Diquark Correlations in a Hadron From Lattice QCD

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The 19th Particles and Nuclei International Conference
July 24–29, 2011
Diquarks

- Diquarks are two-quark systems.
- Diquark models are used to describe baryons as quark+diquark, and for other aspects of hadronic physics.
- They are colored objects and so cannot be isolated.
- Lattice QCD provides a means of studying them.
Good and bad diquarks

- The lowest energy diquarks are formed from two quarks in the color $\overline{3}$ representation, with no spatial excitations, so the flavor-spin wavefunction must be symmetric.
- This leaves two positive-parity diquarks: the spin 0, flavor antisymmetric “good” diquark $(q^T C \gamma_5 q)$ and the spin 1, flavor symmetric “bad” diquark $(q^T C \gamma_i q)$.
- One gluon exchange in a quark model predicts that the bad diquarks have higher energy by $\sim 200$ MeV.
- Instanton interactions also favor good diquarks.
- The remaining diquarks created by $q^T C \Gamma q$ quark bilinears have odd parity and higher energy.

Color antitriplet diquark combined with a static quark to form a color singlet.

Measured two-quark density correlator:

\[ C_{\Gamma}(r_u, r_d, t) = \langle 0 | B_{\Gamma}(0, 2t) J_0^u(r_u, t) J_0^d(r_d, t) B_{\Gamma}^\dagger(0, 0) | 0 \rangle \]

where \( J_0^f = \bar{f} \gamma_0 f \) and \( B_{\Gamma} = \epsilon^{abc} [ u_a^T C \Gamma d_b \pm d_a^T C \Gamma u_b ] s_c \).
Spatial correlations

To isolate the intrinsic diquark correlations, the authors looked at spherical shells $|r_u| = |r_d| = r$. 

$C_\Gamma(r=\alpha=5,10) / C_{\Gamma_0}(r=0.5,0)$
A different approach


- Using gauge-fixed lattices and finite mass strange quarks (degenerate with u and d), the zero-momentum correlator

\[ G(\vec{r}_u, \vec{r}_d, t) = \sum_{\vec{r}} \langle u(\vec{r} + \vec{r}_u, t) d(\vec{r} + \vec{r}_d, t) s(\vec{r}, t) \bar{u}(\vec{0}, 0) \bar{d}(\vec{0}, 0) \bar{s}(\vec{0}, 0) \rangle \]

was computed for the Λ, Σ, and Σ* baryons.

- This was used to define a wave function

\[ \Psi(\vec{r}_u, \vec{r}_d) = \frac{G(\vec{r}_u, \vec{r}_d, t)}{\sum_{\vec{r}_u, \vec{r}_d} |G(\vec{r}_u, \vec{r}_d, t)|^2}. \]
Motivation

Previous work

Diquark wavefunction

Λ (red) and Σ* (green) in the Coulomb gauge, at $R/a = 4.5$ (left) and $R/a = 2.25$ (right).
Uncorrelated $\rho_2(r_1, r_2) = \rho_1(r_1)\rho_1(r_2)$ plotted this way can give the appearance of a diquark.

Want to show only the clustering induced by the diquark interaction.
Correlation function

\[ C(r_1, r_2) = \frac{\rho_2(r_1, r_2) - \rho_1(r_1)\rho_1(r_2)}{\rho_1(r_1)\rho_1(r_2)} \]

- Is zero if there is no diquark interaction.
- Denominator compensates for presence of static quark at \( r = 0 \).
Lattice measurements

\[ \rho_1(r) \propto \langle 0 | B(0, t_f) J_0^u(r, t) \overline{B}(0, t_i) | 0 \rangle \]

\[ \rho_2(r_1, r_2) \propto \langle 0 | B(0, t_f) J_0^u(r_1, t) J_0^d(r_2, t) \overline{B}(0, t_i) | 0 \rangle \]

- Studied both good diquark \( B = (u^T C \gamma_5 d)s \) and bad diquark \( B = (u^T C \gamma_i d)s \), with degenerate \( u \) and \( d \) quarks and a static quark \( s \).
- Two lattice ensembles:
  1. \( 16^3 \times 32, a = 0.088 \) fm, quenched Wilson quarks with \( m_\pi = 940 \) MeV.
  2. \( 20^3 \times 64, a = 0.124 \) fm, domain wall valence quarks on a staggered sea with \( m_\pi = 293 \) MeV.

Start by looking at \( m_\pi = 940 \) MeV good diquark.
New contributions

Image effects

\[ a^3 \rho_1(r) \]

\[ \rho_1(r) \]

\[ \frac{r}{a} \]

\[ 10^{-5}, 10^{-4}, 10^{-3} \]

Jeremy Green (MIT-CTP)
New contributions

Image effects

\[ \rho_2 \text{ with } R \text{ fixed and } \mathbf{R} \perp \mathbf{r} \]

\[ R/a = 0 \quad \text{red line} \]
\[ R/a = 4 \quad \text{green line} \]
\[ R/a = 6 \quad \text{blue line} \]

\[ R/a = 0 \]
\[ R/a = 4 \]
\[ R/a = 6 \]
Image effects

\[ \rho_2 \text{ with } R \text{ fixed and } \mathbf{R} \parallel \mathbf{r} \]
Image correction

- Deal with image effects by fitting a parameterized function to the data.
- Instead of fitting $f(r_1, r_2)$, fit

$$f_{\text{img}}(r_1, r_2) = \sum_{n_1^i, n_2^i=-1,0,1} f(r_1 + n_1^i L, r_2 + n_2^i L)$$

- Given a good fit, the image effects can be subtracted off.
- Data points most affected by images were excluded from the fit shown here.
- 11 parameter fit, $\chi^2_{\text{dof}} \sim 0.25–1.85$
Image correction

\[ \rho_1(r) \]

\[ \frac{a^3 \rho_1(r)}{r/a} \]

\[ \rho_1(r) \] vs. \[ r/a \]

\[ 10^{-5}, 10^{-4}, 10^{-3} \]
Image correction

\[ \rho_2 \text{ with } R \text{ fixed and } R \perp r \]
Image correction

\[ \rho_2 \text{ with } R \text{ fixed and } \mathbf{R} \parallel \mathbf{r} \]
Comparison of $\rho_1$

$\rho_1 \propto \left( \frac{1}{r} \right)^{0.6}$

$\rho_1 \propto \left( \frac{1}{r} \right)^{1.0}$

$\rho_1 \propto \left( \frac{1}{r} \right)^{1.2}$

$\rho_1 \propto \left( \frac{1}{r} \right)^{1.4}$

$\rho_1 \propto \left( \frac{1}{r} \right)^{1.6}$

$\rho_1 \propto \left( \frac{1}{r} \right)^{1.8}$
Correlation function

$C(r_1, r_2)$

$r_\text{(fm)}$

good $m_\pi = 940 \text{ MeV}$
bad $m_\pi = 940 \text{ MeV}$
good $m_\pi = 293 \text{ MeV}$
bad $m_\pi = 293 \text{ MeV}$

$C$ with $R = 0$
Correlation function

\[ C(r_1, r_2) \]

\[
\begin{align*}
good m_\pi &= 940 \text{ MeV} \\
bad m_\pi &= 940 \text{ MeV} \\
good m_\pi &= 293 \text{ MeV} \\
bad m_\pi &= 293 \text{ MeV}
\end{align*}
\]

\[ C \text{ with } R = 0.2 \text{ fm and } \mathbf{R} \perp \mathbf{r} \]
Correlation function

\[ C(r_1, r_2) \]

with \( R = 0.2 \text{ fm} \) and \( R \parallel r \)

\begin{align*}
good m_\pi &= 940 \text{ MeV} \\
bad m_\pi &= 940 \text{ MeV} \\
good m_\pi &= 293 \text{ MeV} \\
bad m_\pi &= 293 \text{ MeV}
\end{align*}
Correlation function

\[ C(r_1, r_2) \]

\[ C \] with \( R = 0.4 \text{ fm} \) and \( R \perp r \)

\[ \text{good } m_\pi = 940 \text{ MeV} \]
\[ \text{bad } m_\pi = 940 \text{ MeV} \]
\[ \text{good } m_\pi = 293 \text{ MeV} \]
\[ \text{bad } m_\pi = 293 \text{ MeV} \]
Results

Correlation function

$C(r_1, r_2)$

$C$ with $R = 0.4$ fm and $R \parallel r$
Correlation function

\[ C(r_1, r_2) \]

\( m_\pi = 940 \text{ MeV} \)

\( m_\pi = 293 \text{ MeV} \)

\( R = 0.4 \text{ fm} \)

\[ C \] from fit versus \( r \) (in fm), with \( R = 0.4 \text{ fm} \).
Summary

- Good and bad diquark compared at $m_\pi = 940$ MeV and $m_\pi = 293$ MeV.
- Good diquark has stronger correlation as expected.
- Difference between good and bad diquark is greater at smaller pion mass.