Distortion of photon-correlation functions in detection systems with paralyzable dead-time effects

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We introduce an analytical model of the influence of paralyzable dead time on registered photon-correlation functions. Distortions of correlation functions in the case of the Poisson point process, the doubly stochastic Poisson point process, and the pairwise point process are calculated. The model permits the analysis of detection systems with constant and random dead times. The results of the analytical model are tested by computer simulation.

1. INTRODUCTION

One of the main distortion factors in measurements of photon-correlation functions is event loss, which depends on whether an event falls into a dead period. In a paralyzable system all events that occur within less than the dead time of another event of the input process are lost. In most papers that have dealt with distortions as the result of dead time, dead time has been considered constant and the input process considered Poisson or associatively with Poisson (e.g., see Refs. 1 and 2). Previous studies of correlation functions were not successful because research on the first-order statistical quantities were affected by the dead-time effects. Schatzel[3,4] obtained expressions for correlation functions in the case of a paralyzable constant and fluctuating dead times, but unfortunately these expressions cannot be applied for random dead time. Cho and Morris[5] considered an expression for the correlation function of a general renewal process and derived an equation valid when dead time is greater than twice the sample time.

The analysis of dead-time distortion in the case of arbitrary processes and random dead time is difficult and requires a more general mathematical model of point processes[6,7] and systems of measurement. We propose such a model, which is developed in the framework of arbitrary random point-process transformations.[8]

2. ANALYTICAL MODEL

A random point process in time and space can be defined as a stochastic process that consists of a collection of events (points). Each event has a well-defined time and position. We denote the times when events occur as $\zeta_1, \zeta_2, \ldots$. The interval when a point process is registered is the counting interval $G = [T_1, T_2]$, $T_1 \leqslant \zeta_1 \leqslant \zeta_2 \leqslant \ldots \leqslant T_2$. An important quantity in the study of random point processes is the probability-generating functional (PGF). The PGF is defined as

$$L[u, G] = \left\langle \prod_{j=1}^{v} [1 + u(\zeta_j)] \right\rangle_{\zeta, v}, \tag{1}$$

where $u(\zeta)$ is a trial function that is used as some formal parameter and $\langle \ldots \rangle_{\zeta, v}$ denotes averaging along the times of occurrence of events and along the number v of points in G.

The characteristics of random point processes can be obtained from symmetric probability densities $\pi_k(t_1, \ldots, t_k; G)$, $k = 0, 1, \ldots$, which denote the probability of the appearance of exactly k events in the counting interval G at moments $t_1, \ldots, t_k$. Using averaging along the times of occurrence and the number of points in G, we can rewrite Eq. (1) as

$$L[u; G] = \sum_{k=0}^{\infty} \frac{1}{k!} \int_G \pi_k(t_1, \ldots, t_k; G) \times \prod_{j=1}^{k} [1 + u(t_j)] \mathrm{d}t_1 \ldots \mathrm{d}t_k . \tag{2}$$

We then use functional differentiation to obtain the densities from the PGF,[9] so

$$\frac{\delta L[u; G]}{\delta u(t_1)} = \left\{ \frac{\delta}{\delta \lambda} L[u(\cdot) + \lambda \delta(\cdot - t_1); G] \right\} \bigg|_{\lambda=0}, \tag{3}$$

$$\frac{\delta L^i[u; G]}{\delta u(t_1) \ldots \delta u(t_i)} = \frac{\delta}{\delta u(t_i)} \left\{ \frac{\delta L^{i-1}[u; G]}{u(t_1) \ldots \delta u(t_{i-1})} \right\}, \tag{4}$$

where $\delta(x)$ is the Dirac delta function.

Applying Eqs. (3) and (4) to Eq. (2), we have

$$\frac{\delta L^i[u; G]}{\delta u(t_1) \ldots \delta u(t_i)} = \sum_{k=i}^{\infty} \frac{k(k-1) \ldots (k-i+1)}{k!} \times \int_G \pi_k(t_1, \ldots, t_i, t_{i+1}, \ldots, t_k; G) \times \prod_{j=i+1}^{k} [1 + u(t_j)] \mathrm{d}t_{i+1} \ldots \mathrm{d}t_k . \tag{5}$$

If we then put $u(\cdot) = -1$ into Eq. (5), we obtain

$$\pi_i(t_1, \ldots, t_i) = \frac{\delta^i L[u; G]}{\delta u(t_1) \ldots \delta u(t_i)} \bigg|_{u(\cdot)=-1}, \qquad i = 1, 2, \ldots .$$

We can introduce other systems with probability densities based on the PGF. One of the most important sys-

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tems can be defined as the ith-order functional derivative of $L[u; G]$ when $u(\cdot) = 0$ (Refs. 8 and 9):

$$f_i(t_1, \ldots, t_i) = \left.\frac{\delta^i L[u; G]}{\delta u(t_1) \ldots \delta u(t_i)}\right|_{u(\cdot)=0}, \qquad i = 1, 2, \ldots, \tag{6}$$

where the first derivative $f_1(t_1)$ is intensity and the second derivative $f_2(t_1, t_2)$ is the correlation function.

Let us consider input photoevents as an arbitrary point process A with the PGF $L[v; G]$. $B(x)$ is a dead-time distribution function of a paralyzable detection system. Events of input processes A occur at $t_1, t_2, \ldots$; events of recording process C occur at times $\tau_1, \tau_2, \ldots$. Moreover, we consider process Q, in which the moments $x_1, x_2, \ldots$ describe the ends of dead-time intervals. The counting intervals of process A and C are the same and are equal to $G = [T_1, T_2]$. The counting interval for process Q is $X = [T_1, \infty)$.

The PGF of moments $(\tau_1, \tau_2, \ldots)$, $(t_1, t_2, \ldots)$, and $(x_1, x_2, \ldots)$ can be represented as

$$\begin{aligned} L[v; G; u; G; w; X] = \sum_{k=0}^{\infty} \frac{1}{k!} \int_{G^k} \int_{X^k} \pi_k(\tau_1, \ldots, \tau_k; G) \prod_{j=1}^{k} [1 + v(\tau_j)][1 + w(x_j)] \\ \times L^C_{|2k}[u; G \mid \tau_1, \ldots, \tau_k, x_1, \ldots, x_k] \mathrm{d}B(x_1 - \tau_1) \ldots \mathrm{d}B(x_k - \tau_k) \mathrm{d}\tau_1 \ldots \mathrm{d}\tau_k, \end{aligned} \tag{7}$$

where $v(\cdot)$, $u(\cdot)$, and $w(\cdot)$ are trial functions of processes A, C, and Q; $L^C_{|2k}[u; G \mid \tau_1, \ldots, \tau_k, x_1, \ldots, x_k]$ is the PGF of process C, provided that exactly k events appear at moments $\tau_1, \ldots, \tau_k$, and the corresponding dead-time intervals finish at moments $x_1, \ldots, x_k$.

The set of events of process C is almost the same as the set of process A, with the exception of events that appear in dead-time intervals. Therefore for process C we must remove from the set of photoevents that occur at $\tau_1, \ldots, \tau_k$ all events that belong to area G', where G' is defined by

$$G' = \bigcup_{m=1}^{k} (\tau_m, x_m]. \tag{8}$$

The set of remaining events will represent process C.

We can remove events by transformating the trial function to zero in area G'. Hence we can find

$$L^C_{|2k}[u; G \mid \tau_1, \ldots, \tau_k, x_1, \ldots, x_k] = \prod_{j=1}^{k} [1 + u(\tau_j)]|_{u(\tau)=0, \tau \in G'}. \tag{9}$$

We substitute Eq. (9) into Eq. (7) and assume that $v(\cdot t) = w(\cdot) = 0$, so the PGF of process C takes the form

$$\begin{aligned} L^C[u; G] = \sum_{k=0}^{\infty} \frac{1}{k!} \int_{G^k} \int_{X^k} \pi_k(\tau_1, \ldots, \tau_k; G) \\ \times \prod_{j=1}^{k} \left\{ 1 + u(\tau_j) \prod_{\substack{m=1 \\ m \neq j}}^{k} [\theta(\tau_m - \tau_j) + \theta(\tau_j - x_m)] \right\} \mathrm{d}B(x_1 - \tau_1) \ldots \mathrm{d}B(x_k - \tau_k) \mathrm{d}\tau_1 \ldots \mathrm{d}\tau_k, \\ \theta(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geqslant 0 \end{cases}. \end{aligned} \tag{10}$$

From Eq. (10) we can obtain the characteristics of a photoevent process after a detection system with paralyzable dead time. The output photoevent intensity is the first functional derivative of Eq. (10), whereas the correlation function is the second functional derivative of Eq. (10) when $u(\cdot) = 0$.

The first functional derivative of Eq. (10) is given by Eq. (3):

$$\begin{aligned} \frac{\delta L^C[u; G]}{\delta u(t_1)} = \sum_{k=1}^{\infty} \frac{k}{k!} \int_{G^{k-1}} \int_{X^k} \pi_k(\tau_1, \ldots, \tau_{k-1}, t_1; G) \\ \times \prod_{j=1}^{k-1} \left\{ 1 + u(\tau_j) \prod_{\substack{m=1 \\ m \neq j}}^{k-1} [\theta(\tau_m - \tau_j) + \theta(\tau_j - x_m)][\theta(t_1 - \tau_j) + \theta(\tau_j - x_k)] \right\} \\ \times \prod_{m=1}^{k-1} [\theta(\tau_m - t) + \theta(t_1 - x_m)] \mathrm{d}B(x_1 - \tau_1) \ldots \mathrm{d}B(x_{k-1} - \tau_{k-1}) \mathrm{d}B(x_k - t_1) \mathrm{d}\tau_1 \ldots \mathrm{d}\tau_{k-1}. \end{aligned} \tag{11}$$

Substituting $u(\cdot) = 0$ into Eq. (11), we obtain in accordance with Eq. (5)

$$\begin{aligned} f_1^C(t_1) = \sum_{k=1}^{\infty} \frac{k}{k!} \int_{G^{k-1}} \pi_k(\tau_1, \ldots, \tau_{k-1}, t_1; G) \\ \times \int_{X^k} \prod_{m=1}^{k-1} [\theta(\tau_m - t_1) + \theta(t_1 - x_m)] \mathrm{d}B(x_1 - \tau_1) \ldots \mathrm{d}B(x_{k-1} - \tau_{k-1}) \mathrm{d}B(x_k - t_1) \mathrm{d}\tau_1 \ldots \mathrm{d}\tau_{k-1} \\ = \sum_{k=1}^{\infty} \frac{k}{k\,!} \int_{G^{k-1}} \pi_k(\tau_1, \ldots, \tau_{k-1}, t_1; G) \prod_{m=1}^{k-1} [\theta(\tau_m - t_1) + B(t_1 - \tau_m)] \mathrm{d}\tau_1 \ldots \mathrm{d}\tau_{k-1}. \end{aligned} \tag{12}$$

Based on Eq. (5), we can rewrite Eq. (12) as

$$f_1^C(t_1) = \left.\frac{\delta L[v; G]}{\delta v(t_1)}\right|_{v(\cdot)=\theta(\cdot-t_1)+B(t_1-\cdot)-1} . \tag{13}$$

The second functional derivative of Eq. (10) is obtained by differentiation of Eq. (12):

$$\begin{aligned}\frac{\delta^2 L^C[u; G]}{\delta u(t_1)\delta u(t_2)} = {} & \sum_{k=2}^{\infty} \frac{k(k-1)}{k!} \int_{G^{k-2}} \int_{X^k} \pi_k(\tau_1, \ldots, \tau_{k-2}, t_1, t_2; G) \\ & \times \prod_{j=1}^{k-2} \left\{ 1 + u(\tau_j) \prod_{\substack{m=1 \\ m\neq j}}^{k-2} [\theta(\tau_m - \tau_j) + \theta(\tau_j - x_m)][\theta(t_1 - \tau_j) + \theta(\tau_j - x_{k-1})][\theta(t_2 - \tau_j) + \theta(\tau_j - x_k)] \right\} \\ & \times \prod_{m=1}^{k-2} [\theta(\tau_m - t_1) + \theta(t_1 - x_m)][\theta(t_2 - t_1) + \theta(t_1 - x_k)] \prod_{m=1}^{k-2} [\theta(\tau_m - t_2) + \theta(t_2 - x_m)] \\ & \times [\theta(t_1 - t_2) + \theta(t_2 - x_{k-1})]\mathrm{d}B(x_1 - \tau_1)\ldots \mathrm{d}B(x_{k-2} - \tau_{k-2})\mathrm{d}B(x_{k-1} - t_1)\mathrm{d}B(x_k - t_2)\mathrm{d}\tau_1 \ldots \mathrm{d}\tau_{k-2} . \end{aligned} \tag{14}$$

Substituting $u(\cdot) = 0$ into Eq. (14) yields

$$\begin{aligned} f_2^C(t_1, t_2) &= \left.\frac{\delta^2 L^C[u; G]}{\delta u(t_1)\delta u(t_2)}\right|_{u(\cdot)=0} \\ &= \sum_{k=2}^{\infty} \frac{k(k-1)}{k!} \int_{G^{k-2}} \pi_k(\tau_1, \ldots, \tau_{k-2}, t_1, t_2; G) \\ &\quad \times \prod_{m=1}^{k-2} \left\{ \int_X [\theta(\tau_m - t_1) + \theta(t_1 - x_m)] \right. \\ &\quad \left. \times [\theta(\tau_m - t_2) + \theta(t_2 - x_m)]\mathrm{d}B(x_m - \tau_m) \right\} \\ &\quad \times [\theta(t_2 - t_1) + B(t_1 - t_2)] \\ &\quad \times [\theta(t_1 - t_2) + B(t_2 - t_1)]\mathrm{d}\tau_1 \ldots \mathrm{d}\tau_{k-2} . \end{aligned} \tag{15}$$

If we suppose that $t_2 \geqslant t_1$, then

$$\begin{aligned} f_2^C(t_1, t_2) = B(t_2 - t_1) & \sum_{k=2}^{\infty} \frac{k(k-1)}{k!} \\ & \times \int_{G^{k-2}} \pi_k(\tau_1, \ldots, \tau_{k-2}, t_1, t_2; G) \\ & \times \prod_{m=1}^{k-2} [\theta(\tau_m - t_2) + \theta(\tau_m - t_1)B(t_2 - \tau_m) \\ & + B(t_1 - x_m)]\mathrm{d}\tau_1 \ldots \mathrm{d}\tau_{k-2} . \end{aligned} \tag{16}$$

Then, using Eq. (5), we can rewrite Eq. (16) as

$$\begin{aligned} f_2^C(t_1, t_2) = B(t_2 - t_1) & \left.\frac{\delta^2 L[v; G]}{\delta v(t_1)\delta v(t_2)}\right|_{u(\cdot)} \\ & = \theta(\cdot - t_2) + \theta(\cdot - t_1)B(t_2 - \cdot) + B(t_1 - \cdot) - 1 . \end{aligned} \tag{17}$$

Equation (17) is our main result in the analysis of the influence of paralyzable dead-time on PCF's. It should be noted that Eq. (17) can be applied to any input process and to any distribution function of dead time.

3. APPLICATIONS

We evaluated our result in Eq. (17) by analyzing three different types of point process: the Poisson point process, the doubly stochastic Poisson point process (DSPP), and the pairwise point process.

The DSPP can be used as a mathematical model of photoevents in the case of registration of laser radiation. The PGF of the DSPP is[10]

$$L[v, G] = \left\langle \exp \int_G \xi(\tau)v(\tau)\mathrm{d}\tau \right\rangle_{\xi} , \tag{18}$$

where $\langle \ldots \rangle_{\xi}$ is the averaging along the realization of the stochastic process $\xi(t)$; $\xi(t) \geqslant 0$ for $t \in G$.

By application of Eqs. (3) and (4) to Eq. (18) we have

$$\frac{\delta^2 L[v; G]}{\delta v(t_1)\delta v(t_2)} = \left\langle \xi(t_1)\xi(t_2)\exp \int_G \xi(\tau)v(\tau)\mathrm{d}\tau \right\rangle_{\xi} . \tag{19}$$

From Eq. (17) one can obtain

$$\begin{aligned} f_2^C(t_1, t_2) = B(t_2 - t_1) & \left\langle \xi(t_1)\xi(t_2)\exp \int_G \xi(\tau)[B(t_1 - \tau) \right. \\ & \left. + B(t_2 - \tau)\theta(\tau - t_1) + \theta(\tau - t_2) - 1]\mathrm{d}\tau \right\rangle_{\xi} . \end{aligned} \tag{20}$$

When $T_1 = -\infty$ for a stationary process, Eq. (20) can be rewritten as

$$\begin{aligned} f_2^C(\tau) = B(\tau) & \left\langle \xi(0)\xi(\tau)\exp\left\{ -\int_{-\infty}^{0} \xi(t)[1 - B(-t)]\mathrm{d}t \right.\right. \\ & \left.\left. - \int_0^{\tau} \xi(t)[1 - B(\tau - t)]\mathrm{d}t \right\} \right\rangle_{\xi} . \end{aligned} \tag{21}$$

In particular, the Poisson point process is a DSPP with deterministic function $\xi(t)$, which coincides with the point-process intensity $f_1(t)$. The Poisson point process serves as the representation of photocount time sequences when laser radiation of constant intensity is detected. It is known that for the Poisson point process the correlation function is $f_2(t_1, t_2) = f_1(t_1)f_1(t_2)$, but, according to Eq. (20), the correlation function of the Poisson point process after a detection system with paralyzable dead time can be expressed as

$$\begin{aligned} f_2^C(t_1, t_2) = {} & f_1(t_1)f_1(t_2)B(t_2 - t_1) \\ & \times \exp\left\{ -\int_{T_1}^{t_1} f_1(\tau)[1 - B(t_1 - \tau)]\mathrm{d}t \right\} \\ & \times \exp\left\{ -\int_{t_1}^{t_2} f_1(\tau)[1 - B(t_2 - \tau)]\mathrm{d}t \right\} . \end{aligned} \tag{22}$$

A DSPP can also represent the photoevents in the case of registration of laser radiation that is generated when a laser is running above threshold.[11] For this random point process $\xi(t)$ is a Gaussian stochastic process with mean $f_1(t)$ and covariance function $g_2(t_1, t_2)$. This point process is called a pairwise correlated point process, and its PGF is given by[12]

$$L[v, G] = \exp\left[\int_G f_1(\tau)v(\tau)\mathrm{d}\tau + \frac{1}{2}\iint_{G^2} g_2(\tau_1, \tau_2)v(\tau_1)v(\tau_2)\mathrm{d}\tau_1\mathrm{d}\tau_2\right], \quad (23)$$

where $g_2(\tau_1, \tau_2) = f_2(\tau_1, \tau_2) - f_1(\tau_1)f_1(\tau_2)$.

To obtain the correlation function of the registered process we take a second functional derivative of the PGF [Eq. (23)], which can be represented as

$$\frac{\delta^2 L[v; G]}{\delta v(t_1)\delta v(t_2)} = \exp\left[\int_G f_1(\tau)v(\tau)\mathrm{d}\tau + \frac{1}{2}\iint_{G^2} g_2(\tau_1, \tau_2)v(\tau_1)v(\tau_2)\mathrm{d}\tau_1\mathrm{d}\tau_2\right] \times \left\{\left[f_1(t_1) + \int_G g_2(t_1, \tau)v(\tau)\mathrm{d}\tau\right] \times \left[f_1(t_2) + \int_G g_2(t_2, \tau')v(\tau')\mathrm{d}\tau'\right] + g_2(t_1, t_2)\right\}. \quad (24)$$

Then we use the substitution from $v(\tau)$ to $B(t_1 - \tau) + B(t_2 - \tau)1(\tau - t_1) + 1(\tau - t_2) - 1$ and take into account that for $\tau \in [T_1, t_1]$ the function $\theta(\tau - t_1)$ equals zero and for $\tau \in (t_1, t_2]$ the distribution functions $B(t_1 - \tau) = 0$ and $B(t_2 - \tau)\theta(\tau - t_1) = B(t_2 - \tau)$. We obtain

$$\begin{aligned} f_2^C(t_1, t_2) = {} & B(t_2 - t_1)\exp\left\{\int_{T_1}^{t_1} f_1(\tau)[B(t_1 - \tau) - 1]\mathrm{d}\tau + \int_{t_1}^{t_2} f_1(\tau)[B(t_2 - \tau) - 1]\mathrm{d}\tau + \int_{T_1}^{t_1}\int_{\tau_1}^{t_1} g_2(\tau_1, \tau_2)\right. \\ & \times [B(t_1 - \tau_1) - 1][B(t_1 - \tau_2) - 1]\mathrm{d}\tau_1\mathrm{d}\tau_2 + \int_{T_1}^{t_1}\int_{t_1}^{t_2} g_2(\tau_1, \tau_2)[B(t_1 - \tau_1) - 1][B(t_2 - \tau_2) - 1]\mathrm{d}\tau_1\mathrm{d}\tau_2 \\ & \left. + \int_{t_1}^{t_2}\int_{\tau_1}^{t_2} g_2(\tau_1, \tau_2)[B(t_2 - \tau_1) - 1][B(t_2 - \tau_2) - 1]\mathrm{d}\tau_1\mathrm{d}\tau_2\right\}\left(\left\{f_1(t_1) + \int_{T_1}^{t_1} g_2(t_1, \tau)[B(t_1 - \tau) - 1]\mathrm{d}\tau\right.\right. \\ & \left. + \int_{t_1}^{t_2} g_2(t_1, \tau)[B(t_2 - \tau) - 1]\mathrm{d}\tau\right\}\left\{f_1(t_2) + \int_{T_1}^{t_1} g_2(t_2, \tau)[B(t_1 - \tau) - 1]\mathrm{d}\tau\right. \\ & \left.\left. + \int_{t_1}^{t_2} g_2(t_2, \tau)[B(t_2 - \tau) - 1]\mathrm{d}\tau\right\} + g_2(t_1, t_2)\right). \end{aligned} \quad (25)$$

Equation (25) allows us to calculate the influence of the detection system with arbitrary paralyzable dead time on the correlation function of the pairwise point process.

If the dead time is constant and equal to T_m, we can write the distribution function as

$$B(\tau) = \begin{cases} 1 & \tau \geqslant T_m \\ 0 & \tau < T_m \end{cases}, \quad (26)$$

and for stationary pairwise point process with intensity C we can obtain

$$\begin{aligned} f_2^C(\tau) = {} & 1(\tau - T_m)\exp\left\{-2CT_m + 2\int_0^{T_m}(T_m - z)g_2(z)\mathrm{d}z + \int_0^{T_m} z[g_2(z + \tau - T_m) + g_2(z + \tau)]\mathrm{d}z\right\} \\ & \times \left\{\left[C + \int_0^{T_m} g_2(z)\mathrm{d}z + \int_0^{T_m} g_2(z - \tau)\mathrm{d}z\right] \times \left[C + \int_0^{T_m} g_2(z)\mathrm{d}z + \int_0^{T_m} g_2(z + \tau)\mathrm{d}z\right] + g_2(\tau)\right\}. \end{aligned} \quad (27)$$

In photon-correlation experiments it often can be assumed that $f_1(t) = C$ and $g_2(\tau) = X\exp(-Y\tau)$.[13,14] In this case we can rewrite Eq. (27) as

$$\begin{aligned} f_2^C(\tau) = {} & 1(\tau - T_m)\exp\left\{-2CT_m + \frac{2X}{Y^2}[\exp(-YT_m) + YT_m - 1] - \frac{XT_m}{Y}[\exp(-Y\tau) + \exp(-Y\tau - YT_m)]\right. \\ & \left. + \frac{X}{Y^2}[\exp(-Y\tau + YT_m) - \exp(-Y\tau - YT_m)]\right\}\left(\left\{C + \frac{X}{Y}[\exp(-YT_m) - 1][1 - \exp(-YT_m)]\right\}\right. \\ & \left. \times \left\{C + \frac{X}{Y}[\exp(-YT_m) - 1][1 + \exp(-Y\tau)]\right\} + X\exp(-Y\tau)\right). \end{aligned}$$

For a random dead time with the distribution function $B(\tau) = 1 - \exp(-A\tau)$, the correlation function of the pairwise point process can be expressed as

$$f_2^C(\tau) = [1 - \exp(-A\tau)]\exp\left\{\frac{C}{A}[\exp(-A\tau) - 2] + \frac{X}{Y^2 - A^2}[\exp(-A\tau) - \exp(-Y\tau)] + \frac{X}{Y - A}\,\frac{1}{2A}[2 - \exp(-2A\tau)] + \frac{1}{Y + A}[\exp(-A\tau - Y\tau) - 2]\right\}\left(\left\{C - \frac{X}{Y + A} + \frac{X}{Y - A}[\exp(-Y\tau) - \exp(-A\tau)]\right\} \times \left\{C - \frac{X}{Y + A}[1 + \exp(-Y\tau) - \exp(-A\tau - Y\tau)]\right\} + X\exp(-Y\tau)\right).$$

4. SIMULATION EXPERIMENTS

Previously we developed a simulation system for correlation analysis[15] in which we tested our analytical model of paralyzable dead-time distortions for the correlation function of a pairwise point process. The experiments were done when $f_1(t) = 4$ and $g_2(\tau) = 6\exp(-4\tau)$. In Fig. 1 we introduce the results of experiments with a constant dead time T_m. In Fig. 2 the dead time is random with a distribution function $B(\tau) = 1 - \exp(-A\tau)$. For this $B(\tau)$ the average dead time is $1/A$. The solid curves are the calculated curves, and the asterisks represent correlation functions that were obtained in simulation experiments. The number of realizations in each case is 10^3 and the width of the channel is 0.02, with all data in relative units. The simulated values are in good agreement with the calculated curves. These results support our conclusion that paralyzable dead-time distortions of photon-correlation functions can be calculated by use of the analytical model derived in Section 2.

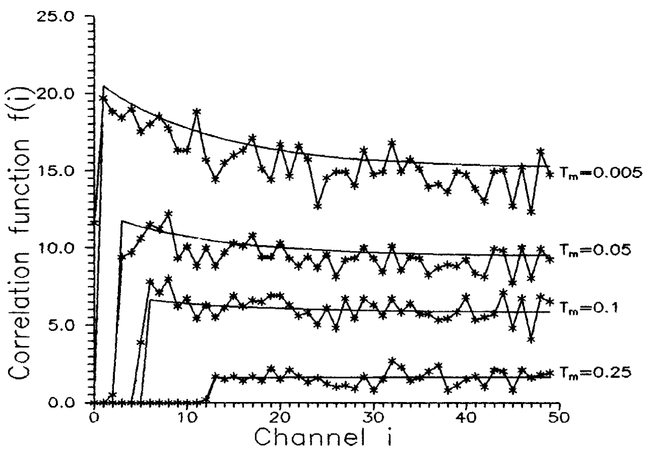

Fig. 1. Correlation function $f(i) = f_2(\tau)$ distorted by a detection system with constant paralyzable dead time T_m. The results of the simulation experiments are marked by asterisks. The calculated data are represented by solid curves. The width of channel is $h = 0.02$, $f_1(t) = 4$, $g_2(\tau) = 6\exp(-4\tau)$, $f_2(\tau) = 16 + 6\exp(-4\tau)$, and $\tau = ih$.

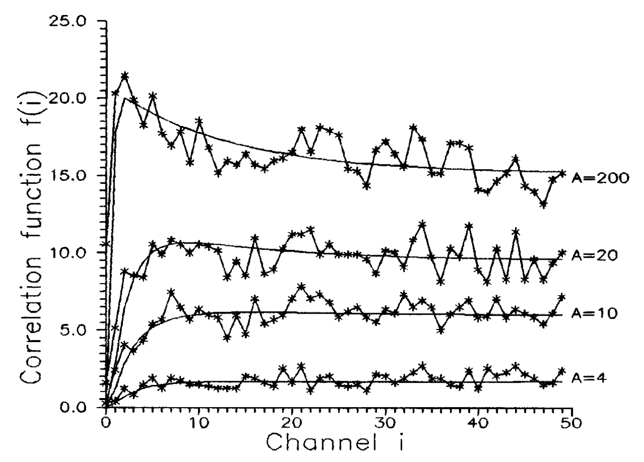

Fig. 2. Correlation function $f(i) = f_2(\tau)$ distorted by a detection system with random paralyzable dead time. The distribution function $B(\tau) = 1 - \exp(-A\tau)$. The results of the simulation experiments are marked by asterisks. The calculated data are represented by solid curves. The width of channel $h = 0.02$, $f_1(t) = 4$, $g_2(\tau) = 6\exp(-4\tau)$, $f_2(\tau) = 16 + 6\exp(-4\tau)$, and $\tau = ih$.

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