Tracing Remnants of the Baryon Vector Current Anomaly in Neutron Radiative $\beta$-Decay

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Harvey, Hill, and Hill (2007, 2008): “New” interactions which involve $N$, $\gamma$, and $W^{\pm}$ or $Z^0$ emerge from gauging the axial anomaly of QCD under the SU(2)$_L \times$U(1)$_Y$ symmetry of the Standard Model at low energies. They are P- and T-odd, and their couplings can be complex with “new” physics.

To discover them, enter radiative neutron (or nuclear) $\beta$-decay. A triple-product correlation in momenta, probed by an asymmetry $A_{HHH}$, can be generated by this new source of CP violation.

How big can the effect be? What “new physics” can it probe? A T-odd decay correlation can be mimicked by non-T-violating final-state interactions. How large is $A_{FSI}$?

The difference between $|A_{HHH}|$ and $|A_{FSI}|$ is the nominal “window” on new physics
Anomalous interactions at low energies

Harvey, Hill, and Hill: When we consider the gauge invariance of the Wess-Zumino-Witten (WZW) term in the QCD chiral Lagrangian under the full electroweak gauge group of the SM \((\text{SU}(2)_L \times \text{U}(1)_Y))\); the vector current is no longer conserved.

In the presence of spin one fields which couple to baryon number, the baryon vector current is anomalous and new contact interactions of pseudo-Chern-Simons form result. Promoting these “spin one fields” to vector mesons of QCD, we have the new interactions, e.g.,

\[ \sim \epsilon^{\mu \nu \rho \sigma} \omega_\mu Z_\nu F_\rho \sigma \quad \text{and} \quad \sim \epsilon^{\mu \nu \rho \sigma} \rho^\pm_\mu W^\mp_\nu F_\rho \sigma \]

[Harvey, Hill, and Hill (2007, 2008)]

At low energy we adopt an effective action of form [Harvey, Hill, and Hill (2007)]

\[
S_{\text{HHH}} = \sum_f \frac{eG_F}{\sqrt{2}} \bar{\xi} \int d^4x \epsilon^{\mu \nu \rho \sigma} \bar{N} \gamma_\mu N \bar{\nu}_L^f \gamma_\nu \nu_L^f F_\rho \sigma
\]

with the estimate \( \xi = \frac{N_c}{12\pi^2} \frac{g^2_\omega}{m^2_\omega} \frac{eG_F}{\sqrt{2}} \), and will use its charged-current analogue.

Note in a chiral effective theory of nucleons and pions with a complete set of electroweak gauge fields, such terms first appear in \(N^2\text{LO}\) in the chiral expansion. [Hill (2010)]
Isolating the "HHH" interaction in neutron radiative $\beta$-decay

Its $\varepsilon^{\mu\nu\rho\sigma}$ structure suggests its symmetry properties can be used to isolate it. In $n(P_n) \rightarrow p(P_p) + e^- (L_e) + \bar{\nu}_e (L_\nu) + \gamma (K)$ decay interference with the $V - A$ terms yields to leading recoil and radiative order

$$|M|^2 = -64M^2 \frac{e^2 G_F^2}{2} \text{Im} \bar{\xi} \frac{E_e}{L_e \cdot K} (L_e \times L_\nu) \cdot K + \ldots$$

The pseudo-T-odd interference term is finite as $\omega \rightarrow 0$. Defining $\eta \equiv (L_e \times L_\nu) \cdot K$ we partition phase space into regions of definite sign:

$$A \equiv \frac{(\Gamma_{\eta>0} - \Gamma_{\eta<0})}{(\Gamma_{\eta>0} + \Gamma_{\eta<0})}$$

To leading recoil order, where $\omega^{\text{min}}$ is lowest detectable photon energy,

$$A(\omega^{\text{min}} = 0.01 \text{ MeV}) = -1.4 \cdot 10^{-3} \text{Im} \bar{\xi} (\text{MeV}^{-2}) , \text{Br}(\omega^{\text{min}} = 0.01 \text{ MeV}) = 3.5 \cdot 10^{-3}$$

and

$$A(\omega^{\text{min}} = 0.3 \text{ MeV}) = -1.3 \cdot 10^{-2} \text{Im} \bar{\xi} (\text{MeV}^{-2}) , \text{Br}(\omega^{\text{min}} = 0.3 \text{ MeV}) = 8.6 \cdot 10^{-5}$$
Radiative $\beta$-Decay Studies

To estimate the ability to detect an asymmetry we consider the counting rates from the neutron radiative $\beta$-decay experiment at NIST. The $ep$ double coincidence counting rate was $20 \text{s}^{-1}$ for a quoted neutron flux of $1.1 \times 10^8 \text{cm}^{-2}\text{s}^{-1}$. [Nico et al. (2006); Nico, private communication]

Noting that the flux at the NG-6 end station is some $\sim 20\times$ larger, then in one week of running one could have a stat. error on the asymmetry of $O(10^{-3})$.

V-A universality implies our formulas apply to nuclear radiative $\beta$-decays, too

The asymmetry is sensitive to the energy release. Nuclear radiative $\beta$ decays with larger energy release will have larger asymmetries.

Perhaps one could study $^{19}\text{Ne} \rightarrow ^{19}\text{F}$ radiative $\beta$-decay, e.g., in an atom trap experiment. The asymmetry is bigger because the energy release is larger, and the $^{19}\text{Ne}$ lifetime is 17.2 s.

Such studies could be realized at a rare isotope accelerator such as FRIB at MSU.

To evaluate the viability of these possibilities we must compute the asymmetry induced by final-state interactions....
What new physics can enter?

Of course $\text{Im } \bar{\xi}$ in the SM is zero. However, the baryon vector current should be anomalous in theories in which the gauge fields couple differently to L- and R-handed quarks. Thus we suppose that “HHH” (pCS) interactions have broader support. They should exist in theories beyond the SM which possess $\text{SU}(2)_L \times \text{U}(1)_Y$ symmetry at low energies.

In searching for new matter beyond the SM, the coupling between the new particles and the SM gauge particles can be complex. Some promising candidates for collider searches include: “quirks” [Okun (1980); Kang and Luty (2008)] “dark quarks” [Blennow et al., arXiv:1009.3159v1 (2010)], and in some cases these can generate $\text{Im } \bar{\xi}$ through processes such as:

![Diagram showing process](image-url)
Decay correlations can be motion-reversal-odd only. This means that final-state interactions can mimic T-odd effects.

[Callan and Treiman (1967); Khriplovich and Okun (1967)]

This happens when calculating the interference of the tree level and the anti-Hermitian parts of the one-loop level Feynman diagrams, so that

$$A_{FSI} \sim 2 \text{Re} \left( \sum_s M^*_\text{tree} \cdot i \text{Im}(M_{\text{loop}}) \right).$$

The anti-Hermitian part is generated by putting intermediate particles on their mass shell, and the induced T-odd correlation is made real by executing traces with $\gamma_5$.

A triple momentum correlation has been previously studied in $K^+ \rightarrow \pi^0 l^+ \nu_l \gamma$ decay. There both electromagnetic and strong ($\pi$ mediated) radiative corrections can mimic the T-odd effect, but the em-induced effects are orders of magnitude larger.

[Braguta, Likhoded, and Chalov (2002); Khriplovich and Rudenko, arXiv:1012.0147 (2010); Müller, Kubis, and Meißner (2006)]

The small energy release associated with neutron and nuclear radiative $\beta$-decay imply that only electromagnetic radiative corrections can mimic the T-odd effect. The induced T-odd effects in this case have never been studied before.

Let’s see all the radiative corrections which appear in $O(\alpha)$.
The Complete Family of Two-Particle Cuts

(1) \( P_n \rightarrow P_p \)

(2) \( P_p \rightarrow P_p \)

(3) \( P_p \rightarrow P_p \)

(4) \( P_p \rightarrow P_p \)

(5.1) \( P_n \rightarrow P_p \)

(5.2) \( P_p \rightarrow P_p \)

(6.1) \( P_p \rightarrow P_p \)

(6.2) \( P_p \rightarrow P_p \)

(6.3) \( P_p \rightarrow P_p \)

(7.1) \( P_p \rightarrow P_p \)

(7.2) \( P_p \rightarrow P_p \)

(8.1) \( P_p \rightarrow P_p \)

(8.2) \( P_p \rightarrow P_p \)

(8.3) \( P_p \rightarrow P_p \)
Im($M_{\text{loop}}$) is computed by performing a “Cutkosky cut”, which technically means that one puts the intermediate particles on their mass shell in all the physically allowed ways and then one integrates over the intermediate phase space and sums over spins. Take the loop diagram (5.1) for instance,

This diagram puts the intermediate electron and photon on their mass shells and gives: [Cutkosky, 1960]

$$\text{Im} M_{5.1}^{\gamma e}(n \rightarrow p e \bar{\nu} \gamma) = \sum_s \int \frac{d^4 L'_e d^4 K'}{2(2\pi)^2} \delta^{(4)}(L'_e + K' - L_e - K) \delta^{(+)}(L'_e^2 - M_e^2) \delta^{(+)}(K'^2) \mathcal{M}^*(\gamma' e' \rightarrow \gamma e) \mathcal{M}(n \rightarrow p \bar{\nu} e' \gamma')$$

$$\equiv \frac{1}{8\pi^2} \sum_s \int d\rho_e \mathcal{M}^*(\gamma' e' \rightarrow \gamma e) \mathcal{M}(n \rightarrow p \bar{\nu} e' \gamma')$$
To deal with an intermediate phase space integral in $L'_e$ and $K'$, e.g., we need to apply the “Passarino-Veltman reduction”, which decomposes the integral in terms of combinations of all the independent final momenta that connect to the loop. In diagram 6.2 we have [Passarino and Veltman, 1979]

\[
J_{6.2} = \int d\rho_e \frac{1}{(L_e \cdot K')(P_p \cdot K')}
\]

with

\[
K_{6.2}^\mu = \int d\rho_e \frac{K^{\mu'}}{(L_e \cdot K')(P_p \cdot K')} = a_{6.2}K^\mu + b_{6.2}L_e^\mu + c_{6.2}P_p^\mu,
\]

and

\[
L_{6.2}^{\mu\nu} = \int d\rho_e \frac{K^{\mu'}K^{\nu'}}{(L_e \cdot K')(P_p \cdot K')}
= d_{6.2}g^{\mu\nu} + e_{6.2}P_p^\mu P_p^\nu + f_{6.2}L_e^\mu L_e^\nu + g_{6.2}K^\mu K^\nu + h_{6.2}(P_p^\mu L_e^\nu + L_e^\mu P_p^\nu) + \ldots
\]

where the omitted terms (...) run through all the remaining possible symmetric combinations of the independent momenta.

We fix the coefficients by solving a set of dynamical equations.
For neutron radiative $\beta$ decay, we have 2 tree level diagrams:

As we can see, there are 14 one-loop diagrams in total and 2 tree diagrams, which gives $14 \times 2 = 28$ contributions to $A_{\text{FSI}}$. We have gone through all of them. Several terms are zero in leading recoil order (LRO), but many are not. For a specific example, take the loop diagram (5.1) and the tree diagram (02), after applying the techniques as mentioned, we get:

$$|\mathcal{M}_{5.1.02}|^2_{T-\text{odd}} = 64 M^3 (1 - \lambda^2) \alpha^2 G_F^2 \left[ b_{5.1} \frac{E_e}{l_e \cdot k \omega} - J_{5.1} \frac{1}{l_e \cdot k} - 2 J_{5.1} \frac{E_e}{l_e \cdot k \omega} \right] l_\nu \cdot (l_e \times k)$$

in LRO, where $b_{5.1}$ and $J_{5.1}$ are the coefficients obtained from the Passarino-Veltman reduction.
Infrared divergences can appear in $n \rightarrow pe^- \bar{\nu} \gamma$ in $\mathcal{O}(\alpha^2)$ in the rate. That is, in diagrams (6.3) and (8.2) the momentum of the exchanged photon can go to zero.

However, from the work of Kinoshita and Lee and Nauenberg we know the total decay rate must still be infrared finite. [Kinoshita (1960); Lee and Nauenberg (1964)]

We in fact confirmed this by combining the contributions of both (6.3) and (8.2); the infrared divergences cancel, and indeed the two diagrams combined don’t have leading recoil order contributions.
Numerical Estimates

We want to compute the precise value of the T-odd asymmetry induced by the FSI and compare it to the one induced by the “HHH” interaction. We are almost finished! In the meantime, the numerical computation of $|\mathcal{M}_{5.1.02}|^2_{T-\text{odd}}$ can give us insight. In LRO, we find

$$\mathcal{A}_{\text{FSI}}^{5.1.02}(\omega_{\text{min}} = 0.01 \text{ MeV}) = 3.4 \cdot 10^{-5}$$

$$\mathcal{A}_{\text{FSI}}^{5.1.02}(\omega_{\text{min}} = 0.3 \text{ MeV}) = 1.0 \cdot 10^{-4}$$

Comparing with $\mathcal{A}_{\text{HHH}}$, this estimate of $\mathcal{A}_{\text{FSI}}$ is only $\sim 10x$ smaller than $\mathcal{A}_{\text{HHH}}$ with $\text{Im} \tilde{\xi} \sim \mathcal{O}(1\text{MeV}^{-2})$.

From our detailed computations, we note that $\mathcal{A}_{\text{FSI}}$ is proportional to $(1 - \lambda^2)$, with $\lambda = g_A/g_V = 1.267$ for neutron $\beta$ decay. The observed quenching of the Gamow-Teller strength in nuclear decays can suppress $\mathcal{A}_{\text{FSI}}$ considerably!

Note shell-model calculations determine $g_A^{\text{eff}} = qg_A$ where $q \approx 0.75$. [Wildenthal, Curtin, and Brown (1983); Martínez-Pinedo et al. (1996)]

We plan to study the pattern of $\mathcal{A}_{\text{FSI}}$ with $g_A^{\text{eff}}$ and energy release carefully.
Harvey, Hill, and Hill suggest it is possible to study remnants of the baryon vector current anomaly in the Standard Model in low energy interactions.

We have argued that the new contact (“HHH”, “pCs”) interactions can also appear in theories BSM with $\text{SU}(2)_L \times \text{U}(1)_Y$ electroweak symmetry at low energies.

We have studied how new sources of CP violation connected to such interactions can be probed through a triple-product momentum correlation in neutron and nuclear radiative $\beta$-decay.

The HHH interaction does not involve the nucleon spin; the constraints offered through the study of the pseudo-T-odd, P-odd asymmetry in neutron (or nuclear) radiative $\beta$-decay are complementary but distinct from those from EDMs.