1 Sliding dumbbell

The two spheres are rigidly connected to the rod of negligible mass and are initially at rest on the smooth horizontal surface. A force $F$ is suddenly applied to one sphere in the $y$–direction and imparts an impulse of $10\text{N}s$ during a negligibly short period of time. As the spheres pass the dashed position, calculate the velocity of each one.
2 Carriage and pendulum

The carriage of mass $2m$ is free to roll along the horizontal rails and carries the two spheres, each of mass $m$, mounted on rods of length $l$ and negligible mass. The shaft to which the rods are secured is mounted in the carriage and is free to rotate. If the system is released from rest with the rods in the vertical position where $\theta = 0$, determine the velocity $v_x$ of the carriage and the angular velocity $\dot{\theta}$ of the rods for the instant when $\theta = 180^\circ$. Treat the carriage and the spheres as particles and neglect any friction.
3 Ball on a conical surface

The 0.2 – kg ball and its supporting cord are revolving about the vertical axis on the fixed smooth conical surface with an angular velocity of 4rad/s. The ball is held in the position \( b = 300mm \) by the tension \( T \) in the cord. If the distance \( b \) is reduced to the constant value of 200mm by increasing the tension \( T \) in the cord, compute the new angular velocity \( \omega \) and the work \( U'_{1-2} \) done on the system by \( T \).
4 Basketball collision

During a pregame warmup period, two basketballs collide above the hoop when in the positions shown. Just before impact, ball 1 has a velocity \( v_1 \) which makes a 30° angle with the horizontal. Also, during impact, an imaginary line between the center of mass of the two balls makes a 30° angle with the horizontal. If the velocity \( v_2 \) of ball 2 just before impact has the same magnitude as \( v_1 \), determine the two possible values of the angle \( \theta \), measured from the horizontal, which will cause ball 1 to go directly through the center of the basket. The coefficient of restitution is \( e = 0.8 \).
5 MATLAB - A Taylor Series Sine Expansion

Equation 1 represents the sine function as a Taylor series expansion.

\[
\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots = \sum_{n=0}^{N} \frac{(-1)^n x^{2n+1}}{(2n+1)!}
\]  

1. Write a MATLAB program that uses a function that defines sine as a Taylor series expansion. Pass a vector \( x \) for the sine argument and an integer \( N \) to determine how many terms to include in the approximation \((N \geq 0)\) (hint: use the \texttt{sum} MATLAB function).

2. On the same graph, plot the sine function and your Taylor series expansion sine function for \( N = 2, 5, 10 \) for comparison.

6 Dynamics - Bouncing Ball

When air drag and friction are neglected, the flight of a bouncing ball consists of a series of parabolic trajectories separated by “collisions” with the ground. The equations of motion for each parabolic trajectory are

\[
x(t) = v_x t; \quad y(t) = v_y t - \frac{1}{2} g t^2; \quad 0 < t < t_f
\]

where

\[
v_x = v \cos \theta; \quad v_y = v \sin \theta; \quad t_f = \frac{2v_y}{g}
\]

and \( v \) = initial velocity, and \( \theta \) = initial angle.

Since the ball is not perfectly elastic \((e < 1, \text{ where } e \text{ is the coefficient of restitution})\), the angles before and after impact are not the same (see Figure 1). The equations of motion for each collision are

\[
v'_x = v_x, \quad v'_y = -ev_y
\]

Write a MATLAB program to do the following:

1. Prompt the user for an initial velocity \( v_i \), initial angle \( \theta_i \), and number of bounces \( n \) and store the values for later use in your code.
2. Repeatedly calls a function you wrote to calculate the trajectory for each bounce.

3. Plots the complete trajectory $x(t)$ vs. $y(t)$ for $n = 5$ bounces when $v_i = 20$ m/s, $\theta_i = 80$ degrees, and $e = 0.8$. 

Figure 1: When $e < 1$ the impact angle before and after collision is different.