Multiperiod Pricing for Perishable Products
A Robust Optimization Approach

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4th Annual INFORMS Revenue Management and Pricing Section Conference, Boston
June 10-11, 2004
Agenda

- Description of model
- Literature review
- Terminology and problem formulation
- Robust demand case
- Computational algorithm
- Numerical examples
Description of model

- We study models for competitive pricing in a multi-period, oligopolistic market where each seller has predetermined starting inventory of a single perishable product for sale and demand is uncertain.

- We give theoretical and numerical results for the robust demand model.
Goals

- Modeling of competition in the market.
- Modeling of uncertainty in demand and concept of a robust policy.
- Existence of market equilibrium policies.
- Performance of algorithm for computing equilibrium.
- Numerical study of robust policies.
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Research Areas

- Monopolistic
  - Periodic production review
- Competitive
  - Fixed inventory
Literature review

- Periodic production-review models
  - Monopolist
    - Zipkin (2000)
    - Federgruen and Heching (1997)
    - Chen and Simchi-Levi (2002)
  - Competitive
    - Chan, Shen, Simchi-Levi and Swann (2001)
    - Cachon and Netessine (2003)
    - Vives (1999)
    - Petruzzi and Dada (1999)
Literature review

- Fixed inventory models
  - Non-competitive
    - McGill and van Ryzin (1999)
    - Bitran and Caldentey (2002)
      - Bitran and Mondschein (1997)
      - Bitran, Caldentey and Mondschein (1998)
      - Feng and Gallego (1995)
      - Gallego and Van Ryzin (1994)
      - Zhao and Zheng (2000)
  - Competitive
    - ???
Literature review

- Robust Optimization
  - Soyster (1973)
  - Bertsimas and Sim (2002)
  - Bertsimas and Thiele (2003)
Agenda

- Description of model
- Literature review
- **Terminology and problem formulation**
- Robust demand case
- Computational algorithm
- Numerical examples
## Terminology used

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$i \in I$</td>
<td>seller</td>
</tr>
<tr>
<td>$t \in T$</td>
<td>period</td>
</tr>
<tr>
<td>$C_i^t$</td>
<td>starting inventory of seller $i$</td>
</tr>
<tr>
<td>$p_i^t$</td>
<td>price set by seller $i$ for period $t$</td>
</tr>
<tr>
<td>$D_i^t$</td>
<td>protection level set by seller $i$ for period $t$</td>
</tr>
<tr>
<td>$d_i^t$</td>
<td>amount sold by seller $i$ in period $t$</td>
</tr>
</tbody>
</table>

Protection level $D_i^t$ is the amount of inventory reserved for sale in periods $t + 1$ or later by seller $i$. 
$h_1^t(p_1^t, p_2^t, \ldots, p_I^t)$
Seller $i$

$p_i$

Period 1

Period $t$

Period $T$
Seller 1 \( \ldots \) Seller \( i \) \( \ldots \) Seller \( I \)

\( p_1 \) \( \ldots \) \( p_i \) \( \ldots \) \( p_I \)

Period 1 \( \ldots \) Period \( t \) \( \ldots \) Period \( T \)
Problems

- **Best response problem**
  - What is the any individual seller’s best policy if she somehow knew the policy set by other sellers?

- **Market equilibrium problem**
  - Do there exist Nash equilibrium policies for the market?
    - Is there a set of policies – one for each seller such that no seller has the incentive to unilaterally deviate?
Applications

- Transportation
  - Single leg one way airfare pricing
- Communication
  - Bandwidth pricing
- Energy trading
  - Pricing of future contracts
- Hospitality
  - Advance hotel reservation pricing
Assumptions

- Single product and fixed perishability deadline.
- Perfect information about starting inventories of competitors, demand structure, etc.
- Demand is a function of only prices in current period.
- Objective of all sellers is revenue maximization over time horizon.
- Policies are declared at the beginning of time horizon.
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Robust demand

- The form of the demand function is known but the parameters are uncertain and assumed to be within an uncertainty set.

- Robust policy gives optimal payoff under adverse values of uncertain parameters (It is robust to the variation of parameters within the uncertainty set.)
Uncertainty set

- The uncertainty set for any $\xi_i$ is a general closed and convex set.

- We restrict the uncertainty set for the $\xi_i$'s for all periods jointly to be a Cartesian product of the uncertainty sets for each period separately.

$$u_i = u_i^1 \times u_i^2 \times \cdots \times u_i^T$$
Best response problem

$$\begin{align*}
\max_{p_i, d_i, D_i} & \quad \sum_{t=1}^{T} d_i^t p_i^t \\
\text{such that} & \quad d_i^t \leq h_i^t(p_i^t, \bar{p}_{-i}^t, \xi_i^t) \quad \forall \xi_i^t \in \mathcal{U}_i^t, \forall t \in T \\
& \quad \sum_{\tau=1}^{t} d_i^\tau \leq C_i - D_i^t \quad \forall t \in T \\
& \quad C_i \geq D_i^1 \geq \cdots \geq D_i^T = 0 \\
& \quad p_i^{t_{\min}} \leq p_i^t \leq p_i^{t_{\max}} \quad \forall t \in T \\
& \quad d_i^t \geq d_i^{t_{\min}} \quad \forall t \in T
\end{align*}$$
Conditions

1. Price is bounded.
2. $d_i^t$ is not allowed to fall to zero in any period.
3. $h_i^t()$ is a concave function for any fixed $\xi$.
4. $h_i^t()$ is decreasing in $p_i^t$ for any fixed $\xi$.
5. $-h()$ is strictly monotone for any fixed $\xi$. 
Results

- The best response problem and the variational inequality are equivalent and the variational inequality has a unique solution.
Results

- Joint quasi-variational inequality is equivalent to solving the variational inequalities for all sellers simultaneously.

- Joint quasi-variational inequality gives Nash equilibrium policy.
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Computation of equilibrium

Iterative Learning Algorithm
Convergence result

The iterative learning algorithm converges to an equilibrium pricing policy under some conditions.
Convergence rate

- Number of iterations of algorithm required to converge to a solution $\varepsilon$-close to the equilibrium is

$$O(c \ln(\varepsilon)), \text{ where } c = F(D, A, L)$$

- Each iteration involves solving $N$ robust best response problems

D: diam. of K,
L: Lipsch. Cont. h,
A: s-mon. $-h$
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- **Numerical examples**
Numerical examples

4 sellers $I = \{1, 2, 3, 4\}$
10 time periods $T = \{1, \cdots, 10\}$
Starting inventories $C = \{1000, 1300, 1500, 2800\}$

$$h_i^t(p_i^t, p_{-i}^t, \xi_i^t) = D_{i_{base}}^t - \beta_i^t p_i^t + \sum_{j \in I, j \neq i} \alpha_j^t p_j^t \quad \forall \ i \in I, \ t \in T$$

where $\xi_i^t = (D_{i_{base}}^t, \beta_i^t, \alpha_{-i}^t)$ can take any value in an uncertainty set $\mathcal{U}_i^t$ given by

$$\mathcal{U}_i^t = \left\{ (D_{i_{base}}^t, \beta_i^t, \alpha_{-i}^t) \left| \begin{array}{l} D_{i_{base}}^t = \bar{D}_{i_{base}}^t, \\
\beta_i^t \in (\beta_{i_{min}}^t, \beta_{i_{max}}^t), \\
\alpha_j^t \in (\alpha_{j_{min}}^t, \alpha_{j_{max}}^t) \\
\forall j \neq i \end{array} \right. \right\}$$
Numerical examples

The values of $\beta$ (top) and $\alpha$ (bottom) chosen for the numerical example and the allowed range for uncertainty.
### Numerical examples

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\bar{D}_{i,\text{base}}$</th>
<th>$\beta_{i,\text{min}}^t$</th>
<th>$\beta_{i,\text{nominal}}^t$</th>
<th>$\beta_{i,\text{max}}^t$</th>
<th>$\alpha_{i,\text{min}}^t$</th>
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<td>0.91</td>
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### Ranges for uncertain parameters.
Numerical examples

Convergence of policies through successive iterations for each seller.
Numerical examples

Equilibrium policies for each seller

Equilibrium policy of sellers

Price ($P$)

Period (t)
Numerical examples

<table>
<thead>
<tr>
<th>$t$</th>
<th>Seller 1</th>
<th>Seller 2</th>
<th>Seller 3</th>
<th>Seller 4</th>
</tr>
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<tr>
<td>1</td>
<td>2.13</td>
<td>1.80</td>
<td>1.58</td>
<td>1.52</td>
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<tr>
<td>2</td>
<td>2.32</td>
<td>1.99</td>
<td>1.77</td>
<td>1.70</td>
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<td>3</td>
<td>2.57</td>
<td>2.23</td>
<td>2.01</td>
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<tr>
<td>4</td>
<td>2.62</td>
<td>2.28</td>
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<td>2.63</td>
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<td>2.76</td>
<td>2.52</td>
<td>2.45</td>
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Equilibrium prices
Numerical examples
### Numerical examples

<table>
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<th>$t$</th>
<th>Price</th>
<th>Protection Level</th>
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<td>2.81</td>
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<td>92.85</td>
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<tr>
<td>10</td>
<td>3.12</td>
<td>0.00</td>
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</table>

**Robust Policy for Seller 1.**
Numerical examples

<table>
<thead>
<tr>
<th>$t$</th>
<th>Price</th>
<th>Protection Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.40</td>
<td>822.86</td>
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<tr>
<td>2</td>
<td>4.37</td>
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</table>

Nominal Policy for Seller 1.
Numerical examples
Changing budget
Changing budget
Insights

- Prices are higher in periods of lower price sensitivity.
- Sellers with more starting inventory set lower prices but earn higher revenues.
- Robust policies are less sensitive to variation in uncertain parameters.
- Robust policies give better worst case performance.
- The budget will help decrease variance with small tradeoff in average payoff.
- Iterative learning algorithm converges to equilibrium very fast in practice.
- Starting guess for prices doesn’t affect convergence of algorithm.
Thank you!