PLAUSIBLE EXPLANATION FOR THE $\Delta_{5/2}^+(2000)$ PUZZLE

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References

i) Ju-Jun Xie, A. Martínez Torres, E. Oset and P. González

ii) P. González, E. Oset and J. Vijande

iii) P. González, J. Vijande and A. Valcarce
Motivation

There is a puzzle concerning the PDG-cataloged $\Delta_{5/2}^{+}(2000)\ F_{35}$ (**). Its estimated mass does not correspond to any experimentally extracted resonance.

K. Nakamura et al. (Particle Data Group), JPG 37, 075021 (2010)

\[
\Delta(2000)\ F_{35} \quad I(J^{P}) = \frac{3}{2}(5^{+}) \quad \text{Status: **}
\]

OMITTED FROM SUMMARY TABLE
The latest GWU analysis (ARNDT 06) finds no evidence for this resonance.

\[
\begin{array}{|c|c|c|}
\hline
\text{VALUE (MeV)} & \text{DOCUMENT ID} & \text{TECN} & \text{COMMENT} \\
\hline
\approx 2000 \text{ OUR ESTIMATE} & & & \\
1724 \pm 61 & VRANA & 00 & \text{DPWA Multichannel} \\
1752 \pm 32 & MANLEY & 92 & \pi N \to \pi N & N\pi\pi \\
2200 \pm 125 & CUTKOSKY & 80 & \text{IPWA} \pi N \to \pi N \\
\hline
\end{array}
\]

N. Suzuki et al. (EBAC), PRL 104, 042302 (2010) : $\Delta_{5/2}^{+}(1738)$
3q Model Description

In 3q models the lowest $\Delta_{5/2^+}$ state is the orbital symmetric (56, $2^+$) in the $N=2$ energy band with mass about 1900 MeV: $\Delta_{5/2^+} (1905)$ (****).

<table>
<thead>
<tr>
<th>VALUE (MeV)</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1857.8 ± 1.6</td>
<td>ARNDT 06</td>
<td>DPWA</td>
<td>$\pi N \to \pi N, \eta N$</td>
</tr>
<tr>
<td>1881 ± 18</td>
<td>MANLEY 92</td>
<td>IPWA</td>
<td>$\pi N \to \pi N &amp; N \pi \pi$</td>
</tr>
<tr>
<td>1910 ± 30</td>
<td>CUTKOSKY 80</td>
<td>IPWA</td>
<td>$\pi N \to \pi N$</td>
</tr>
<tr>
<td>1905 ± 20</td>
<td>HOEHLER 79</td>
<td>IPWA</td>
<td>$\pi N \to \pi N$</td>
</tr>
<tr>
<td>1873 ± 77</td>
<td>VRANA 00</td>
<td>DPWA</td>
<td>Multichannel</td>
</tr>
</tbody>
</table>

**Alternative Interpretation**

$\Delta (2000) F_{35}$

$I(J^P) = \frac{3}{2}(\frac{5}{2}^+) \text{ Status: } **$

OMITTED FROM SUMMARY TABLE
The latest GWU analysis (ARNDT 06) finds no evidence for this resonance.

<table>
<thead>
<tr>
<th>$\Delta(2000)$ BREIT-WIGNER MASS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>VALUE (MeV)</strong></td>
</tr>
<tr>
<td>$\approx 2000 \text{ OUR ESTIMATE}$</td>
</tr>
<tr>
<td>1724 ± 61</td>
</tr>
<tr>
<td>1752 ± 32</td>
</tr>
<tr>
<td>2200 ± 125</td>
</tr>
</tbody>
</table>

$\Delta_{5/2^+} (2200)$ may be assigned to a 3q state in the $N = 4$ energy band.

$\Delta_{5/2^+} (1740) \text{ (N.C.) might be a Dynamically Generated Resonance.}$
Candidates to be dominantly DGR (PRC 2008)

Quark model results by Capstick and Isgur, PRD 34, 2809 (1986)
A meson-baryon threshold lies in between the calculated 3q mass and the experimental mass.
An attractive meson-baryon interaction could give rise to a DGR as a bound state below the meson-baryon threshold:

\[ \Lambda (1405), \Delta_{5/2-} (1930), \ldots \]

The identification of \( \Delta_{5/2+}(1740) \) as a \( \pi N(1675) \) bound state could explain why it is only extracted in some data analyses, those reproducing the \( \pi \pi N \) production cross section data.
DESCRIPTION OF $\Delta_{5/2}^+(1740)$ AS A $\pi N(1675)$ BOUND STATE

INDEX

i) $N_{5/2}^-(1675)$ as DGR from the $\rho\Delta$ interaction.

ii) $\Delta_{5/2}^+(1740)$ as a plausible DGR from $\pi(\rho\Delta)_{N(1675)}$ in the Fixed Center Approximation.

iii) $\Delta_{1/2}^+(1750)$ as DGR partner of $\Delta_{5/2}^+(1740)$.

iv) Consistency checks in the $I = 3/2$, 1/2 sectors.

v) Summary
$N_{5/2}^{-(1675)}$ as effective DGR from the $\rho\Delta$ interaction (PRC 2009)

$\Delta_{5/2}^{\,-}, \Delta_{3/2}^{\,-}, \Delta_{1/2}^{\,-}$: Quark Pauli blocked states explained as $\rho\Delta$ bound states
The same $\rho\Delta$ interaction, very attractive in the $I=1/2$ channel, provides an alternative description of $N_{5/2}-(1675)$ as a bound state.

\[ \rho\Delta \rightarrow \rho\Delta \]

\[ V_{ij} = -\frac{1}{4f^2} C_{ij}(k^0 + k^0)' \vec{\varepsilon} \cdot \vec{\varepsilon}' \]

| $\rho\Delta$, $I = \frac{1}{2}$ | $C = 5$ |
| $\rho\Delta$, $I = \frac{3}{2}$ | $C = 2$ |
| $\rho\Delta$, $I = \frac{5}{2}$ | $C = -3$ |
\[ T = \frac{V}{1 - VG \varepsilon', \varepsilon} \]

**G**: Loop function for intermediate \( \rho \Delta \) state

\[
G(s_{\Delta\rho}) = i \int \frac{d^4q}{(2\pi)^4} \frac{2M_\Delta}{[(P - q)^2 - M_\Delta^2 + i\epsilon](q^2 - m_\rho^2 + i\epsilon)}
\]

\[
= \frac{2M_\Delta}{16\pi^2} \left( a(\mu) + \ln \frac{M_\Delta^2}{\mu^2} + \frac{m_\rho^2 - M_\Delta^2 + s_{\Delta\rho}}{2s_{\Delta\rho}} \ln \frac{m_\rho^2}{M_\Delta^2} \right.

+ \frac{q}{\sqrt{s_{\Delta\rho}}} \left[ \ln[s_{\Delta\rho} - (M_\Delta^2 - m_\rho^2) + 2q \sqrt{s_{\Delta\rho}}] \right.

+ \ln[s_{\Delta\rho} + (M_\Delta^2 - m_\rho^2) + 2q \sqrt{s_{\Delta\rho}}] \right.

- \ln[-s_{\Delta\rho} + (M_\Delta^2 - m_\rho^2) + 2q \sqrt{s_{\Delta\rho}}] \left.ight) \]
Fine tuned $a_{\rho\Delta} (\mu = 800 \text{ MeV}) = -2.28$ to get $N_{5/2^-}$ at 1675 MeV.

Note that $\Delta_{5/2^-}(1930)$ is reproduced as well.
Assignments

\[ N_{5/2}^- \ (1675) (***) \quad \Delta_{5/2}^- \ (1930) (***) \]

\[ N_{3/2}^- \ (1700) (***) \quad \Delta_{3/2}^- \ (1940) (*) \]

\[ N_{1/2}^- \ (1650) (***) \quad \Delta_{1/2}^- \ (1900) (**) \]
Fixed Center Approximation (FCA) to the three-body problem

Two Center Model

The projectile is scattered, one by one, by the two particles (fixed centers) in the target which suffers no distortion.

10 – 25 % accuracy in $k^d$

The \( \pi(\rho\Delta)_{N(1675)} \) system: Fixed Center Approximation

Brueckner Formula

\[
T = T_1 + T_2 = \frac{t_1 + t_2 + 2t_1 t_2 G_0}{1 - t_1 t_2 G_0^2}
\]

Parameters: \( a_{\pi\rho}, \quad a_{\pi\Delta} \)

\[
t_1 = \frac{5}{9} t_{\Delta\pi}^{3/2} + \frac{4}{9} t_{\Delta\pi}^{1/2}
\]

\[
t_2 = \frac{5}{6} t_{\rho\pi}^{2} + \frac{1}{6} t_{\rho\pi}^{1}
\]

\[
G_0(s) = \sqrt{\frac{M_{N^*}}{E_{N^*}}} \sqrt{\frac{M_{N^*}}{E'_{N^*}}} \int \frac{d^3 \bar{q}}{(2\pi)^3} F_{N^*}(q)
\times \frac{1}{q^2 - \bar{q}^2 - m^2_{\pi} + i\epsilon}.
\]
$I = 3/2$

$\Delta_{5/2^+, 3/2^+, 1/2^+}$

$a_{\pi\rho} = -2.0$

$a_{\pi\Delta} = -3.0$

$a_{\pi\Delta} = -3.4, -3.0, -2.6$

$a_{\pi\rho} = -2.6, -2.0, -1.4$

It is plausible that $\Delta_{5/2^+}(1740)$ is a DGR from $\pi(\rho\Delta)_{N(1675)}$ (there is a parameters interval giving rise to the bound state)
For $\Delta_{5/2^+}(1600)$ there is a quite lower meson-baryon threshold: $\pi N(1520)$.

The existence of $\Delta_{5/2^+}(1740)$ as a DGR from $\pi(\rho\Delta)_{N(1675)}$ implies that $\Delta_{1/2^+, 3/2^+}$ partners should exist.

$\Delta_{5/2^+} : \pi N(1675)$
$\Delta_{3/2^+} : \pi N(1700)$
$\Delta_{1/2^+} : \pi N(1650)$

<table>
<thead>
<tr>
<th>Name</th>
<th>$J^P$</th>
<th>Estimated mass (MeV)</th>
<th>Extracted mass (MeV)</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta(1740)$</td>
<td>$5/2^+$</td>
<td></td>
<td>$1752 \pm 32$</td>
<td>N.C.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(1724 \pm 61)$</td>
<td></td>
</tr>
<tr>
<td>$\Delta(1600)$</td>
<td>$3/2^+$</td>
<td>$1550$–$1700$</td>
<td>$1706 \pm 10$</td>
<td>***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(1687 \pm 44)$</td>
<td></td>
</tr>
<tr>
<td>$\Delta(1750)$</td>
<td>$1/2^+$</td>
<td>$\approx 1750$</td>
<td>$1744 \pm 36$</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(1721 \pm 61)$</td>
<td></td>
</tr>
</tbody>
</table>

For $\Delta(1600)$ there is a quite lower meson-baryon threshold: $\pi N(1520)$.
\( \Delta_{1/2+}(1750) \) as DGR partner of \( \Delta_{5/2+}(1740) \)

\[
\Delta(1750) \quad P_{31} \\

\begin{array}{ll}
\hline
I(J^P) &= \frac{3}{2}(1^+) \\
\text{Status:} &= * \\
\hline
\end{array}
\]

OMITTED FROM SUMMARY TABLE

The latest GWU analysis (ARNDT 06) finds no evidence for this resonance.

\[
\Delta(1750) \text{ BREIT-WIGNER MASS} \\
\begin{array}{cccc}
\hline
\text{VALUE (MeV)} & \text{OUR ESTIMATE} & \text{DOCUMENT ID} & \text{TECN} & \text{COMMENT} \\
\hline
\approx 1750 & \text{MANLEY} & 92 & IPWA & \pi N \rightarrow \pi N \ & \& \ & \pi \pi \\
1744 & \pm 36 & & & \\
\bullet \bullet \bullet & \text{We do not use the following data for averages, fits, limits, etc.} & & & \\
1712 & \pm 1 & \text{PENNER} & 02C & \text{DPWA Multichannel} \\
1721 & \pm 61 & \text{VRANA} & 00 & \text{DPWA Multichannel} \\
1715.2 & \pm 21.0 & \text{1 CHEW} & 80 & \text{BPWA} \pi^+ p \rightarrow \pi^+ p \\
1778.4 & \pm 9.0 & \text{1 CHEW} & 80 & \text{BPWA} \pi^+ p \rightarrow \pi^+ p \\
\hline
\end{array}
\]

\( \Delta_{1/2+}(1750) \) is a Quark Pauli blocked case as it was \( \Delta_{5/2-}(1930) \)

3q calculated mass is significantly higher than data
Consistency checks in the $I = 3/2, 1/2$ sectors

Further $\Delta$ states assignments

<table>
<thead>
<tr>
<th>$\Delta$</th>
<th>$J^P$</th>
<th>$E$ (MeV)</th>
<th>$T_2^r$ (MeV)</th>
<th>$T_2^p$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta(1905)$</td>
<td>$5/2^+$</td>
<td>1865–1915</td>
<td>1881 ± 18</td>
<td>(1873 ± 77)</td>
</tr>
<tr>
<td>$\Delta(1920)$</td>
<td>$3/2^+$</td>
<td>1900–1970</td>
<td>2014 ± 16</td>
<td>(1889 ± 100)</td>
</tr>
<tr>
<td>$\Delta(1910)$</td>
<td>$1/2^+$</td>
<td>1870–1920</td>
<td>1882 ± 10</td>
<td>(1995 ± 12)</td>
</tr>
</tbody>
</table>
I = 1/2

\[ a_{\pi\Delta} = -3.4 \]
\[ a_{\pi\rho} = -1.4 \]

All results are fully consistent with existing data.
Summary

i) There is a puzzle concerning the PDG cataloged $\Delta_{5/2}^+ (2000)$ (**). Actually this entry may be representing two distinctive non-cataloged resonances: $\Delta_{5/2}^+ (1740)$ and $\Delta_{5/2}^+ (2200)$.

ii) There is a plausible explanation of $\Delta_{5/2}^+ (1740)$ as a Dynamically Generated Resonance (DGR) from $\pi N(1675)$.

iii) The existence of $\Delta_{5/2}^+ (1740)$ provides an explanation to $\Delta_{1/2}^+ (1750)$ (*) as a DGR partner from $\pi N(1650)$.

iv) A complete description of the light-quark bayon spectrum seems feasible from $3q +$ DGR states.
THE END
\[
F_{N^*}(q) = \frac{1}{N} \int_{|\vec{p}|<\Lambda, |\vec{p}-\vec{q}|<\Lambda} \frac{d^3 \vec{p}}{E_{\Delta}(\vec{p})} \frac{M_{\Delta}}{2\omega_{\rho}(\vec{p})} \frac{1}{M_{N^*} - E_{\Delta}(\vec{p}) - \omega_{\rho}(\vec{p}) + i\left(\frac{\Gamma_{\Delta} + \Gamma_{\rho}}{2}\right)} \\
\times \frac{1}{M_{N^*} - E_{\Delta}(\vec{p} - \vec{q}) - \omega_{\rho}(\vec{p} - \vec{q}) + i\left(\frac{\Gamma_{\Delta} + \Gamma_{\rho}}{2}\right)}
\]

\[
N = \int_{|\vec{p}|<\Lambda} \frac{d^3 \vec{p}}{E_{\Delta}(\vec{p})} \left(\frac{M_{\Delta}}{2\omega_{\rho}(\vec{p})}\right)^2 \frac{1}{M_{N^*} - E_{\Delta}(\vec{p}) - \omega_{\rho}(\vec{p}) + i\left(\frac{\Gamma_{\Delta} + \Gamma_{\rho}}{2}\right)}^2
\times \frac{1}{M_{N^*} - E_{\Delta}(\vec{p} - \vec{q}) - \omega_{\rho}(\vec{p} - \vec{q}) + i\left(\frac{\Gamma_{\Delta} + \Gamma_{\rho}}{2}\right)}^2
\]
Harmonic Oscillator Model

Baryon (3q system): two relative (Jacobi) coordinates.

\[ V_{ij}^{h.o.} \implies E_\rho, E_\lambda \]

\[ E^{h.o.} = (2n_\rho + l_\rho + \frac{3}{2})\omega + (2n_\lambda + l_\lambda + \frac{3}{2})\omega \equiv (N + 3)\omega \]

Band Number:

\[ N \equiv 2(n_\rho + n_\lambda) + l_\rho + l_\lambda \geq L \]

Parity:

\[ P = (-)^{l_\rho + l_\lambda} \equiv (-)^N \]
\( \Delta(1750) \ P_{31} \)

The \( \Delta(1750) \) is the lowest state with \( J^P = \frac{1}{2}^+ \)

\[ L_{\text{min}} = 0 \]

\( N = 0, L = 0 \) is symmetry forbidden

(\( I=3/2, S=1/2, \) orbitally mixed symmetric)

\[ \downarrow \]

\( N = 2 \) : \( (70, 0^+) \ S = 1/2 \)

**Quark Pauli Blocking** is responsible for high mass prediction.

Existence of \( \Delta_{1/2+}(1750) \) supports the existence of \( \Delta_{5/2+}(1740) \) and vice versa.
$\Delta(1930)$

The $\Delta(1930)$ is the lowest state with

\[
L_{\text{min}} = 1 \implies N \geq 1
\]

$N=1, \ L=1$ is symmetry forbidden

(T=3/2, S=3/2, orbitally symmetric)

\[
\downarrow
\]

$N = 3 \implies (56, 1^-)_3$

Quark Pauli Blocking is responsible for high mass prediction.
Phenomenological Models

CMB (CUTKOSKY):

\[ \pi N \rightarrow \pi N \]

Channels: \( \pi N + (NR)\pi \pi N + (\pi \Delta, \rho N, \eta N, \sigma N, \omega N, \pi N^*, \rho \Delta) \)

\[ \pi N \rightarrow \pi N + \pi N \rightarrow \pi \pi N \]

Channels: \( \pi N + (NR)\pi \pi N + (\pi \Delta, \rho N, \rho_3 N, \sigma N, \pi N^*) \)

+ Specific (\( \eta N \) for \( S_{11}, \ldots, \rho \Delta \) for \( P_{31}, D_{35}, F_{37} \))

KSU (MANLEY):

Pitt - ANL (VRANA): Expanded version of CMB + \( \pi N \rightarrow \pi NN \) + \( \pi N \rightarrow \eta N \)

VPI-GWU (ARNDT):

\[ \pi N \rightarrow \pi N + \pi N \rightarrow \eta N \]

Channels: \( \pi N + \eta N + \pi \Delta + \rho N \)
Since 2006: EBAC (Excited Baryon Analysis Center)

JLMS Model:

\[ \pi N \rightarrow \pi N \quad + \quad \pi N \rightarrow \eta N \]

Channels: \( \pi N, \eta N, \pi \Delta, \rho N, \sigma N \) + bare \( N^* \)

Bare \( N^* \) states: quark-core components of the nucleon resonances.

Program: Extraction of \( N^* \) from the world data of meson production reactions induced by pions, photons and electrons:

\[ \pi N \rightarrow \pi N, \pi N \rightarrow \eta N, \pi N \rightarrow \pi \pi N, \gamma N \rightarrow \pi N, \gamma N \rightarrow \pi \pi N, eN \rightarrow e\pi N \]
For $\Delta(1930)$ (***), only the phenomenological analyses including the $\rho\Delta$ channel extract the resonance.

**CMB (CUTKOSKY):**

\[ \pi N \rightarrow \pi N \]

Channels: $\pi N + (NR)\pi \pi N + (\pi\Delta, \rho N, \eta N, \sigma N, \omega N, \pi N^*, \rho\Delta)$

**KSU (MANLEY):**

\[ \pi N \rightarrow \pi N + \pi N \rightarrow \pi\pi N \]

Channels: $\pi N + (NR)\pi \pi N + (\pi\Delta, \rho N, \rho_3 N, \sigma N, \pi N^*) +$ Specific ($\eta N$ for $S_{11}, ..., \rho\Delta$ for $P_{31}, D_{35}, F_{37}$))

**Pitt - ANL (VRANA):** Expanded version of CMB + \[ \pi N \rightarrow \eta N \]

**VPI-GWU (ARNDT):**

\[ \pi N \rightarrow \pi N + \pi N \rightarrow \eta N \]

Channels: $\pi N + \eta N + \pi\Delta + \rho N$