Gluon Saturation in QCD at High Energy: Beyond Leading Logarithms

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Kinematical regimes of DIS

Saturation scale $Q_s(Y)$.

For $Q > Q_s(Y)$: dilute regime: leading twist pQCD.

For $Q < Q_s(Y)$: gluon saturation: dense regime, with a breakdown of the collinear factorization.
Examples of processes described with gluon saturation

- DIS at low-$x$: inclusive and diffractive structure functions, DVCS, exclusive VM production, ... 

- Forward single inclusive particle production in pp or dA at RHIC.  
  Albacete, Marquet (2010)

- Forward *monojet* production in dA at RHIC.  
  Albacete, Marquet (2010)

- Ab initio calculations for the earliest stages of HIC, resumming high density effects, high energy leading logs, and secular divergences ($\sim$ plasma instabilities).  
  Gelis, Venugopalan *et al.* (2006-2011)
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Basics of gluon saturation and BK

Dipole factorization of DIS at LO/LL order

\[ \sigma_{T,L}^{\gamma^*p}(Y, Q^2) = 2 \int d^2r \sum_f \int_0^1 dz |\Psi_{T,L}^f(r, z, Q^2)|^2 \int d^2b \ N(x, y, Y) \]

Virtual photon wave-function (known): \( \Psi_{T,L}^f(r, z, Q^2) \).
Dipole-target elastic scattering amplitude: \( N(x, y, Y) \),
with a dipole of size \( x - y = r \) and impact parameter \( b = (x + y)/2 \).

Nikolaev, Zakharov (1991)
Balitsky Kovchegov (BK) equation at LL

\[ \partial_Y N(x, y, Y) = \bar{\alpha} \int \frac{d^2z}{2\pi} \frac{(x-y)^2}{(x-z)^2(z-y)^2} \left[ N(x, z, Y) + N(z, y, Y) - N(x, y, Y)N(x, z, Y)N(z, y, Y) \right] \]

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\[ \partial_Y N(x, y, Y) = \tilde{\alpha} \int \frac{d^2 z}{2\pi} \frac{(x-y)^2}{(x-z)^2(z-y)^2} \left[ N(x, z, Y) + N(z, y, Y) - N(x, y, Y) - N(x, z, Y)N(z, y, Y) \right] \]


Linear part: BFKL equation in the dipole formalism

Mueller (1994)
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\]


Linear part: BFKL equation in the dipole formalism
Mueller (1994)

Non-linear term: gluon saturation in the target
Unitarizes the dipole target amplitude: \( N(x, y, Y) < 1 \).
Three different regimes for the solutions of BK: the saturated regime, the universal dilute regime (or geometric scaling window), and the initial condition dominated dilute regime, which is progressively invaded by the universal one. \[ L \sim \log 1/r^2 \sim \log k_{\perp}^2. \]
Universal asymptotic behavior of $Q_s(Y)$:

\[
\log(Q_s(Y)^2) = \nu_c \tilde{\alpha} Y - c_l \log(\tilde{\alpha} Y) + \text{Const.} - \frac{c_{-1/2}}{\sqrt{\tilde{\alpha} Y}} + \mathcal{O}\left(\frac{1}{\tilde{\alpha} Y}\right)
\]

with $\nu_c \approx 4.88$, $c_l \approx 2.39$, $c_{-1/2} \approx 2.74$ and $\tilde{\alpha} = N_c \alpha_s / \pi$. Only Const. and $\mathcal{O}(\tilde{\alpha} Y)^{-1}$ terms sensitive to initial conditions.

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Only Const. and $O(\tilde{\alpha} Y)^{-1}$ terms sensitive to initial conditions.


Problem: $\nu_c$ too large $\Rightarrow$ incompatibility of the fixed coupling LL BK equation with HERA data for $F_2$. 
Results from the BK equation at LL accuracy

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Problem: $ν_c$ too large $⇒$ incompatibility of the fixed coupling LL BK equation with HERA data for $F_2$.

$⇒$ Need to consider running coupling and/or NLL corrections.
Towards gluon saturation at NLO

The appropriate running coupling prescription has been derived from quark loop corrections. → rcBK equation.
Balitsky (2006)
(see also Kovchegov, Weigert (2006))

And now the full NLL BK equation and the NLO DIS impact factor are known.
Balitsky, Chirilli (2007-2010)
⇒ We should move on to full NLO/NLL phenomenology.

Problem: Large NLL corrections to BK in the (anti-)collinear limits
⇒ Need for resummations.
Such resummations have already been done for NLL BFKL.
Ciafaloni, Colferai, Salam, Stašto (1998-2007)
Altarelli, Ball, Forte (2000-2008)
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Running coupling and NLL effects

DIS phenomenology with rcbK

Comparison with data on $F_{2r}$ and $\sigma_{rc}$

Fit including heavy quarks

Fits of the HERA data for the reduced DIS cross-section $\sigma_r$ and its charm contribution $\sigma_{rc}$, with numerical solutions of the running coupling BK equation. Albacete, Armesto, Milhano, Quiroga, Salgado (2011) 3 (+3) fit parameters for the initial condition. And another one rescaling $\Lambda_{QCD}$!
Universality, running coupling and higher orders

Running coupling effects drastically modify the solutions of BK, to a new universality class.

Universal asymptotic behavior of $Q_s(Y)$ with running coupling:

$$\log \frac{Q_s^2(Y)}{\Lambda_{QCD}^2} = C_{1/2} \sqrt{Y} + C_{1/6} Y^{1/6} + C_0$$

$$+ C_{-1/6} Y^{-1/6} + C_{-1/3} Y^{-1/3} + O \left( Y^{-1/2} \right).$$

G.B., arXiv:1008.0498
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- $C_{1/2}$, $C_{1/6}$ and $C_{-1/6}$ depend only on the LL BK kernel.
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- $C_0$ and $C_{-1/3}$ depend only on LL and NLL terms in BK.
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- $C_{1/2}$, $C_{1/6}$ and $C_{-1/6}$ depend only on the LL BK kernel.
- $C_0$ and $C_{-1/3}$ depend only on LL and NLL terms in BK.
- At $\mathcal{O}(Y^{-1/2})$: sensitivity to the initial condition and to the NNLL contributions to BK.

G.B., arXiv:1008.0498
Numerical studies of NLL and resummations effects

Toy model: BFKL equation in momentum space with running coupling and saturation boundary, with kernel at LL, NLL, or NLL $\oplus$ collinear resummations.

Avsar, Stašto, Triantafyllopoulos, Zaslavsky (2011)
Improvement of kinematics

Choice: high-energy factorization along $k^+$, at $k^+ = k_f^+$. 
$\Rightarrow$ strong ordering $k^+$ of the gluon cascade.
However, leading logs require simultaneous ordering in $k^+$ and $k^-$. 

"Rapidity" interval associated with the target (or matrix element):

$Y^+ = \log \left( \frac{2k^-_{\text{target}} k_f^+}{Q_0^2} \right)$. 

$\partial_{Y^+} N_{xy}(Y^+) = \bar{\alpha} \int \frac{d^2 z}{2\pi} \frac{(x-y)^2}{(x-z)^2(z-y)^2} \left[ N_{xz}(Y^+-\Delta) 
+ N_{zy}(Y^+-\Delta) - N_{xy}(Y^+) - N_{xz}(Y^+-\Delta) N_{zy}(Y^+-\Delta) \right]$ 

With $\Delta = \log \left[ \max \left( (x-y)^2, (x-z)^2, (z-y)^2 \right) / (x-y)^2 \right]$.

The shift by $\Delta$ restores the missing $k^-$ ordering, and resums the largest NLO contribution to BK.

G.B., *in preparation* (see also Motyka, Staśto (2009))
Conclusion

- Running coupling effects are essential for studies of gluon saturation since they lead to a different universality class of solutions.
- Excellent fits of DIS data are obtained from rcBK, if $\Lambda_{QCD}$ is allowed to change significantly.
- Such rescaling of $\Lambda_{QCD}$ has been shown both analytically and numerically to be the main effect of the missing NLL contributions.
- The full NLO BK equation is known, and the main obstacle preventing NLO/NLL gluon saturation phenomenology is the need for collinear resummations.
- The purely kinematical part of the collinear resummations has been performed. The missing piece is the resummation of non-eikonal gluon emissions.