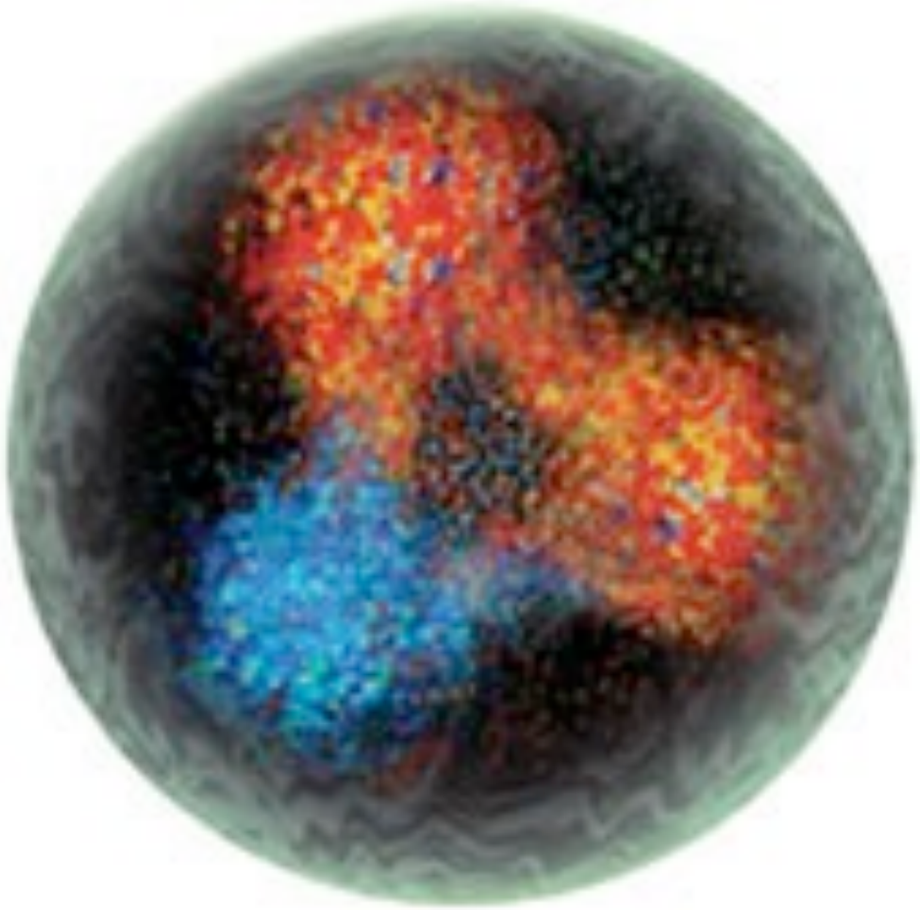
Massachusetts Institute of Technology

Electromagnetic Polarizabilities: Lattice QCD in Background Fields

Brian Tiburzi



PANIC11 RUTHERFORD CENTENNIAL

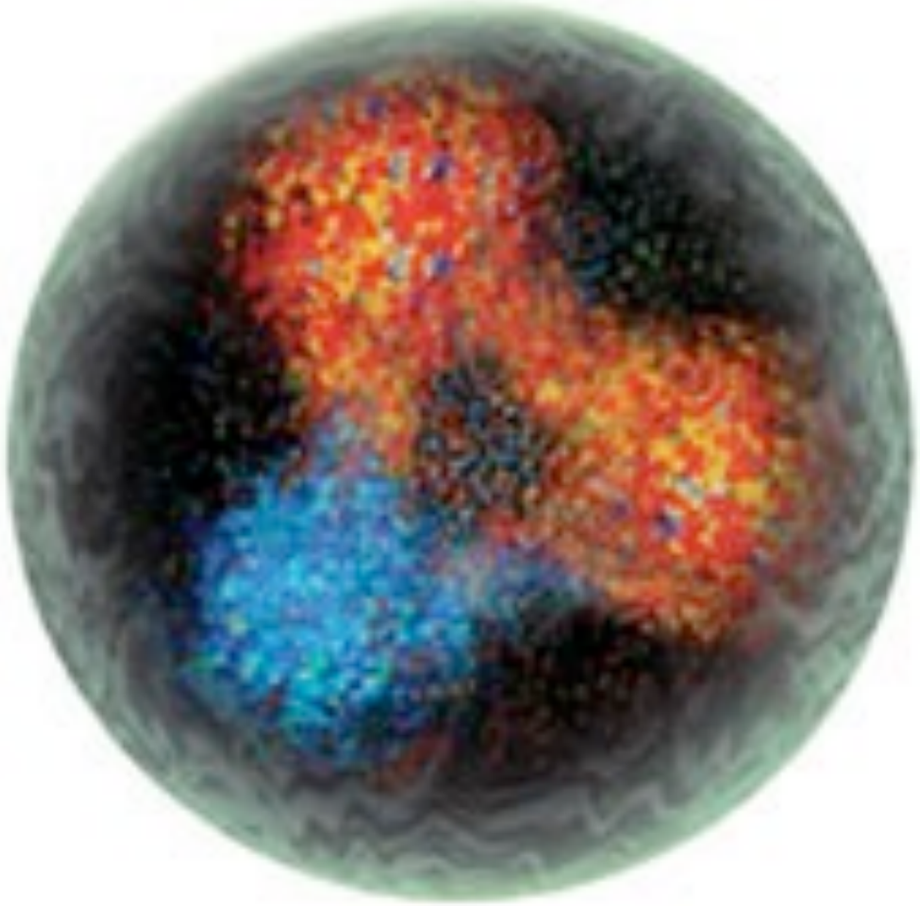
28 July, 2011

Points of Focus

- **Electromagnetic Polarizabilities & Low Energy QCD**
  Ramifications of spontaneous chiral symmetry breaking
  Confrontations with experiment

- **Lattice QCD**
  Numerical solution from first principles

- **Background Field Method**
  Technique to explore the QCD response to external conditions



Ramifications of Spontaneous Chiral Symmetry Breaking

$$\mathcal{L} = \overline{\psi}_L \not{D} \psi_L + \overline{\psi}_R \not{D} \psi_R + m_q \left(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L \right)$$

$$\langle \overline{\psi}_L \psi_R \rangle \neq 0$$

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

-bosons

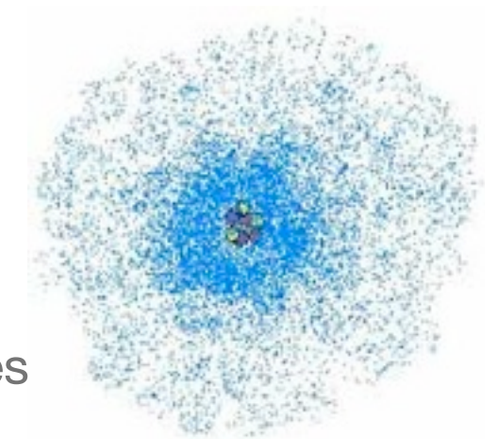
$$m_\pi^2 = \lambda \, m_q \langle \overline{\psi}_L \psi_R \rangle + \ldots$$

- **Chiral Perturbation Theory: *effective* theory of low-energy QCD about chiral limit**

Spontaneous and explicit symmetry breaking pattern constrains form of pion-pion, pion-nucleon interactions

These interactions organized in terms of powers of momentum and/or quark mass

- **E.g. Electromagnetic Polarizabilities**



$$\vec{p} = -\alpha_E \vec{E} \qquad H = -\frac{1}{2} \alpha_E \vec{E}^{\,2} \qquad \alpha_E^{H-\text{atom}} = \frac{27}{8\pi} \left(\frac{4}{3} \pi a_B^3 \right)$$

In external EM fields, hadrons too polarize but against strong chromodynamics forces

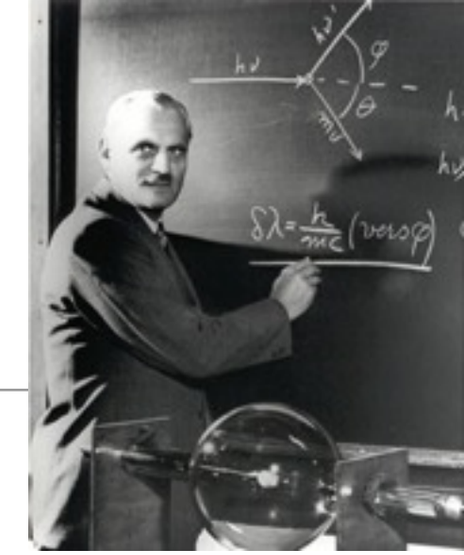
In terms of quarks ... need confinement (bag?) $\quad \alpha_E^{\text{Hadron}} = \mathcal{N} \, \alpha_{\text{f.s.}} \left(\frac{4}{3} \pi \, [\texttt{fm}]^3 \right)$

- **Chiral Perturbation Theory: pion cloud deforms, values well constrained**

$$\alpha_E^N \sim -\frac{e^2 \langle N | \pi N \rangle \langle \pi N | N \rangle}{E_N - E_{\pi N}} = e^2 \frac{g_{\pi N N}^2}{m_\pi M_N^2}$$

$$n \to \pi^- + p \to n$$

Chiral Confrontations



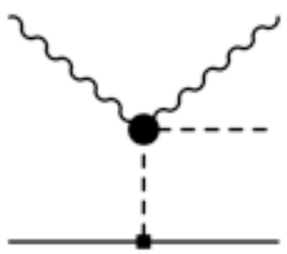
- Polarizabilities accessible in Compton scattering (caveats)

π^+

One Loop **(Holstein)** $(\alpha - \beta)_{\pi^+} = 5.4\,[10^{-4}\,\mathrm{fm}^3]$

Two Loop **(Gasser, Ivanov Sainio)** $(\alpha - \beta)_{\pi^+} = 5.7 \pm 1.0\,[10^{-4}\,\mathrm{fm}^3]$

Mainz Extraction $\gamma + p \to \gamma + \pi^+ + n$



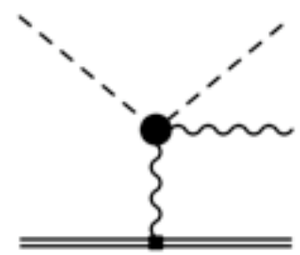
Isolate from hadronic background

Extrapolate to pion pole

Born terms dominate

$$(\alpha - \beta)_{\pi^+} = 11.6 \pm 1.5_{\mathrm{st}} \pm 3.0_{\mathrm{sys}} \pm 0.5_{\mathrm{model}}\,[10^{-4}\,\mathrm{fm}^3]$$

Compass $\pi^- + Pb \to \gamma + \pi^- + Pb$



High statistics

Photon pole

Born terms dominate

DELAYED

N

Theory: include delta resonance?
isovector magnetic?

Phenomenology: low energy > 100 MeV
neutron from deuteron

New data taken off deuterium 60,100 MeV

Comprehensive runs at 65 MeV approved

Turn to Lattice QCD

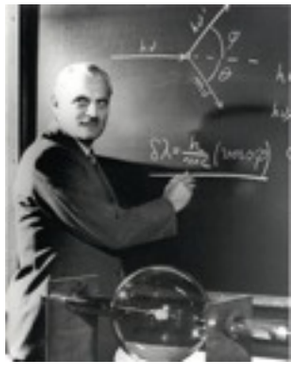
- Timely computations for long-standing discrepancies
- Lattice has stable pion and neutron targets
- Crucial test of low-energy QCD from first principles

CHIRAL CORRECTIONS $m_\pi^2 \log m_\pi^2$

CHIRAL DOMINANCE $1/m_\pi$

But can we do Compton scattering on the lattice?

$$T^{\mu\nu}(k',k) = \int_{x,y} e^{ik\cdot y - ik'\cdot x} \langle H | T\Big\{ J^\mu(x) J^\nu(y) \Big\} | H \rangle$$



$+$

$=?$

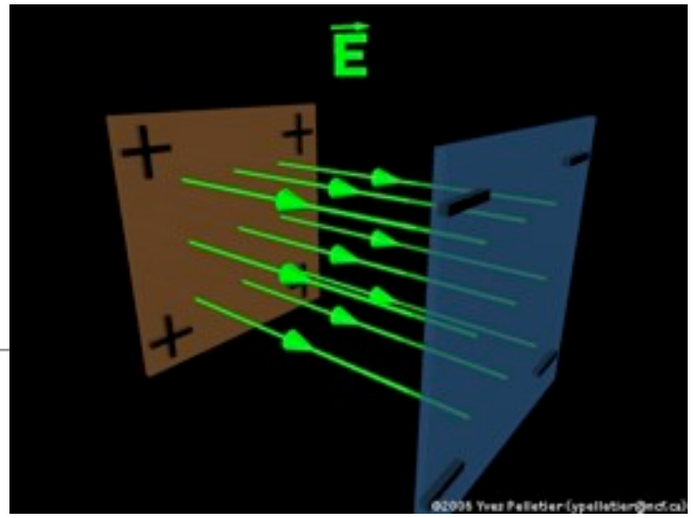
- Four-point functions: two current insertions?... one is difficult enough

  photon (vector meson) hadron scattering

- Momentum quantization: need limit of vanishing momentum

$$\psi(x+L) = \psi(x) \qquad k = \frac{2\pi n}{L} \sim 400\,\mathtt{MeV}$$

Lattice QCD in Background Fields



Method basics are basic

Detmold, Tiburzi, Walker-Loud, PRD 2006

- Measure hadronic correlation functions in classical electromagnetic fields
- Study field strength dependence to determine parameters in effective action

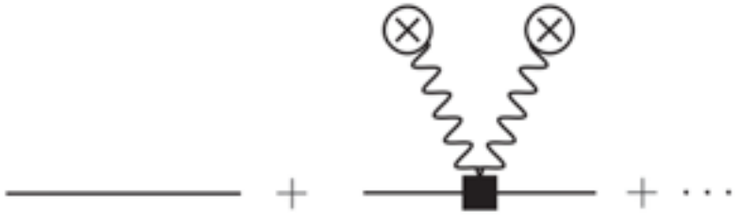
E.g. neutral pion in electric field

$$A_\mu = -\mathcal{E} x_4 \delta_{\mu 3}$$

Effective action $\mathcal{L}(\vec{p}=0, x_4) = \frac{1}{2}\pi^0 \left[-\partial_4\partial_4 + m_\pi^2 + m_\pi \alpha_E \mathcal{E}^2\right] \pi^0$

$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x},\tau)\chi^\dagger(\vec{0},0)\rangle = Z\exp(-E\tau) \qquad E = m_\pi + \frac{1}{2}\alpha_E \mathcal{E}^2 + \ldots$$

Electric field strength dependence of energy yields polarizability



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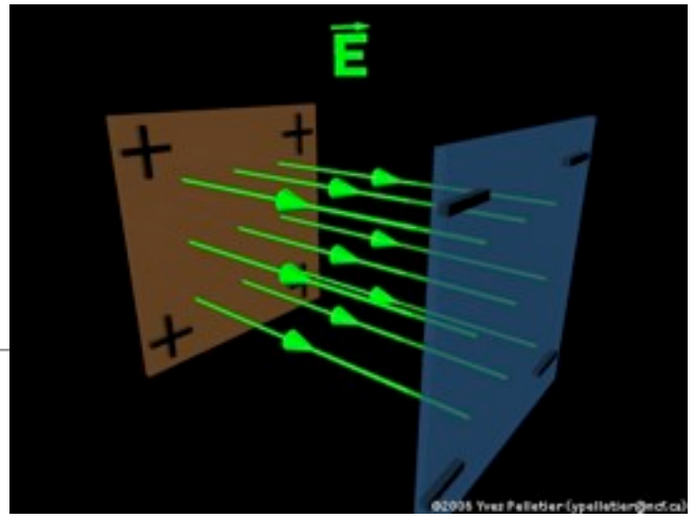

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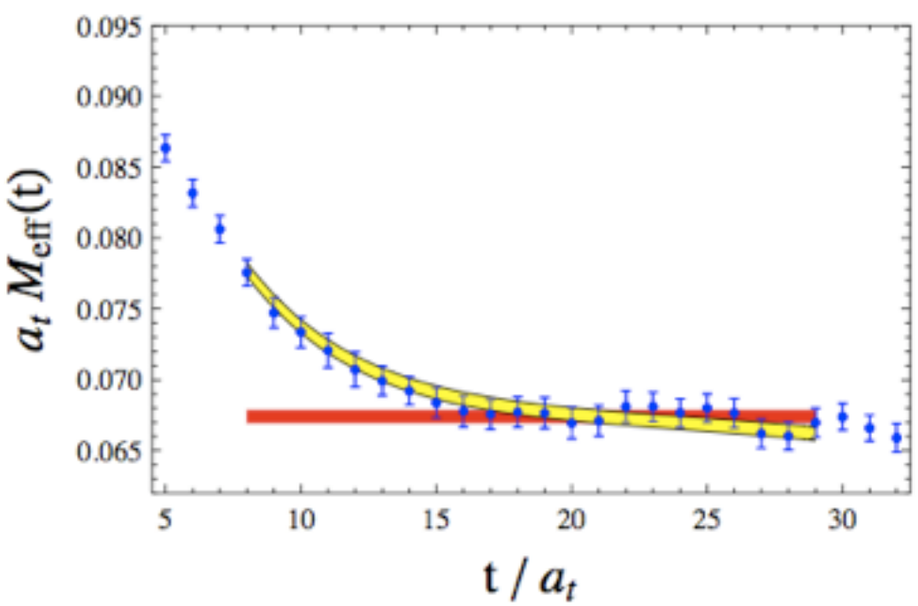

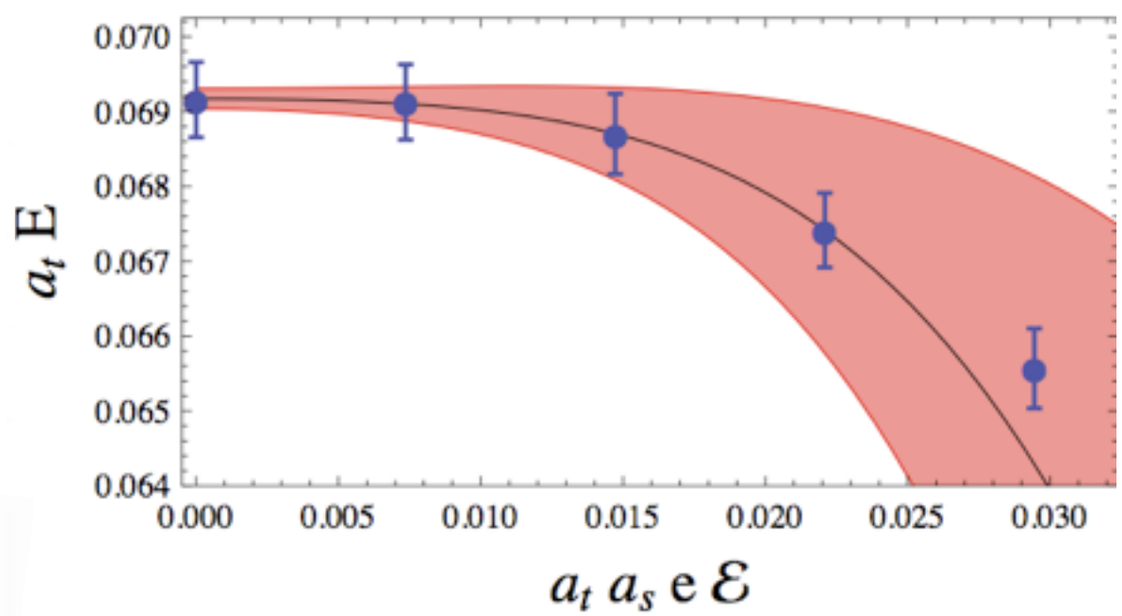

$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x},\tau)\chi^\dagger(\vec{0},0)\rangle = Z\exp(-E\tau)$$

Anisotropic clover lattices (HadSpec)

$20^3 \times 128 \qquad m_\pi = 390\,\mathtt{MeV}$

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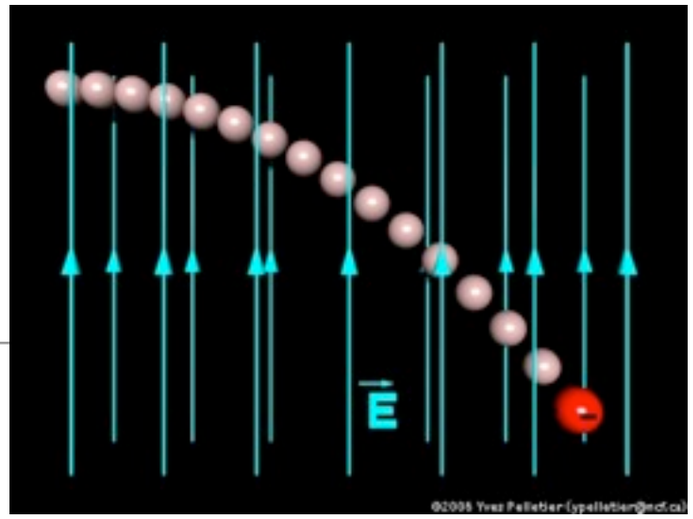

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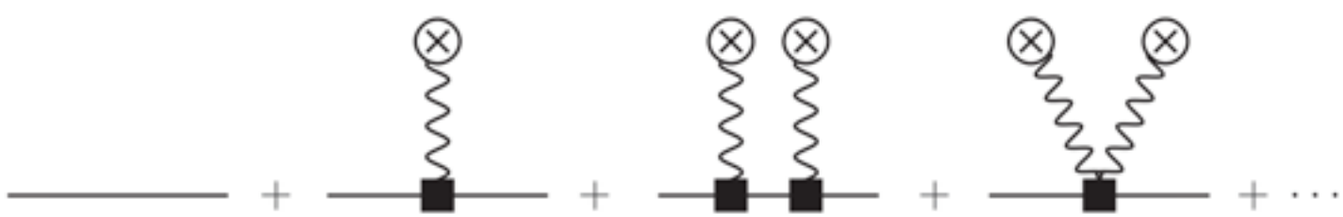

Electric field strength dependence of *rest energy* yields polarizability

$$G(\tau) = \langle \tau | \frac{1}{2\mathcal{H} + E^2} | 0 \rangle = \frac{1}{2} \int_0^\infty ds\, e^{-\frac{1}{2} s E^2} \langle \tau | e^{-s\mathcal{H}} | 0 \rangle$$

Schwinger PR 1951
Tiburzi NuPHA 2008

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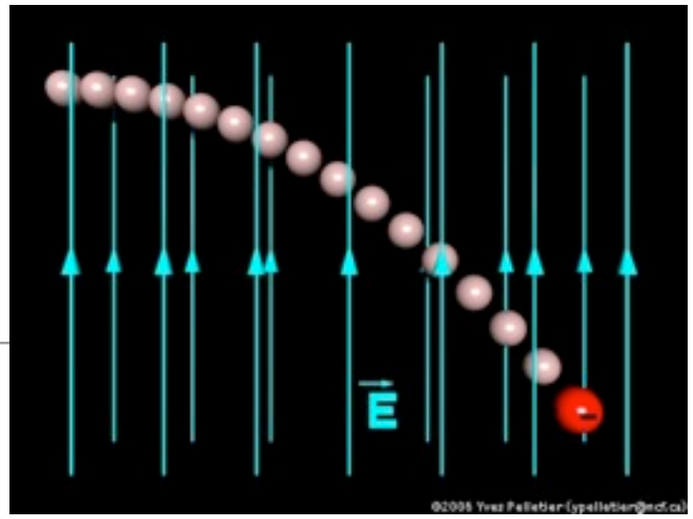

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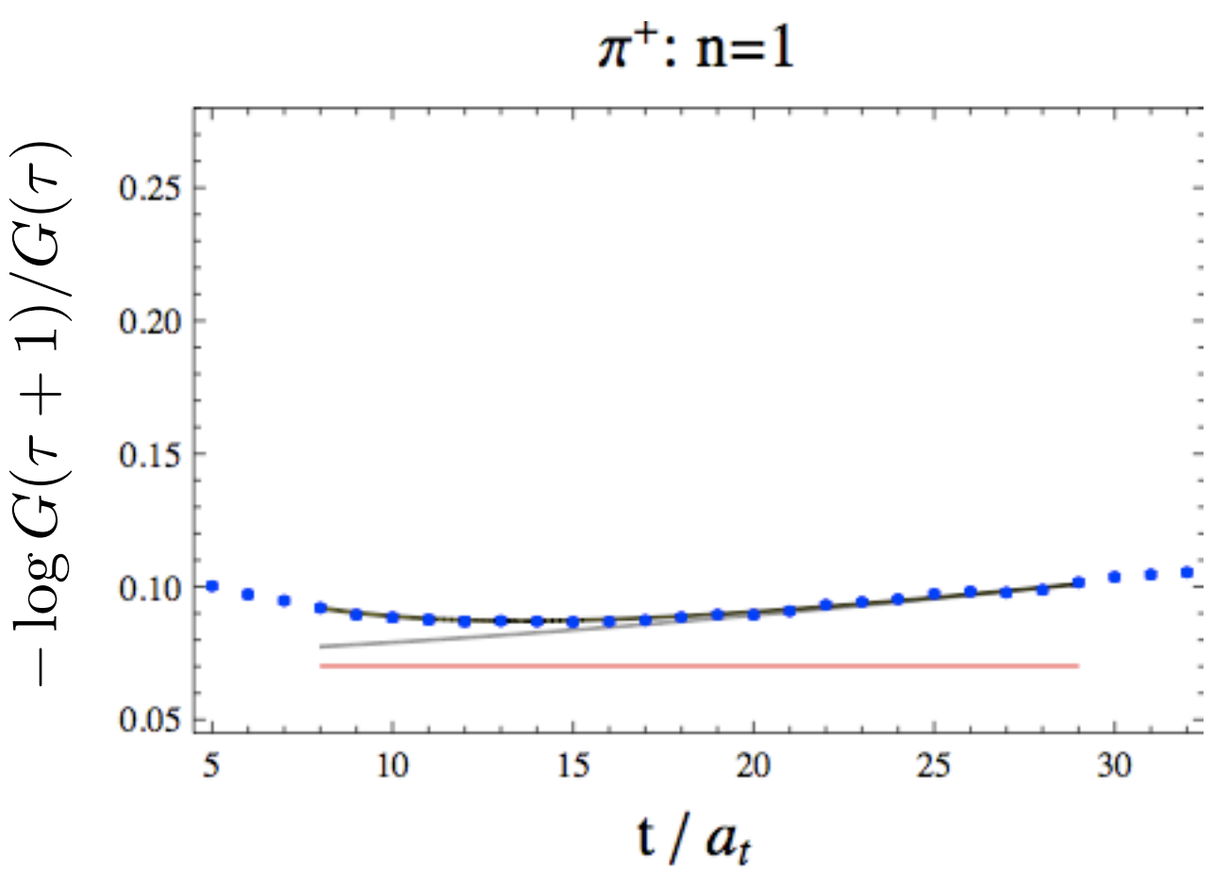

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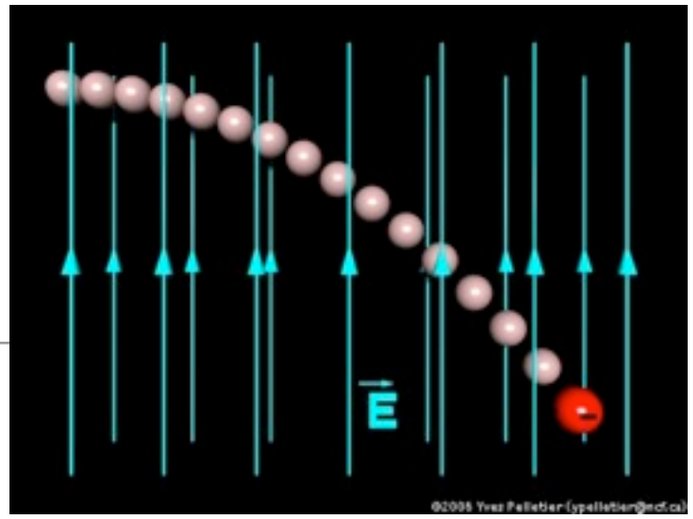

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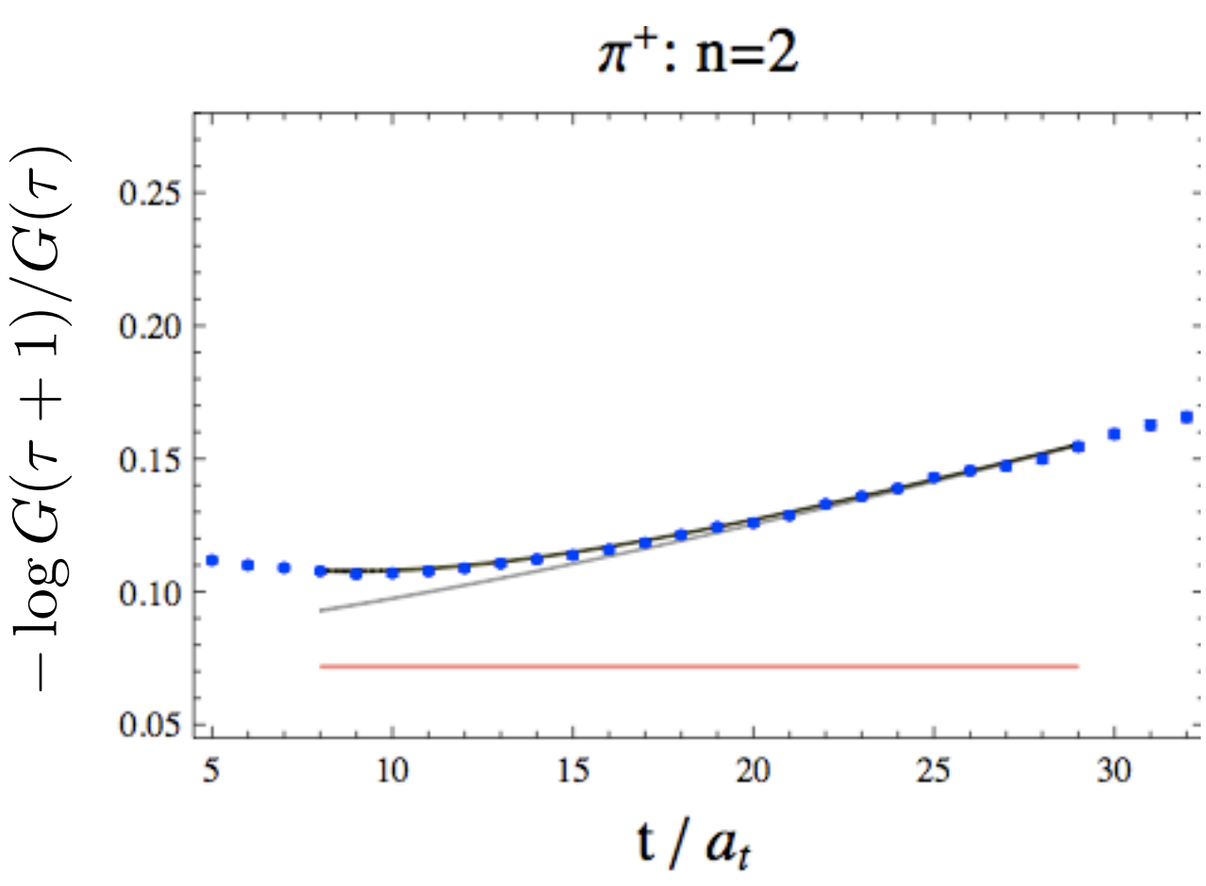

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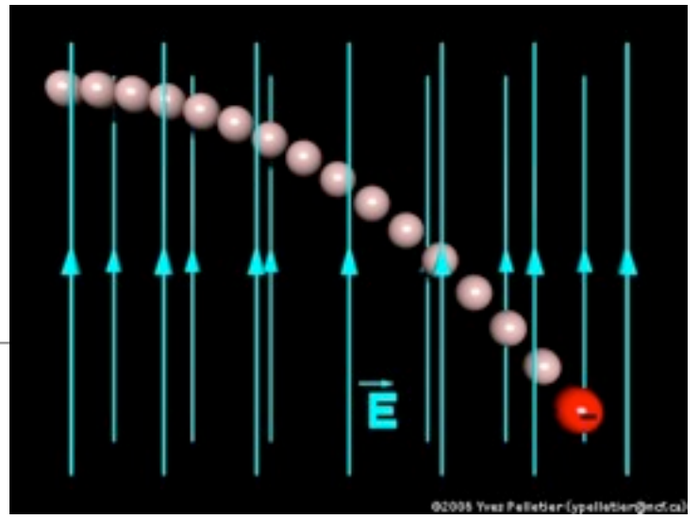

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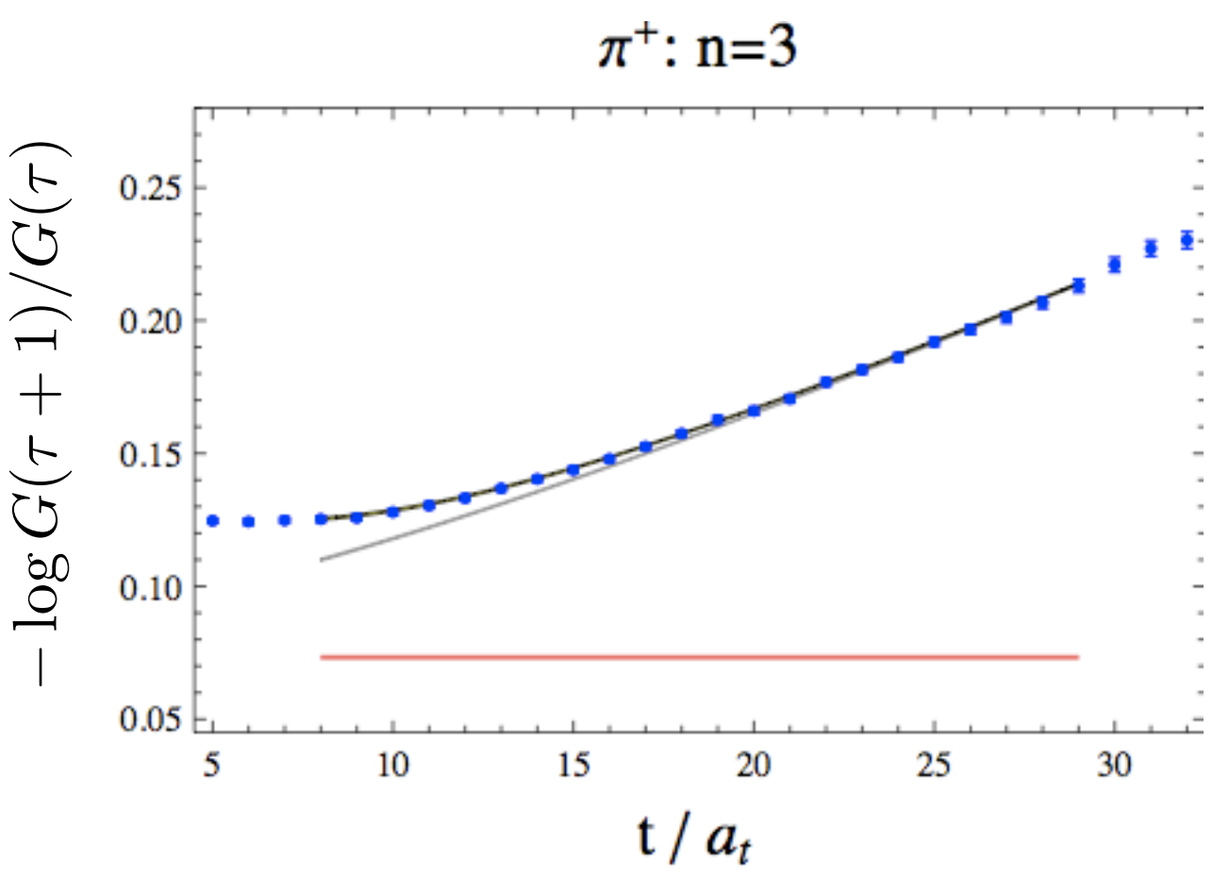

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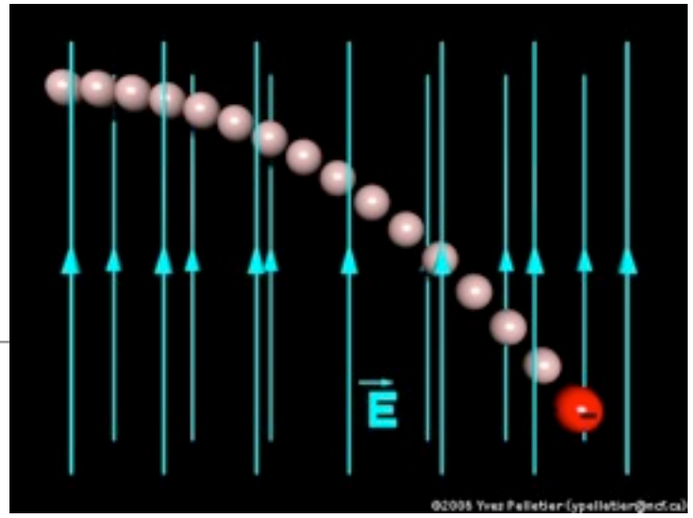

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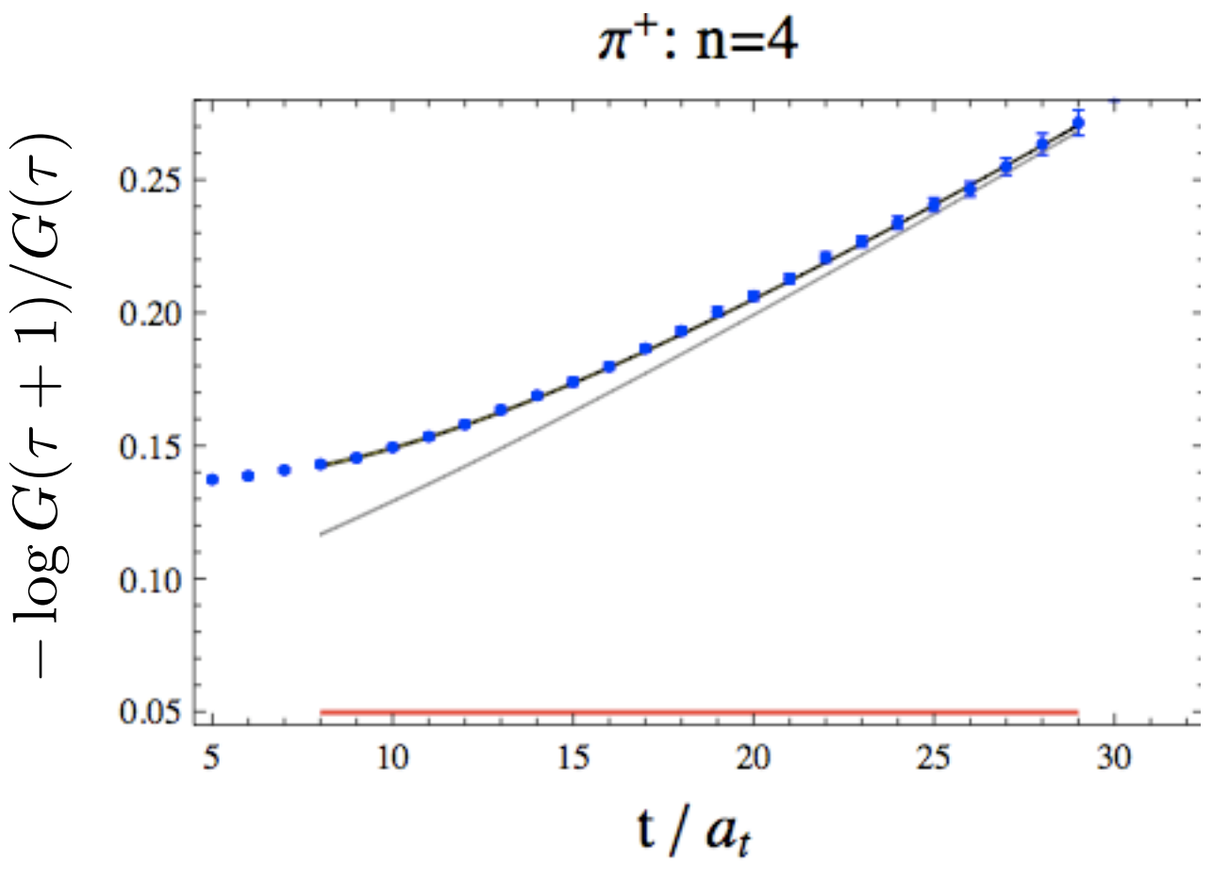

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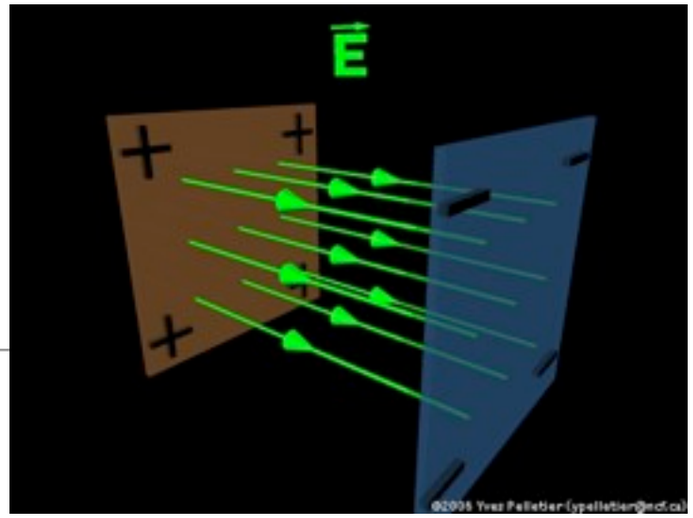

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E.g. neutron in electric field

Detmold, Tiburzi, Walker-Loud, PRD 2010

Effective action $\mathcal{L} = N^\dagger \left[\gamma_\mu \partial_\mu + E - \frac{\mu}{4M_N} \sigma_{\mu\nu} F_{\mu\nu} \right] N$

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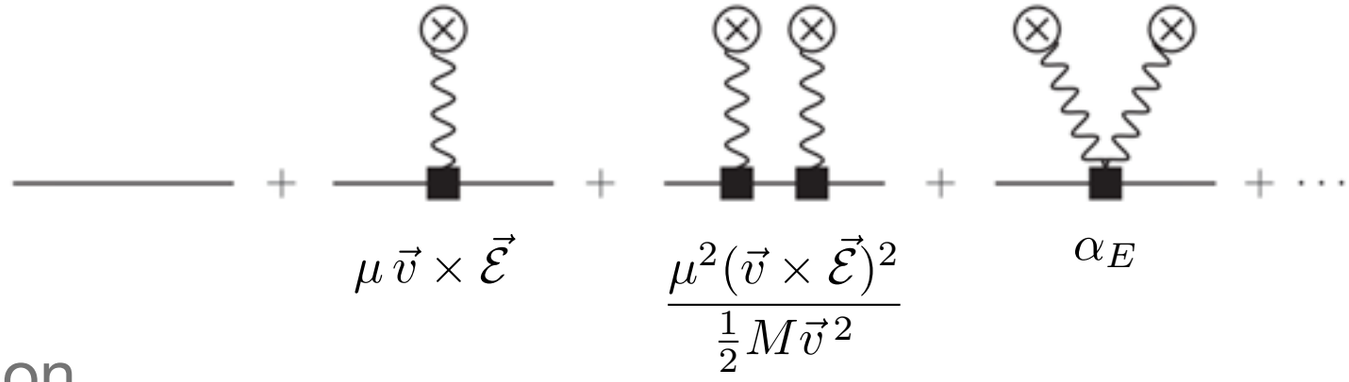

Unpolarized correlation function

$$G(\tau) = Z e^{-E_{\rm eff}\,\tau} \qquad E_{\rm eff} = M + \frac{1}{2}\mathcal{E}^2 \left(\alpha_E - \frac{\mu^2}{4M^3} \right) + \ldots$$

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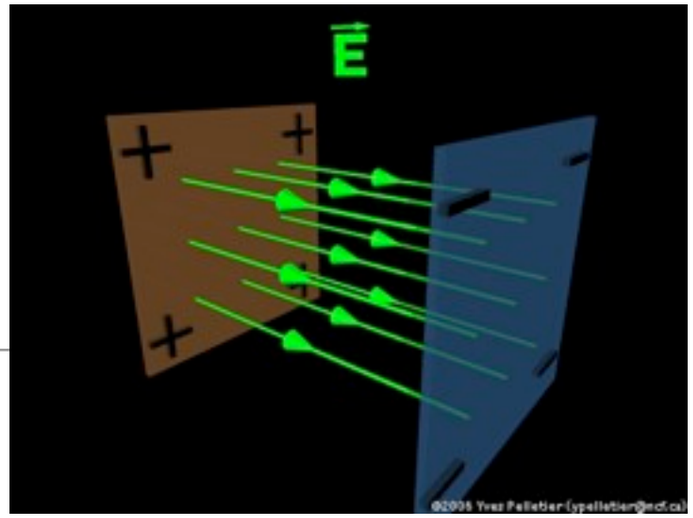

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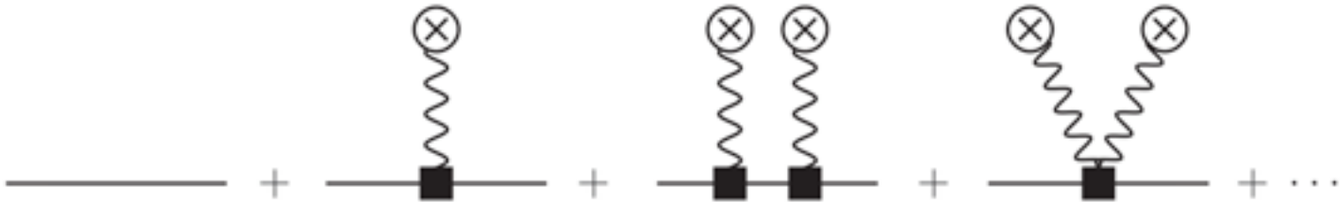
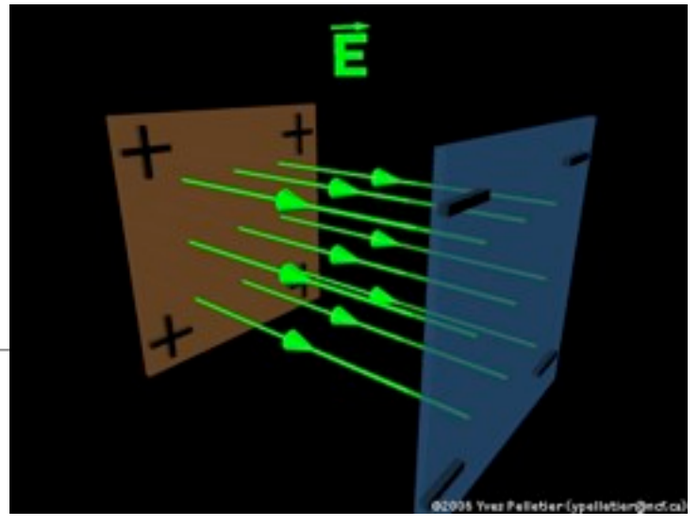


Magnetic Field: spin projected correlators $$G_\pm(\tau) = \mathrm{Tr}\left[\Sigma_\pm G(\tau)\right]$$

$$\sigma_{\mu\nu} F_{\mu\nu} = \vec{S} \cdot \vec{B} \qquad \Sigma_\pm = \frac{1}{2}(1 \pm S_3)$$

Differing energies $$E_\pm = M\left(1 \pm \frac{\mu B}{2M^2}\right)$$

Lattice QCD in Background Fields



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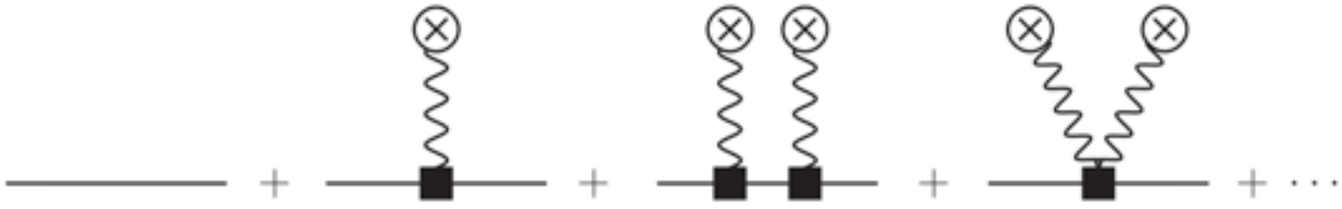
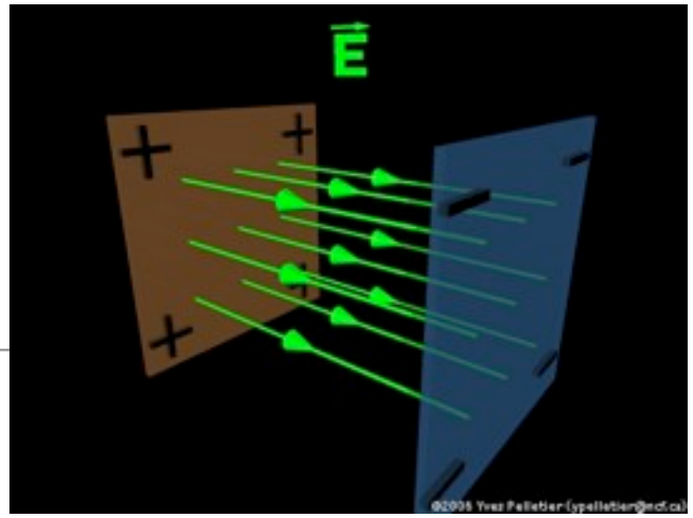


Electric Field: boost projected correlators $G_\pm(\tau) = \mathrm{Tr}\left[\mathcal{P}_\pm G(\tau)\right]$

$\sigma_{\mu\nu} F_{\mu\nu} = \vec{K} \cdot \vec{\mathcal{E}}$ $\quad \mathcal{P}_\pm = \frac{1}{2}(1 \pm K_3)$ $\quad$ Differing amplitudes $Z_\pm = Z\left(1 \pm \frac{\mu \mathcal{E}}{2M^2}\right)$

Lattice QCD in Background Fields



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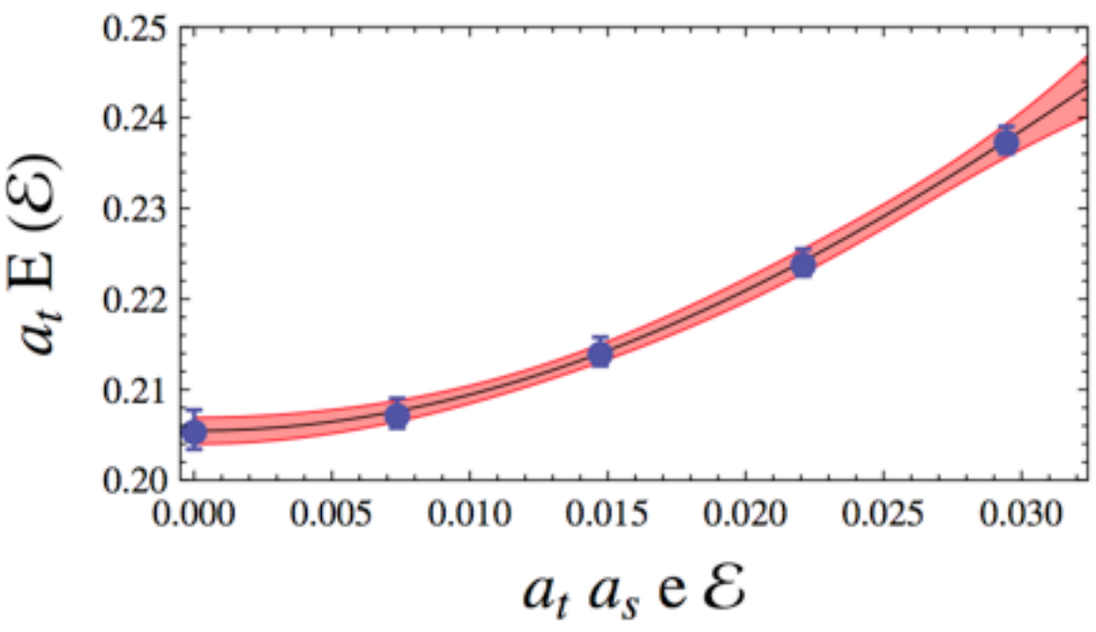


$$\mathrm{Tr}[\mathcal{P}_\pm G(\tau)] = Z\left(1 \pm \frac{\mu\mathcal{E}}{2M_N^2}\right)\exp(-\tau E_{\mathrm{eff}})$$

Simultaneous fit to boost projected correlators

$$\mu_n = -1.6(1)\,[\mu_N]$$
$$(\mu_n)_{\mathrm{exp}} = -1.9\,[\mu_N]$$

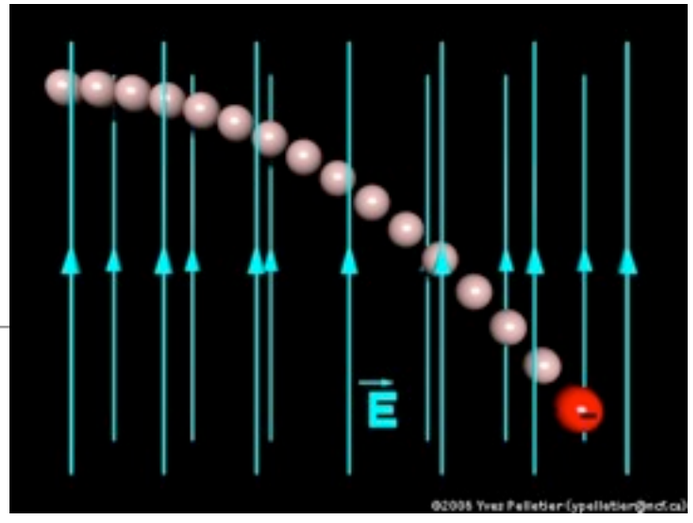
$$\alpha_E^n = 3(1)\times 10^{-4}\,\mathtt{fm}^3$$
$$(\alpha_E^n)_{\mathrm{exp}} = 11(2)\times 10^{-4}\,\mathtt{fm}^3$$

Our results have known systematic errors

$$E = M + \frac{1}{2}\mathcal{E}^2\alpha_E + \ldots$$

Not E_{eff} which includes *Born term*

Proton in Background Electric Field



Charge and anomalous magnetic moment

$$G_\pm(\tau) = Z\left(1 \pm \frac{\kappa\mathcal{E}}{2M^2}\right)\langle\tau|\frac{1}{2\mathcal{H} + E^2 \pm Q\mathcal{E}}|0\rangle$$

Boost projection

- First results for proton

$$\alpha_E^p = 2(1)\times 10^{-4}\,\mathtt{fm}^3$$
$$(\alpha_E^p)_{\rm exp} = 12(1)\times 10^{-4}\,\mathtt{fm}^3$$

Known systematic errors:
pion mass, lattice spacing, lattice volume, ...

$$\alpha_E \sim \frac{1}{m_\pi} \qquad \mu \sim \mu_0 + m_\pi$$

Can test ChPT directly

Relativistic contribution
from magnetic moment

$$\mu_p = 2.6(1)\,[\mu_N]$$
$$(\mu_p)_{\rm exp} = 2.8\,[\mu_N]$$

Known solution:

EMC Detmold, Tiburzi, Walker-Loud, PRD 81 (2010)

Spontaneous chiral symmetry breaking has fundamental consequences for the low-energy structure of hadrons. Confirmation from both experiment and lattice would be a milestone.

Into Focus

- Electromagnetic polarizabilities probe low-energy hadron structure at a fundamental level
- Discrepancies with chiral dynamics motivate their study from first principles
- External field method allows lattice determination; progress made, much work to be done

$$\vec{E} \longrightarrow \vec{B} \longrightarrow \ldots$$

Thanks to: SciDAC Scientific Discovery through Advanced Computing USQCD