzin & Vulcano (2003)
  - Simulation-based optimization on networks
- * Dubey, Gallego, Iyengar, Phillips (2003)
  - Deterministic choice-based LP

A Traditional Network RM Model

Deterministic linear program (DLP)

$$\max \sum_{j=1}^{n} r_j y_j$$ Max. total revenue

$$s.t. \quad Ay \le x \quad : \pi$$ Consumption less than capacity

$$0 \le y \le E[D]$$ Allocation less than mean demand

Decision variables: y_j = Capacity allocated to product j

Primal solution y^* and dual solution π used in a variety of ways to control availability (e.g. bid price control, DAVN, etc.)

Comments on the DLP

- It <u>approximates</u> a much more complex problem
  - Stochastic demand
  - Dynamic decision making over time
  - The “correct” model is really a large-scale dynamic program (DP), but such a model is essentially impossible to solve exactly

- Why use the DLP?
  - Asymptotically optimal (when capacity and demand are scaled up)
  - Good empirical performance
  - Outputs useful in various decomposition heuristics (e.g. DAVN, DP decomposition)

What is the choice equivalent model?

(Dubey, Gallego, Iyengar, Phillips (2003))

$$\max \sum_{S \subseteq N} R(S)\, t(S)$$

Max. total revenue

$$s.t. \quad \sum_{S \subseteq N} AP(S)\, t(S) \le x \quad : \pi$$

Consumption less than capacity

$$\sum_{S \subseteq N} t(S) \le T \quad : \sigma$$

Total time sets offered less than horizon length

S = subset of available (open) products (the offer set)

Decision variables: $t(S)$ = Total <u>time</u> subset S is offered

$$R(S) = \sum_{j \in S} r_j P_j(S)$$

Revenue rate

$$P(S) = (P_1(S), \ldots, P_n(S))$$

Product purchase rates

This LP has an exponential number of variables. How do we solve it?

- Gallego et al. show for special classes of choice models, LP can be solved relatively efficiently using column generation

$$\max_{S \subseteq N} \{ R(S) - AP(S)\pi \} \overset{?}{\geq} \sigma$$

Ex: Customers belong to L segment and each segment l

- Has a disjoint *consideration set* C_l
- Makes multinomial logit (MNL) choice among products in C_l

Then columns can be generated by a simple ranking procedure (linear complexity in $|C_l|$)

Comments on the choice-based DLP

- It too <u>approximates</u> a much more complex problem
  - Stochastic demand
  - Dynamic decision making over time
  - The “correct” model is really a large-scale dynamic program (DP), but again such a model is essentially impossible to solve exactly
- How good is the choice-based DLP solution?
  - Empirically
  - Theoretically
- How can its outputs be used to generate more realistic controls ala DAVN?
- What does it say about the structure of optimal offer sets?

Asymptotic optimality

Theorem: Consider scaled problem in which capacity and time are both increased by the same factor θ ….

Capacity $= \theta\, x$

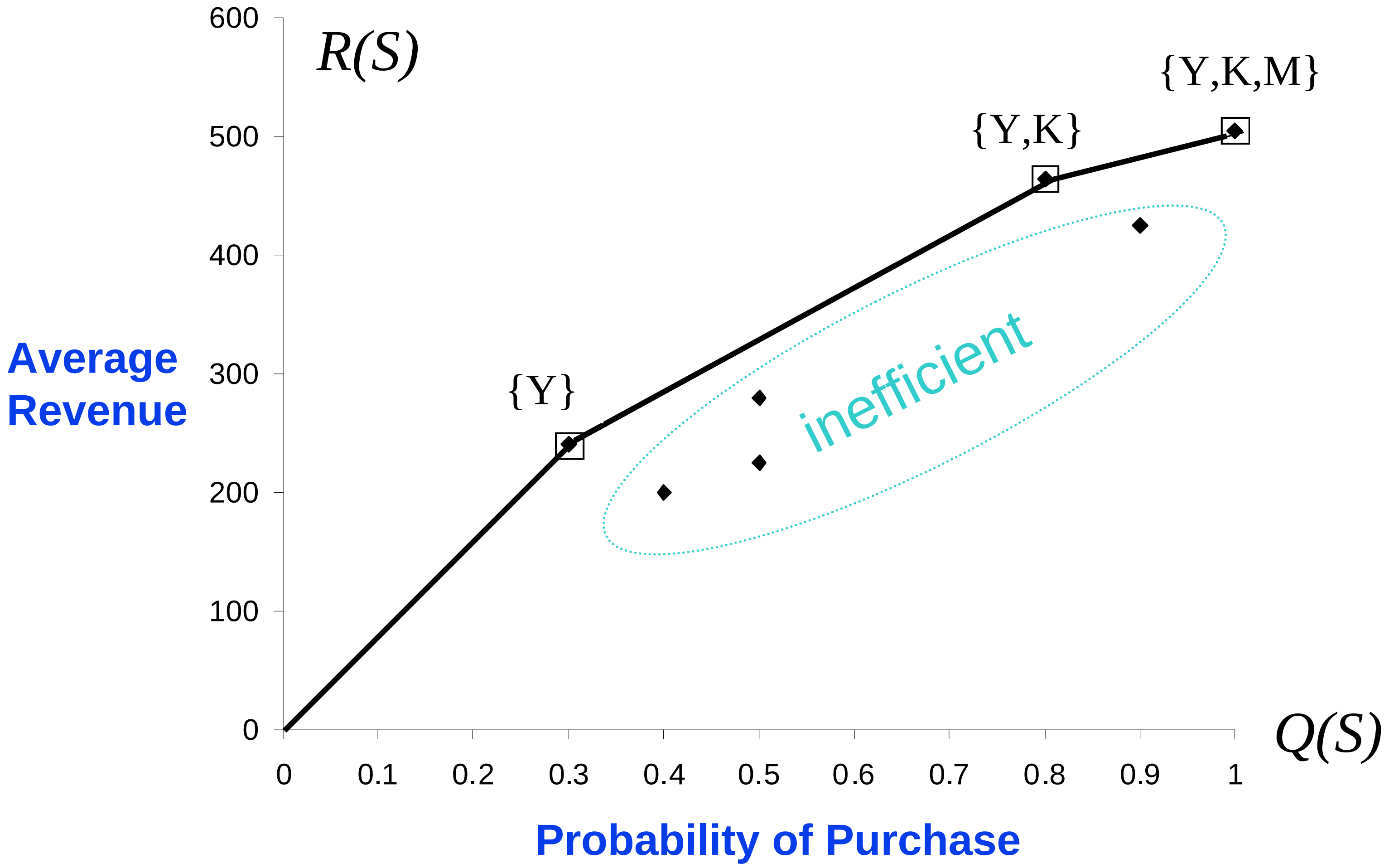
Time horizon $= \theta\, T$

Let $t^*(S)$ denote the optimal solution to the original choice-based LP. Then the solution $\theta\, t^*(S)$ is asymptotically optimal for the corresponding stochastic dynamic network choice problem (suitably defined).

What is the structure of the optimal offer sets?

“Efficient sets” in the single-leg choice model

Talluri and van Ryzin, *Management Science, 2004*



In this example there are 3 efficient set: {Y}, {Y,K}, {Y,K,M}
These are the only combinations we should consider!

A technical definition of efficiency…

Definition 1 *A set T is* <u>*inefficient*</u> *if there exist probabilities $\alpha(S), \forall S \subseteq N$ (including the null set $S = \emptyset$) with $\sum_{S \subseteq N} \alpha(S) = 1$ such that either*

$$Q(T) \geq \sum_{S \subseteq N} \alpha(S) Q(S) \quad \textit{and} \quad R(T) < \sum_{S \subseteq N} \alpha(S) R(S),$$

Otherwise, the set T is <u>*efficient*</u>*.*

…i.e., if a “mixture” of other sets consumes less capacity yet produces more revenue than T, then T is inefficient.

What is an efficient set on a network?

$$R(S) = \sum_{j \in S} r_j P_j(S) \qquad Q_i(S) = \sum_{j \in S} a_{ij} P_j(S),$$

Revenue rate

Consumption rate

Definition: An offer set of network products T is <u>inefficient</u> if there exits convex weights $\alpha(S)$ such that….

$$\begin{aligned} R(T) &< \sum_S \alpha(S) R(S) \\ Q(T) &\geq \sum_S \alpha(S) Q(S) \\ \alpha(S) &\geq 0 \quad \forall S \\ \sum_S \alpha(S) &= 1 \end{aligned}$$

Produces less revenue

Consumes as much capacity on each resource

Theorem: Efficient sets are the only ones used in the solution to the choice-based DLP. Moreover, asymptotically, inefficient sets are never optimal to use for the stochastic dynamic network choice problem.

Consequences:

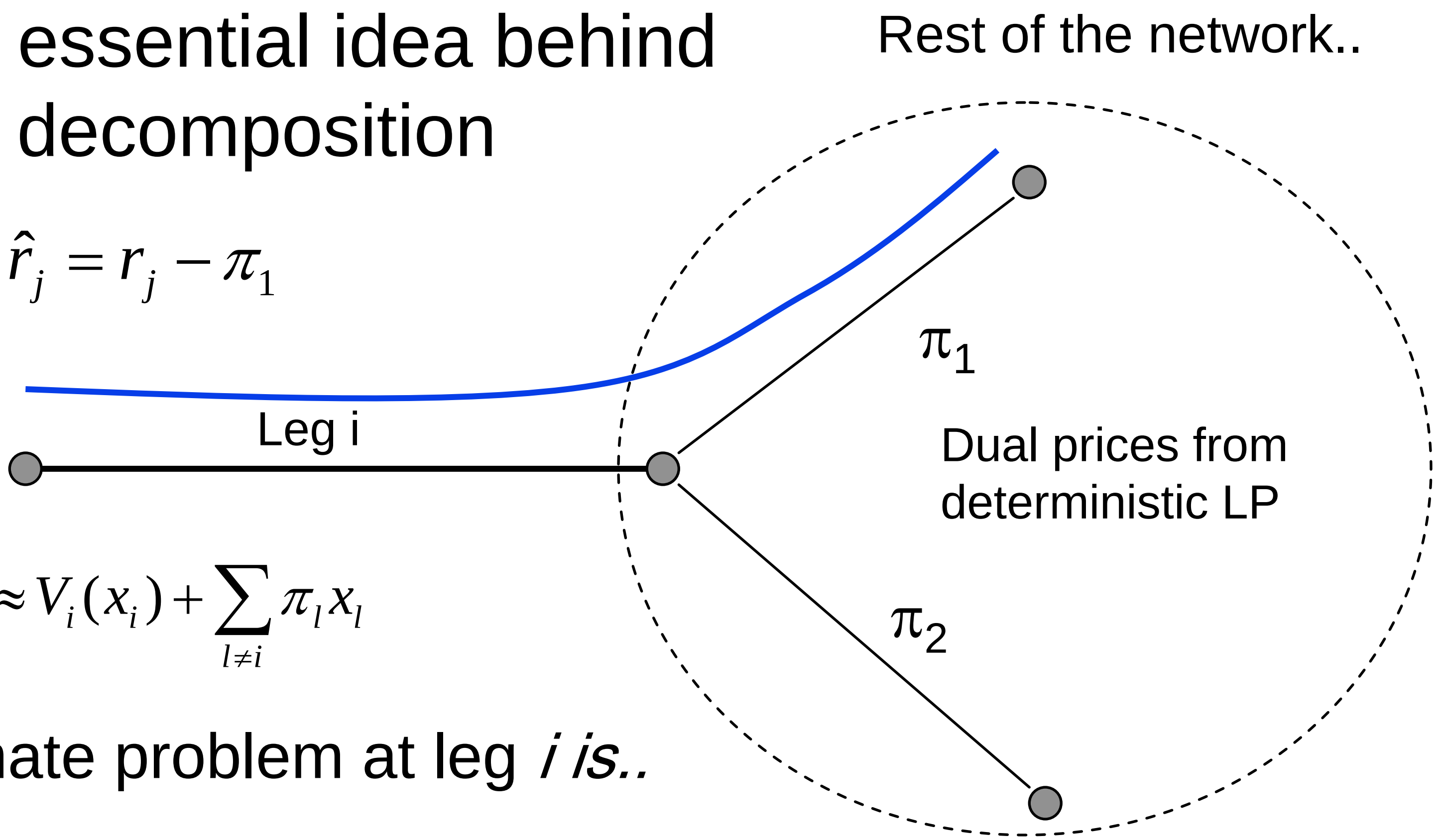
- Suggests we restrict our attention only to efficient sets.
- Efficient sets can be identified off-line

How do we use the choice-based DLP solution?

- Directly apply time variables $t^*(S)$
  (Gallego et al. 2004)

- Discard primal solution, but use dual information in a decomposition heuristic
  (van Ryzin & Liu 2004)

Recall the essential idea behind DAVN/DP decomposition



$$\hat{r}_j = r_j - \pi_1$$

$$V(x) \approx V_i(x_i) + \sum_{l \neq i} \pi_l x_l$$

So approximate problem at leg *i is..*

$$V_t^i(x) = \max_{u \in \{0,1\}^n} \left\{ \sum_{j \in A_i} \lambda_j (\hat{r}_j u_j + V_{t-1}^i(x - u_j)) + (1 - \lambda) V_{t-1}^i(x) \right\}$$

$$\lambda = \sum_{j=1}^{n} \lambda_j$$

$$= \sum_{j \in A_i} \lambda_j \max_{u_j \in \{0,1\}} \left\{ (\hat{r}_j - \Delta V_{t-1}^i(x)) u_j \right\} + V_{t-1}^i(x)$$

Solving the DP leads to control policy for leg i….

- Protection levels for leg i
- Dynamic bid prices for leg i

Repeat for each leg *i* to approximate complete network control policy

- Protection levels on each leg (DAVN)
- Dynamic bid prices on each leg (DP decomposition)

Suggests decomposition heuristic for network choice problem

1. Solve choice-based DLP to obtain optimal dual value π
2. For each leg i, approximate value function by

$$V(x) \approx V_i(x_i) + \sum_{l \neq i} \pi_l x_l$$

3. Solve n one-dimensional choice-based DPs to determine each $V_i(x_i)$
4. Form separable value function approximation by

$$V(x) \approx \sum_{i=1}^{n} V_i(x_i)$$

5. Compute dynamic offer sets from this approximation (simple ranking procedure for MNL choice model)

Some advantages of the method

- Computationally efficient (only need to solve m one-dimensional DPs)

- Produces true dynamic (capacity and time-dependent) offer sets decisions

- Promising improvements in our preliminary tests so far.

Ex: Two-fare class network, 7 legs, disjoint consideration sets with MNL choice model defined on each set

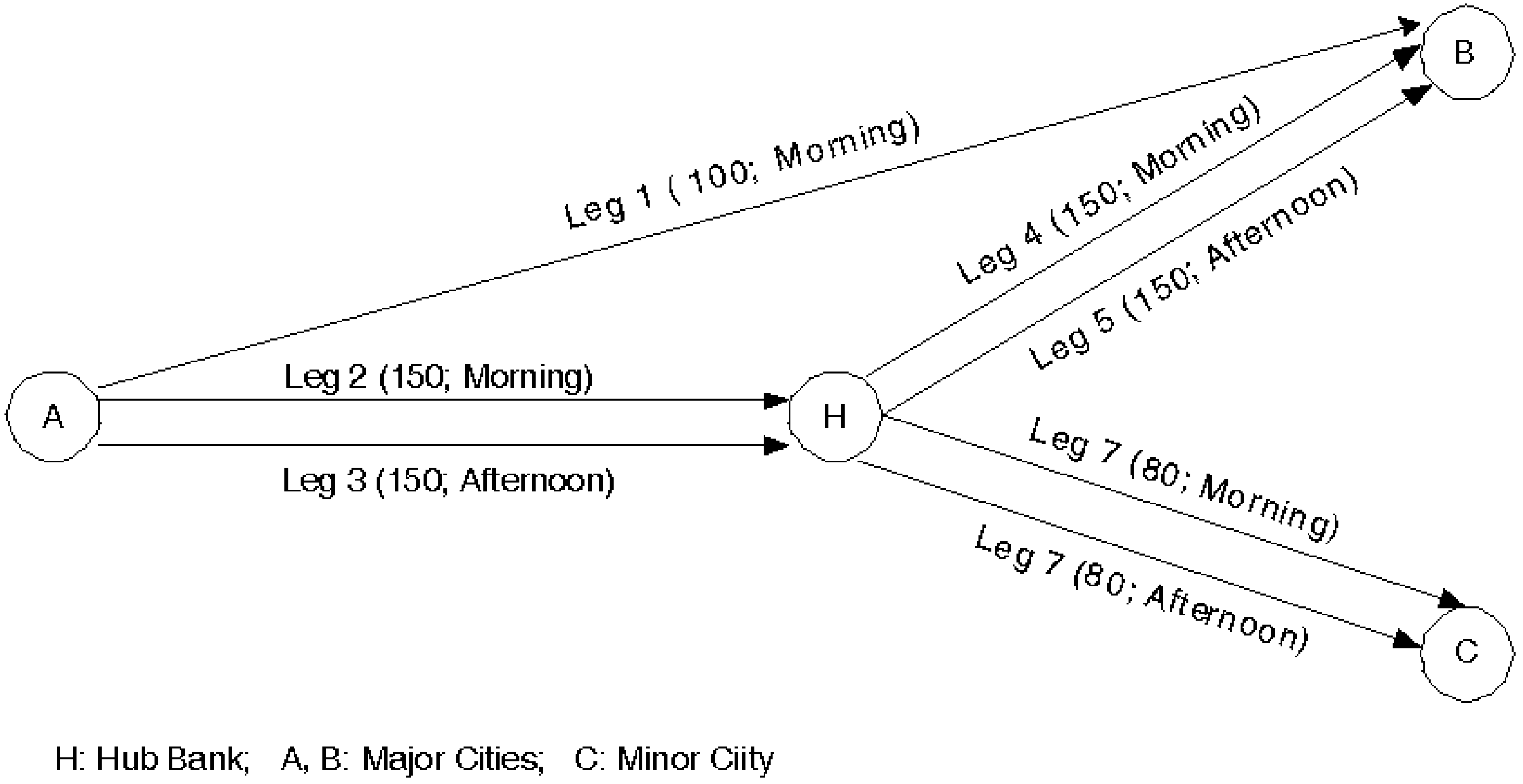


Figure 1: Two-fare classes airline network

22 Products

Product	Legs	Class	Revenue	Products	Legs	Class	Revenue
1	1	1	1000	12	1	2	500
2	2	1	400	13	2	2	200
3	3	1	400	14	3	2	200
4	4	1	300	15	4	2	150
5	5	1	300	16	5	2	150
6	6	1	500	17	6	2	250
7	7	1	500	18	7	2	250
8	{2,4}	1	600	19	{2,4}	2	300
9	{3,5}	1	600	20	{3,5}	2	300
10	{2,6}	1	700	21	{2,6}	2	350
11	{3,7}	1	700	22	{3,7}	2	350

Table 3: Product descriptions and revenues

Customer segments and preferences

Segment	Probability	Description	Consideration Set	Preference Value
1	0.08	AB Business	{1,8,9}	10,5,5
2	0.2	AB Leisure	{19,20,12}	10,10,5
3	0.05	AH Business	{2,3}	10,10
4	0.2	AH Leisure	{13,14}	10,10
5	0.1	HB Business	{4,5}	10,10
6	0.15	HB Leisure	{15,16}	10,5
7	0.02	HC Business	{6,7}	10,5
8	0.05	HC Leisure	{17,18}	10,10
9	0.02	AC Business	{10,11}	10,5
10	0.04	AC Leisure	{21,22}	10,10

Table 4: Segments and their characteristics

Performance comparisons without reoptimization

Less substitution

Lower load factors

α	v_0	UB Rev.	Heur. Rev.	Deter. Rev.	Indep. Rev.	BP Rev.	% Gap (Heur.)	% Gain (Deter.)	% Gain (Indep.)	% Gain (BP)
0.6	(0,0)	186400	173099	176391	178247	147166	-7.14	-1.87	-2.89	17.62
	(1,5)	181835	178527	170889	173263	153737	-1.82	4.47	3.04	16.12
	(5,10)	166017	162944	157739	160058	147739	-1.86	3.30	1.80	10.29
	(10,20)	149798	145954	142690	142477	138529	-2.57	2.29	2.44	5.36
0.8	(0,0)	227200	221181	216638	211798	191747	-2.65	2.10	4.43	15.35
	(1,5)	216062	212708	207150	202949	186844	-1.55	2.68	4.81	13.84
	(5,10)	194500	190035	186545	182256	169073	-2.30	1.87	4.27	12.40
	(10,20)	165560	164142	162609	160192	151124	-0.86	0.94	2.47	8.61
1.0	(0,0)	256000	250020	246568	235457	214114	-2.34	1.40	6.19	16.77
	(1,5)	244110	241207	235039	221974	209286	-1.19	2.62	8.66	15.25
	(5,10)	213833	212713	210270	199507	193513	-0.52	1.16	6.62	9.92
	(10,20)	171071	170963	170329	170484	170261	0.37	0.51	0.28	0.41
1.2	(0,0)	284000	269776	274193	254077	243492	-5.01	-1.61	6.18	10.79
	(1,5)	267429	264850	261102	239542	224190	-0.96	1.44	10.57	18.14
	(5,10)	217738	217946	217382	212847	211952	0.10	0.26	2.40	2.83
	(10,20)	171071	171381	171176	171120	171120	0.18	0.12	0.15	0.15
1.4	(0,0)	309000	306870	300841	271411	262543	-0.69	2.00	13.06	16.88
	(1,5)	269588	269789	268999	253301	251865	0.07	0.29	6.51	7.12
	(5,10)	217738	218090	217745	215627	215619	0.16	0.16	1.14	1.15
	(10,20)	171071	171381	171178	171120	171120	0.18	0.12	0.15	0.15

Table 5: Simulation results under the static estimate marginal capacity for disjoint segments

Performance comparisons with reoptimization (5 reopts)

Lower load factors

Less substitution

α	v_0	UB Rev.	Heur. Rev.	Deter. Rev.	Indep. Rev.	BP Rev.	% Gap (Heur.)	% Gain (Deter.)	% Gain (Indep.)	% Gain (BP)
0.6	(0,0)	186400	181886	181710	178814	174087	-2.42	0.10	1.72	4.48
	(1,5)	181835	179515	177088	173951	170340	-1.28	1.37	3.20	5.39
	(5,10)	166017	163501	161804	160303	157520	-1.52	1.05	2.00	3.80
	(10,20)	149798	146813	146047	142083	140695	-1.99	0.52	3.33	4.35
0.8	(0,0)	227200	222486	221168	212516	208565	-2.07	0.60	4.69	6.67
	(1,5)	216062	213631	211361	202426	199815	-1.13	1.07	5.54	6.91
	(5,10)	194500	191715	190913	181608	179613	-1.43	0.42	5.57	6.74
	(10,20)	165560	164013	163789	159644	158129	-0.93	0.14	2.74	3.72
1.0	(0,0)	256000	253452	251790	235695	232784	-1.00	0.66	7.53	8.88
	(1,5)	244110	241604	240051	221256	218956	-1.03	0.65	9.20	10.34
	(5,10)	213833	212302	211667	199037	197395	-0.72	0.30	6.66	7.55
	(10,20)	171071	170515	170447	170024	169782	-0.32	0.04	0.29	0.43
1.2	(0,0)	284000	280648	279699	253864	251321	-1.18	0.34	10.55	11.67
	(1,5)	267429	264605	263498	239018	236508	-1.06	0.42	10.71	11.88
	(5,10)	217738	217341	217357	212417	211486	-0.18	-0.01	2.32	2.77
	(10,20)	171071	170913	170976	170676	170676	-0.09	-0.04	0.14	0.14
1.4	(0,0)	309000	306623	304754	271434	268626	-0.77	0.61	12.96	14.14
	(1,5)	269588	269183	268924	252973	251629	-0.15	0.10	6.41	6.98
	(5,10)	217738	217497	217578	215087	215077	-0.11	-0.04	1.12	1.13
	(10,20)	171071	170915	170977	170677	170677	-0.09	-0.04	0.14	0.14

Table 6: Simulation results under the dynamic estimate marginal capacity for disjoint segments

Toward a more complete understanding of choice on a network

Traditional DLP Model ➡ Choice-based DLP Model

Asymptotic optimality of DLP ➡ Asymptotic optimality of choice-based DLP

Efficient sets on a single-leg ➡ Efficient sets on a network

Decomposition heuristics ➡ Choice-based decomposition heuristics