Gluon Saturation in QCD at High Energy: Beyond Leading Logarithms

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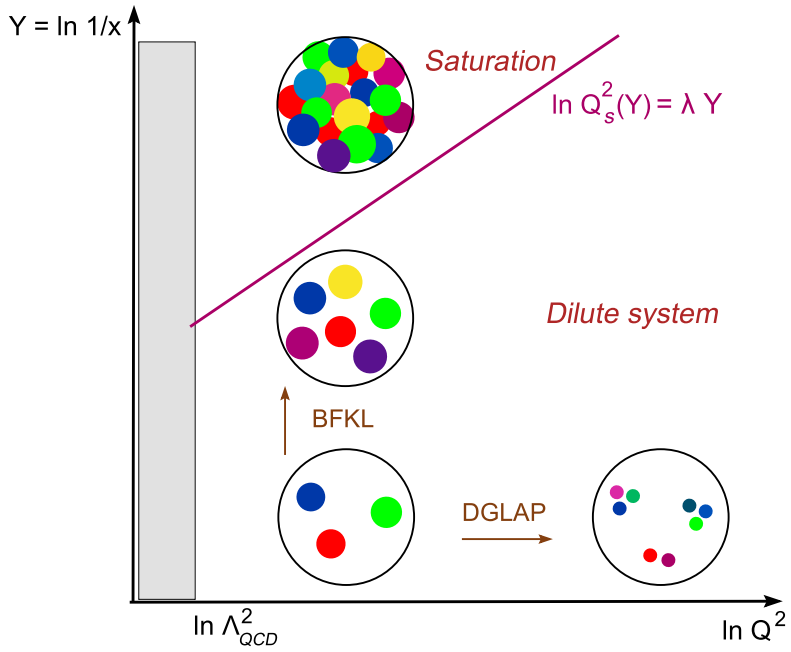
PANIC 2011

MIT, Cambridge, MA, july 25

Kinematical regimes of DIS



Saturation scale $Q_s(Y)$.

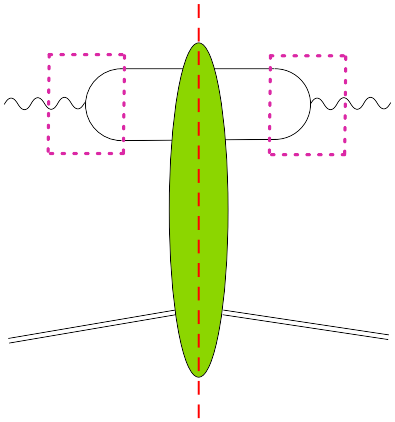
For $Q > Q_s(Y)$: dilute regime: leading twist pQCD.

For $Q < Q_s(Y)$: gluon saturation: dense regime, with a breakdown of the collinear factorization.

Examples of processes described with gluon saturation

- DIS at low-x: inclusive and diffractive structure functions, DVCS, exculsive VM production, ...
- Forward single inclusive particle production in pp or dA at RHIC.
  Albacete, Marquet (2010)
- Forward *monojet* production in dA at RHIC.
  Albacete, Marquet (2010)
- Ab initio calculations for the earliest stages of HIC, resumming high density effects, high energy leading logs, and secular divergences ($\sim$ plasma instabilities).
  Gelis, Venugopalan *et al.* (2006-2011)

Dipole factorization of DIS at LO/LL order



$$\sigma_{T,L}^{\gamma^* p}(Y, Q^2) = 2 \int d^2\mathbf{r} \sum_f \int_0^1 \mathrm{d}z |\Psi_{T,L}^f(\mathbf{r}, z, Q^2)|^2 \int d^2\mathbf{b}\ N(\mathbf{x}, \mathbf{y}, Y)$$

Virtual photon wave-function (known): $\Psi_{T,L}^f(\mathbf{r}, z, Q^2)$.

Dipole-target elastic scattering amplitude: $N(\mathbf{x}, \mathbf{y}, Y)$,

with a dipole of size $\mathbf{x} - \mathbf{y} = \mathbf{r}$ and impact parameter $\mathbf{b} = (\mathbf{x} + \mathbf{y})/2$.

Nikolaev, Zakharov (1991)

Balitsky Kovchegov (BK) equation at LL

$$\partial_Y N(\mathbf{x},\mathbf{y},Y) = \bar{\alpha}\int \frac{d^2\mathbf{z}}{2\pi}\,\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}\Big[N(\mathbf{x},\mathbf{z},Y) + N(\mathbf{z},\mathbf{y},Y) - N(\mathbf{x},\mathbf{y},Y) - N(\mathbf{x},\mathbf{z},Y)\,N(\mathbf{z},\mathbf{y},Y)\Big]$$

Balitsky (1996), Kovchegov (1999-2000)

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Linear part: BFKL equation in the dipole formalism

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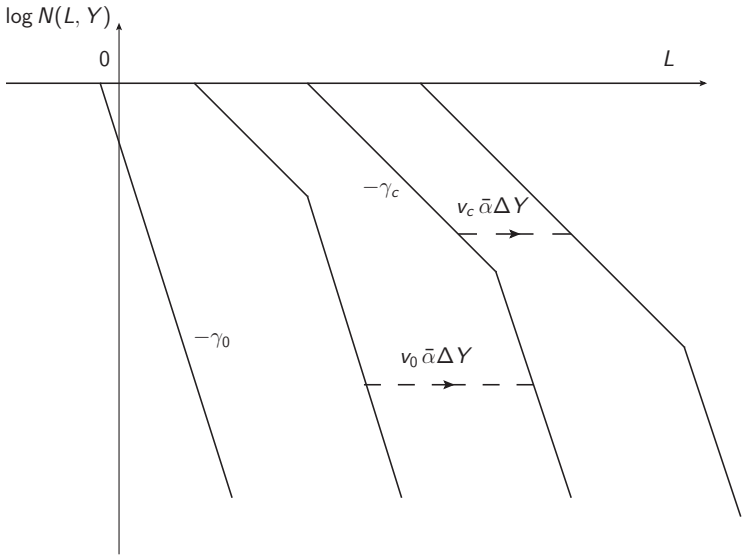
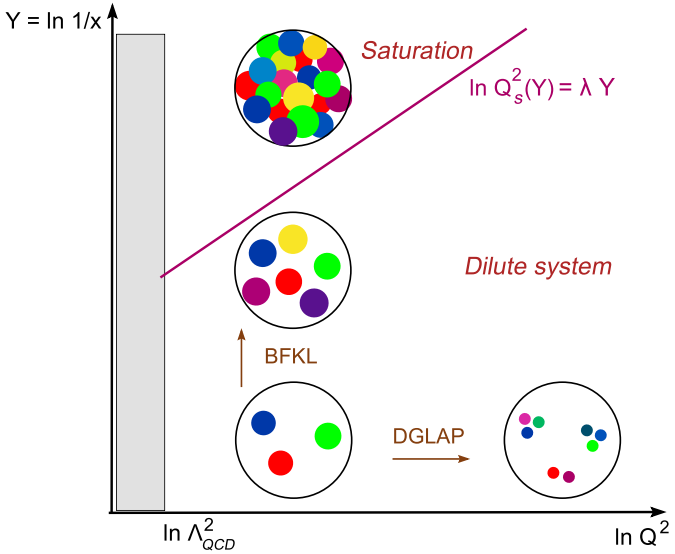
Non-linear term: gluon saturation in the target

Unitarizes the dipole target amplitude: $N(\mathbf{x},\mathbf{y},Y) < 1$.

Universality of the solutions of the BK equation





Three different regimes for the solutions of BK:
the saturated regime, the universal dilute regime (or geometric scaling window), and the initial condition dominated dilute regime, which is progressively invaded by the universal one.
$L \sim \log 1/\mathbf{r}^2 \sim \log {k_\perp}^2$.

Results from the BK equation at LL accuracy

Universal asymptotic behavior of $Q_s(Y)$:

$$\log(Q_s(Y)^2) \quad = \quad v_c\,\bar{\alpha}\,Y - c_l \log(\bar{\alpha}\,Y) + \mathrm{Const.} - \frac{c_{-1/2}}{\sqrt{\bar{\alpha}\,Y}} + \mathcal{O}\left(\frac{1}{\bar{\alpha}\,Y}\right)$$

with $v_c \simeq 4.88$, $c_l \simeq 2.39$, $c_{-1/2} \simeq 2.74$ and $\bar{\alpha} = N_c \alpha_s/\pi$.
Only Const. and $\mathcal{O}(\bar{\alpha}\,Y)^{-1}$ terms sensitive to initial conditions.

Mueller, Triantafyllopoulos (2002)

Munier, Peschanski (2004).

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Problem: v_c too large $\Rightarrow$ incompatibility of the fixed coupling LL BK equation with HERA data for F_2.

$\Rightarrow$ Need to consider running coupling and/or NLL corrections.

Towards gluon saturation at NLO

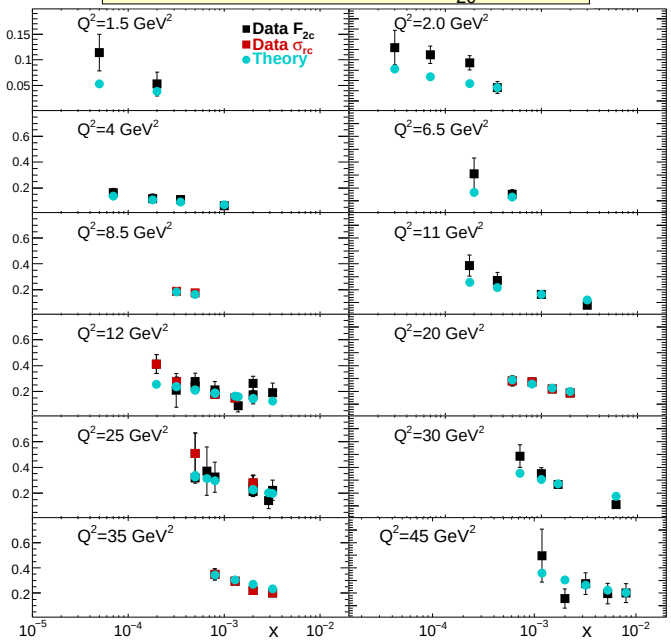
The appropriate running coupling prescription has been derived from quark loop corrections. $\rightarrow$ rcBK equation.
Balitsky (2006)
(see also Kovchegov, Weigert (2006))

And now the full NLL BK equation and the NLO DIS impact factor are known.
Balitsky, Chirilli (2007-2010)
$\Rightarrow$ We should move on to full NLO/NLL phenomenology.

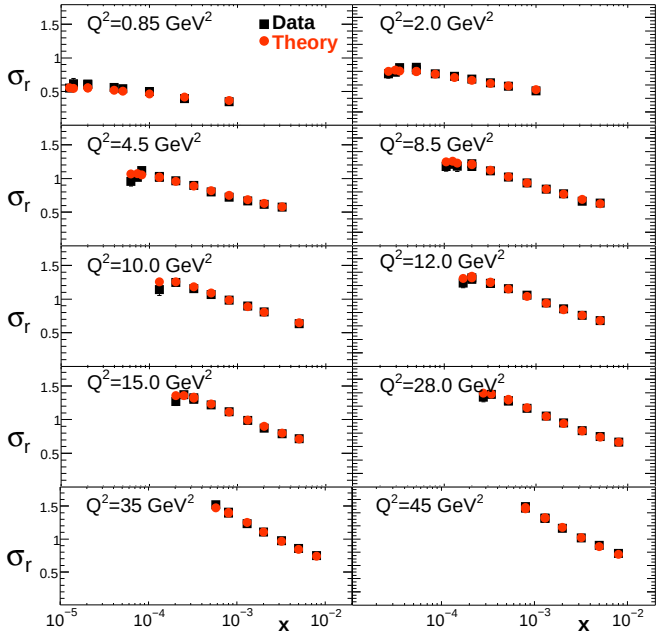
Problem: Large NLL corrections to BK in the (anti-)collinear limits
$\Rightarrow$ Need for resummations.
Such resummations have already been done for NLL BFKL.
Ciafaloni, Colferai, Salam, Staśto (1998-2007)
Altarelli, Ball, Forte (2000-2008)

DIS phenomenology with rcBK

Comparison with data on F_{2c} and σ_{rc}



Fit including heavy quarks



Fits of the HERA data for the reduced DIS cross-section σ_r and its charm contribution σ_{rc},
with numerical solutions of the running coupling BK equation.
Albacete, Armesto, Milhano, Quiroga, Salgado (2011)
3 (+3) fit parameters for the initial condition.
And another one rescaling Λ_{QCD}!

Universality, running coupling and higher orders

Running coupling effects drastically modify the solutions of BK, to a new universality class.

Universal asymptotic behavior of $Q_s(Y)$ with running coupling:

$$\log \frac{Q_s^2(Y)}{\Lambda_{QCD}^2} = C_{1/2}\sqrt{Y} + C_{1/6}\, Y^{1/6} + C_0 + C_{-1/6}\, Y^{-1/6} + C_{-1/3}\, Y^{-1/3} + \mathcal{O}\left(Y^{-1/2}\right).$$

G.B., arXiv:1008.0498

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- $C_{1/2}$, $C_{1/6}$ and $C_{-1/6}$ depend only on the LL BK kernel.
- C_0 and $C_{-1/3}$ depend only on LL and NLL terms in BK.
- At $\mathcal{O}\left(Y^{-1/2}\right)$: sensitivity to the initial condition and to the NNLL contributions to BK.

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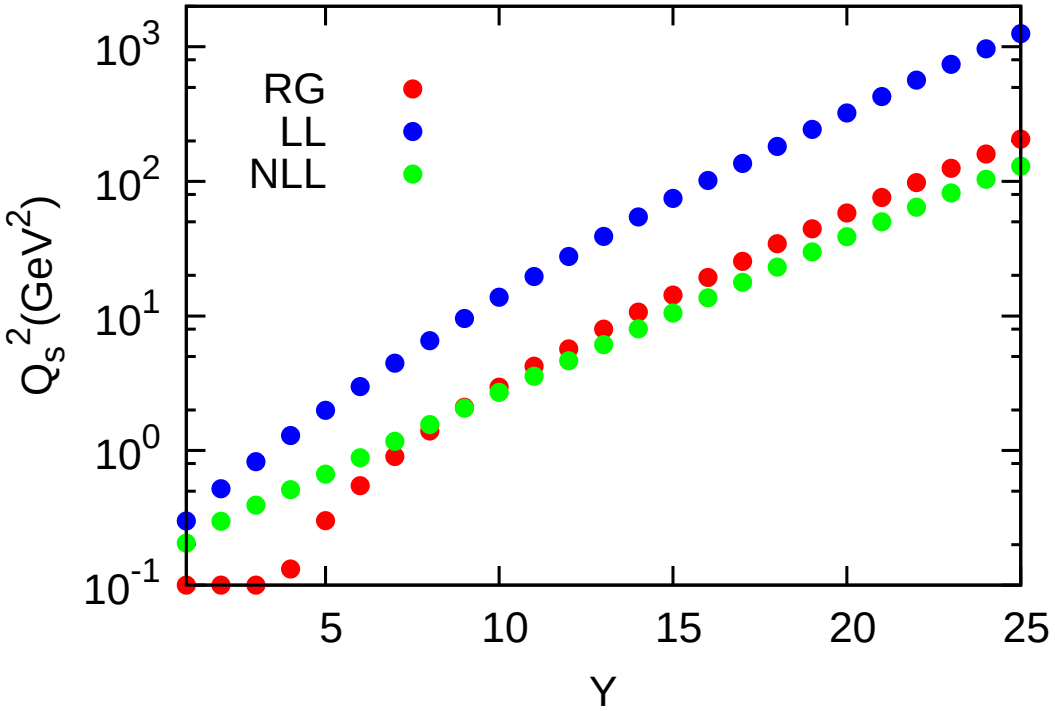
Numerical studies of NLL and resummations effects



Toy model: BFKL equation in momentum space with running coupling and saturation boundary,
with kernel at LL, NLL, or NLL $\oplus$ collinear resummations.
Avsar, Stašto, Triantafyllopoulos, Zaslavsky (2011)

Improvement of kinematics

Choice: high-energy factorization along k^+, at $k^+ = k_f^+$.
$\Rightarrow$ strong ordering k^+ of the gluon cascade.
However, leading logs require simultaneous ordering in k^+ and k^-.

"Rapidity" interval associated with the target (or matrix element):
$Y^+ = \log\left(\frac{2k^-_{\rm target}k^+_{\rm f}}{Q_0^2}\right)$.

$$\partial_{Y^+} N_{\mathbf{xy}}(Y^+) = \bar{\alpha}\int \frac{d^2\mathbf{z}}{2\pi}\,\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}\,\Big[N_{\mathbf{xz}}(Y^+-\Delta)$$
$$+N_{\mathbf{zy}}(Y^+-\Delta) - N_{\mathbf{xy}}(Y^+) - N_{\mathbf{xz}}(Y^+-\Delta)\,N_{\mathbf{zy}}(Y^+-\Delta)\Big]$$

With $\Delta = \log\left[\max\left((\mathbf{x}-\mathbf{y})^2,(\mathbf{x}-\mathbf{z})^2,(\mathbf{z}-\mathbf{y})^2\right)/(\mathbf{x}-\mathbf{y})^2\right]$.

The shift by Δ restores the missing k^- ordering, and resums the largest NLO contribution to BK.

G.B., *in preparation* (see also Motyka, Staśto (2009))

Conclusion

- Running coupling effects are essential for studies of gluon saturation since they lead to a different universality class of solutions.
- Excellent fits of DIS data are obtained from rcBK, if Λ_{QCD} is allowed to change significantly.
- Such rescaling of Λ_{QCD} has been shown both analytically and numerically to be the main effect of the missing NLL contributions.
- The full NLO BK equation is known, and the main obstacle preventing NLO/NLL gluon saturation phenomenology is the need for collinear resummations.
- The purely kinematical part of the collinear resummations has been performed. The missing piece is the resummation of non-eikonal gluon emissions.