Electromagnetic Polarizabilities: Lattice QCD in Background Fields

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Points of Focus

• Electromagnetic Polarizabilities & Low Energy QCD
  Ramifications of spontaneous chiral symmetry breaking
  Confrontations with experiment

• Lattice QCD
  Numerical solution from first principles

• Background Field Method
  Technique to explore the QCD response to external conditions
Ramifications of Spontaneous Chiral Symmetry Breaking

\[ L = \bar{\psi}_L \slashed{D} \psi_L + \bar{\psi}_R \slashed{D} \psi_R + m_q (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) \]

\[ SU(2)_L \times SU(2)_R \to SU(2)_V \]

**Chiral Perturbation Theory:** *effective theory of low-energy QCD about chiral limit*

Spontaneous and explicit symmetry breaking pattern constrains form of pion-pion, pion-nucleon interactions

These interactions organized in terms of powers of momentum and/or quark mass

**E.g. Electromagnetic Polarizabilities**

\[ \rho = -\alpha_E \vec{E} \quad H = -\frac{1}{2} \alpha_E \vec{E}^2 \quad \alpha_E^{H-\text{atom}} = \frac{27}{8\pi} \left( \frac{4}{3} \pi a_B^3 \right) \]

In external EM fields, hadrons too polarize but against strong chromodynamics forces

In terms of quarks ... need confinement (bag?) \( \alpha_E^{\text{Hadron}} = N \alpha_{f.s.} \left( \frac{4}{3} \pi [\text{fm}]^3 \right) \)

**Chiral Perturbation Theory:** pion cloud deforms, values well constrained

\[ \alpha_E^N \sim -\frac{e^2 \langle N|\pi N\rangle\langle\pi N|N\rangle}{E_N - E_{\pi N}} = e^2 \frac{g_{\pi NN}^2}{m_\pi M_N^2} \]

\[ n \to \pi^- + p \to n \]
Chiral Confrontations

- Polarizabilities accessible in Compton scattering (caveats)

\[ \pi^+ \]

One Loop (Holstein)
\[ (\alpha - \beta)_{\pi^+} = 5.4 \times 10^{-4} \text{ fm}^3 \]

Two Loop (Gasser, Ivanov Sainio)
\[ (\alpha - \beta)_{\pi^+} = 5.7 \pm 1.0 \times 10^{-4} \text{ fm}^3 \]

Mainz Extraction
\[ \gamma + p \rightarrow \gamma + \pi^+ + n \]
- Isolate from hadronic background
- Extrapolate to pion pole
- Born terms dominate

\[ (\alpha - \beta)_{\pi^+} = 11.6 \pm 1.5_{\text{st}} \pm 3.0_{\text{sys}} \pm 0.5_{\text{model}} \times 10^{-4} \text{ fm}^3 \]

Compass
\[ \pi^- + Pb \rightarrow \gamma + \pi^- + Pb \]
- High statistics
- Photon pole
- Born terms dominate

\[ N \]

**Theory:** include delta resonance? isovector magnetic?

**Phenomenology:** low energy > 100 MeV
- neutron from deuteron

New data taken off deuterium 60, 100 MeV

Comprehensive runs at 65 MeV approved
Turn to Lattice QCD

- Timely computations for long-standing discrepancies
- Lattice has stable pion and neutron targets
- Crucial test of low-energy QCD from first principles

But can we do Compton scattering on the lattice?

\[ T^{\mu\nu}(k', k) = \int_{x,y} e^{ik'\cdot y - ik\cdot x} \langle H | T \left\{ J_\mu(x) J_\nu(y) \right\} | H \rangle \]

- Four-point functions: two current insertions?... one is difficult enough photon (vector meson) hadron scattering
- Momentum quantization: need limit of vanishing momentum

\[ \psi(x + L) = \psi(x) \quad k = \frac{2\pi n}{L} \sim 400 \text{ MeV} \]
Lattice QCD in Background Fields

**Method basics are basic**  
Detmold, Tiburzi, Walker-Loud, PRD 2006

- Measure hadronic correlation functions in classical electromagnetic fields
- Study field strength dependence to determine parameters in effective action

**E.g. neutral pion in electric field**

Effective action

\[ \mathcal{L}(\vec{p} = 0, x_4) = \frac{1}{2} \pi^0 \left[ -\partial_4 \partial_4 + m^2_{\pi} + m_{\pi} \alpha_E \mathcal{E}^2 \right] \pi^0 \]

\[ A_\mu = -\mathcal{E} x_4 \delta_{\mu 3} \]

\[ G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x}, \tau) \chi^\dagger(\vec{0}, 0) \rangle = Z \exp(-E \tau) \]

\[ E = m_{\pi} + \frac{1}{2} \alpha_E \mathcal{E}^2 + \ldots \]

Electric field strength dependence of energy yields polarizability
Lattice QCD in Background Fields

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E.g. neutral pion in electric field

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Detmold, Tiburzi, Walker-Loud, PRD 2008

Anisotropic clover lattices (HadSpec)

\(20^3 \times 128 \quad m_\pi = 390 \text{ MeV}\)
Lattice QCD in Background Fields

**Method basics are basic**

- Measure hadronic correlation functions in classical electromagnetic fields
- Study field strength dependence to determine parameters in effective action

**E.g. charged pion in electric field**

Effective action

\[
\mathcal{L}(\vec{p} = 0, x_4) = \pi^+ \left[ -\partial_4 \partial_4 + \mathcal{E}^2 x_4^2 + E(E)^2 \right] \pi^-
\]

\[
G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x}, \tau) \chi^\dagger(0, 0) \rangle
\]

Electric field strength dependence of *rest energy* yields polarizability

\[
G(\tau) = \langle \tau | \frac{1}{2\mathcal{H} + E^2} | 0 \rangle = \frac{1}{2} \int_0^\infty ds \, e^{-\frac{1}{2} s E^2} \langle \tau | e^{-s \mathcal{H}} | 0 \rangle
\]

Schwinger PR 1951

Tiburzi NuPH A 2008
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### E.g. charged pion in electric field

\[ G(\tau) = \left\langle \tau \left| \frac{1}{2\mathcal{H} + E^2} \right| 0 \right\rangle = \frac{1}{2} \int_0^\infty ds \ e^{-\frac{1}{2} s E^2} \left\langle \tau \left| e^{-s\mathcal{H}} \right| 0 \right\rangle \]

Detmold, Tiburzi, Walker-Loud, PRD 2008

Anisotropic clover lattices (HadSpec)

\[ 20^3 \times 128 \quad m_\pi = 390 \text{ MeV} \]

\[ \pi^+: n=1 \]

Schwinger PR 1951

Tiburzi NuPHA 2008
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\[ \pi^+: n=2 \]

Schwinger PR 1951
Tiburzi NuPHA 2008
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Anisotropic clover lattices (HadSpec)

\[ m_\pi = 390 \text{ MeV} \]

\[ G(\tau) = \langle \tau | \frac{1}{2\mathcal{H} + E^2} | 0 \rangle = \frac{1}{2} \int_0^\infty ds \, e^{-\frac{1}{2} s E^2} \langle \tau | e^{-s\mathcal{H}} | 0 \rangle \]

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Detmold, Tiburzi, Walker-Loud, PRD 2008

Anisotropic clover lattices (HadSpec)

\[ 20^3 \times 128 \quad m_\pi = 390 \text{ MeV} \]

\[ \pi^+: n=4 \]

\[ \frac{\log G(\tau + 1)}{G(\tau)} \]

Schwinger PR 1951

Tiburzi NuPHA 2008
Lattice QCD in Background Fields

Method basics are basic

- Measure hadronic correlation functions in classical electromagnetic fields
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E.g. neutron in electric field

Effective action

\[
\mathcal{L} = N^\dagger \left[ \gamma_\mu \partial_\mu + E - \frac{\mu}{4M_N} \sigma_{\mu\nu} F_{\mu\nu} \right] N
\]

Unpolarized correlation function

\[
G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x}, \tau) \chi^\dagger(\vec{0}, 0) \rangle
\]

Unpolarized correlation function

\[
G(\tau) = Z e^{-E_{\text{eff}} \tau}
\]

\[
E_{\text{eff}} = M + \frac{1}{2} \mathcal{E}^2 \left( \alpha_E - \frac{\mu^2}{4M^3} \right) + \ldots
\]

Detmold, Tiburzi, Walker-Loud, PRD 2010
Lattice QCD in Background Fields

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**E.g. neutron in electric field**

Detmold, Tiburzi, Walker-Loud, PRD 2010

Effective action \( \mathcal{L} = N^\dagger \left[ \gamma_\mu \partial_\mu + E - \frac{\mu}{4M_N} \sigma_{\mu\nu} F_{\mu\nu} \right] N \)

\[ G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x}, \tau) \chi^\dagger(\vec{0}, 0) \rangle \]

Magnetic Field: spin projected correlators

\[ \sigma_{\mu\nu} F_{\mu\nu} = \vec{S} \cdot \vec{B} \quad \Sigma_\pm = \frac{1}{2} (1 \pm S_3) \quad \text{Differing energies} \quad E_\pm = M \left( 1 \pm \frac{\mu B}{2M^2} \right) \]
Lattice QCD in Background Fields

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**E.g. neutron in electric field**
Detmold, Tiburzi, Walker-Loud, PRD 2010

Effective action
\[
\mathcal{L} = N^\dagger \left[ \gamma_\mu \partial_\mu + E - \frac{\mu}{4M_N} \sigma_{\mu\nu} F_{\mu\nu} \right] N
\]

\[
G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x}, \tau) \chi^\dagger(\vec{0}, 0) \rangle
\]

Electric Field: boost projected correlators

\[
\sigma_{\mu\nu} F_{\mu\nu} = \vec{K} \cdot \vec{E}
\]

\[
P_\pm = \frac{1}{2} (1 \pm K_3)
\]

Differing amplitudes

\[
Z_\pm = Z \left(1 \pm \frac{\mu E}{2M^2} \right)
\]
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Method basics are basic

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E.g. neutron in electric field  
Detmold, Tiburzi, Walker-Loud, PRD 2010

\[
\text{Tr}[\mathcal{P}_\pm G(\tau)] = Z \left( 1 \pm \frac{\mu E}{2M_N^2} \right) \exp(-\tau E_{\text{eff}})
\]

Simultaneous fit to boost projected correlators

\[
\mu_n = -1.6(1) [\mu_N] \\
(\mu_n)_{\text{exp}} = -1.9 [\mu_N]
\]

\[
\alpha^n_E = 3(1) \times 10^{-4} \text{ fm}^3 \\
(\alpha^n_E)_{\text{exp}} = 11(2) \times 10^{-4} \text{ fm}^3
\]

Our results have known systematic errors

Not \( E_{\text{eff}} \) which includes Born term
Proton in Background Electric Field

Charge and anomalous magnetic moment

\[ G_{\pm}(\tau) = Z \left( 1 \pm \frac{\kappa E}{2M^2} \right) \langle \tau \left| \frac{1}{2\mathcal{H} + E^2 \pm QE} \right| 0 \rangle \]

- First results for proton
  \[ \alpha_E^p = 2(1) \times 10^{-4} \text{ fm}^3 \]
  \[ (\alpha_E^p)_{\text{exp}} = 12(1) \times 10^{-4} \text{ fm}^3 \]

Relativistic contribution from magnetic moment

Known systematic errors:
- pion mass, lattice spacing, lattice volume, ...

Known solution:

\[ \frac{\alpha_E}{\mu} \sim \frac{1}{m_\pi} \quad \mu \sim \mu_0 + m_\pi \]

Spontaneous chiral symmetry breaking has fundamental consequences for the low-energy structure of hadrons. Confirmation from both experiment and lattice would be a milestone.

Into Focus

- Electromagnetic polarizabilities probe low-energy hadron structure at a fundamental level
- Discrepancies with chiral dynamics motivate their study from first principles
- External field method allows lattice determination; progress made, much work to be done

\[ \vec{E} \rightarrow \vec{B} \rightarrow \ldots \]