Diquark Correlations in a Hadron From Lattice QCD

Jeremy Green[a]
Michael Engelhardt[b] John Negele[a] Patrick Varilly[c]

[a]Center for Theoretical Physics
Massachusetts Institute of Technology

[b]Physics Department
New Mexico State University

[c]Department of Physics
University of California, Berkeley

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Diquarks

- Diquarks are two-quark systems.
- Diquark models are used to describe baryons as quark+diquark, and for other aspects of hadronic physics.
- They are colored objects and so cannot be isolated.
- Lattice QCD provides a means of studying them.

Good and bad diquarks

- The lowest energy diquarks are formed from two quarks in the color $\overline{\mathbf{3}}$ representation, with no spatial excitations, so the flavor-spin wavefunction must be symmetric.
- This leaves two positive-parity diquarks: the spin 0, flavor antisymmetric "good" diquark ($q^T C\gamma_5 q$) and the spin 1, flavor symmetric "bad" diquark ($q^T C\gamma_i q$).
- One gluon exchange in a quark model predicts that the bad diquarks have higher energy by ~ 200 MeV.
- Instanton interactions also favor good diquarks.
- The remaining diquarks created by $q^T C\Gamma q$ quark bilinears have odd parity and higher energy.

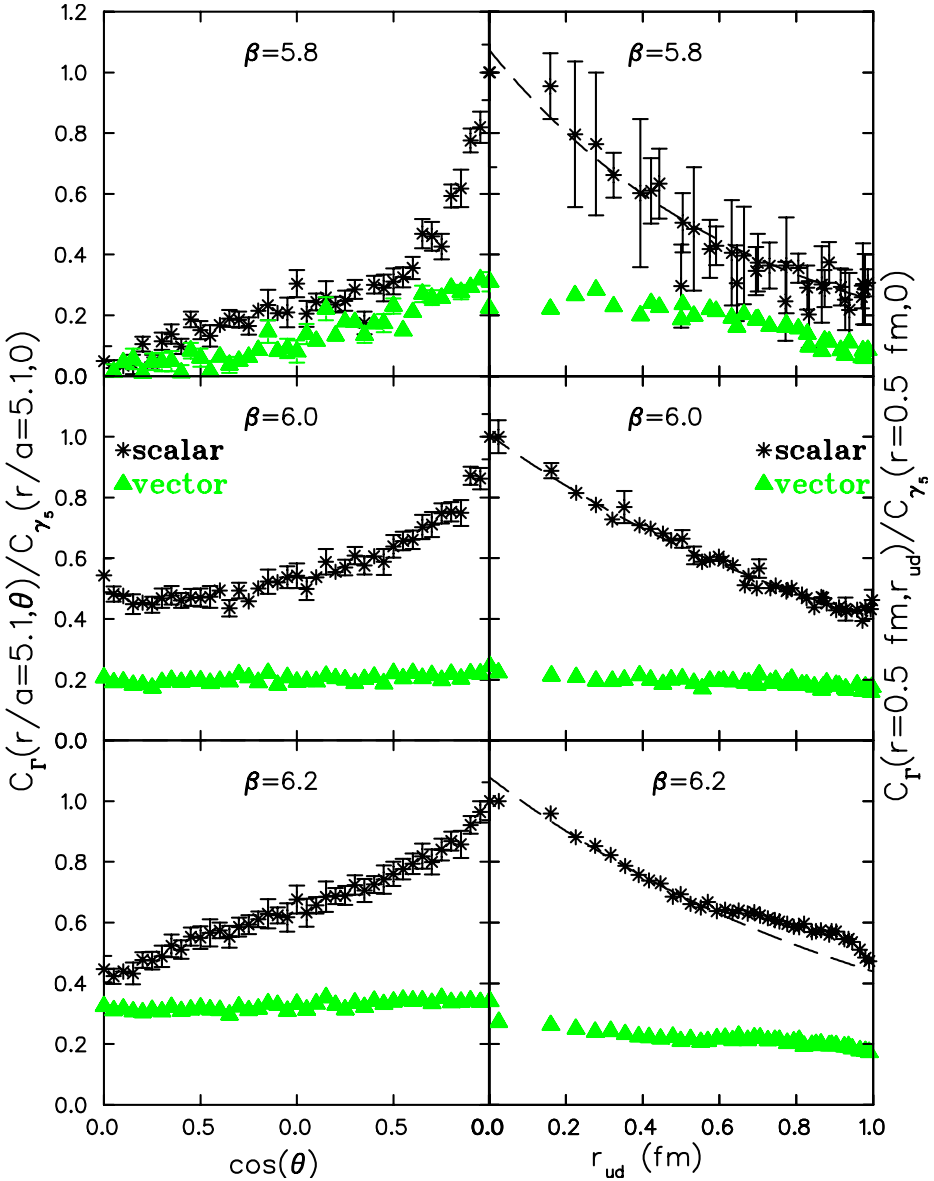
Previous studies

- C. Alexandrou, Ph. de Forcrand and B. Lucini, Phys. Rev. Lett. **97**, 222002 (2006) [arXiv:hep-lat/0609004].
- Color antitriplet diquark combined with a static quark to form a color singlet.
- Measured two-quark density correlator:

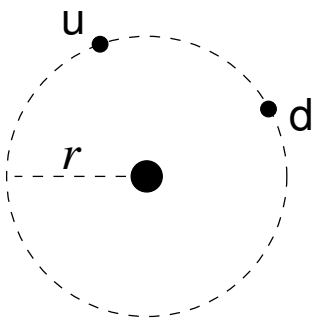
$$C_\Gamma(\mathbf{r}_u, \mathbf{r}_d, t) = \langle 0 | B_\Gamma(\mathbf{0}, 2t) J_0^u(\mathbf{r}_u, t) J_0^d(\mathbf{r}_d, t) B_\Gamma^\dagger(\mathbf{0}, 0) | 0 \rangle$$

where $J_0^f = \overline{f}\gamma_0 f$ and $B_\Gamma = \epsilon^{abc}\left[u_a^T C\Gamma d_b \pm d_a^T C\Gamma u_b\right] s_c$.

Spatial correlations



To isolate the intrinsic diquark correlations, the authors looked at spherical shells $|\mathbf{r}_u| = |\mathbf{r}_d| = r$.



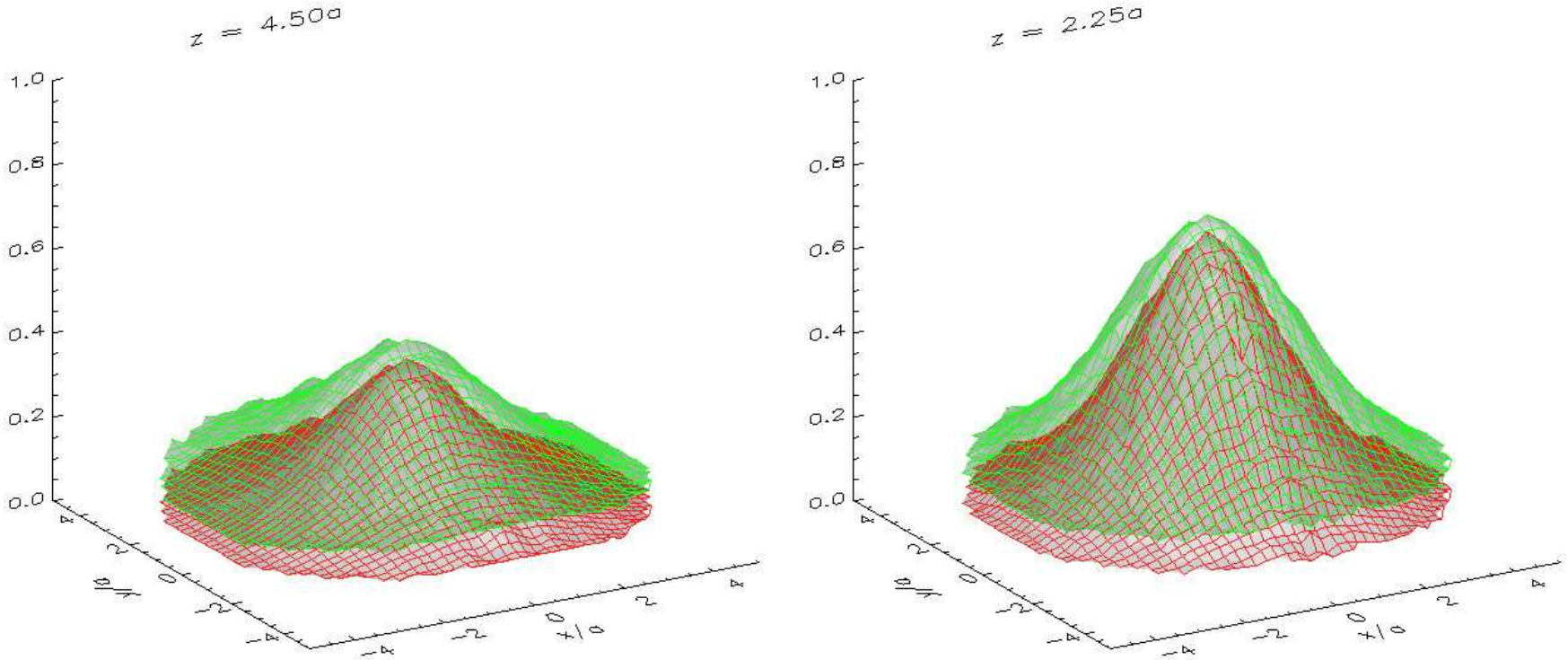
A different approach

- R. Babich, N. Garron, C. Hoelbling, J. Howard, L. Lellouch and C. Rebbi, Phys. Rev. D **76**, 074021 (2007) [arXiv:hep-lat/0701023].
- Using gauge-fixed lattices and finite mass strange quarks (degenerate with u and d), the zero-momentum correlator

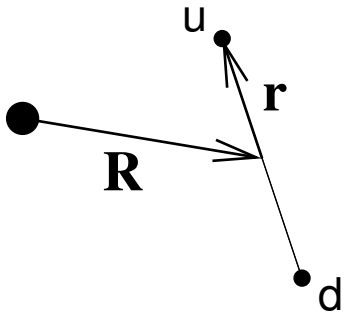
$$G(\vec{r}_u, \vec{r}_d, t) = \sum_{\vec{r}} \langle u(\vec{r} + \vec{r}_u, t) d(\vec{r} + \vec{r}_d, t) s(\vec{r}, t) \bar{u}(\vec{0}, 0) \bar{d}(\vec{0}, 0) \bar{s}(\vec{0}, 0) \rangle$$

  was computed for the Λ, Σ, and Σ^* baryons.
- This was used to define a wave function $\Psi(\vec{r}_u, \vec{r}_d) = \frac{G(\vec{r}_u, \vec{r}_d, t)}{\sum_{\vec{r}_u, \vec{r}_d} |G(\vec{r}_u, \vec{r}_d, t)|^2}$.

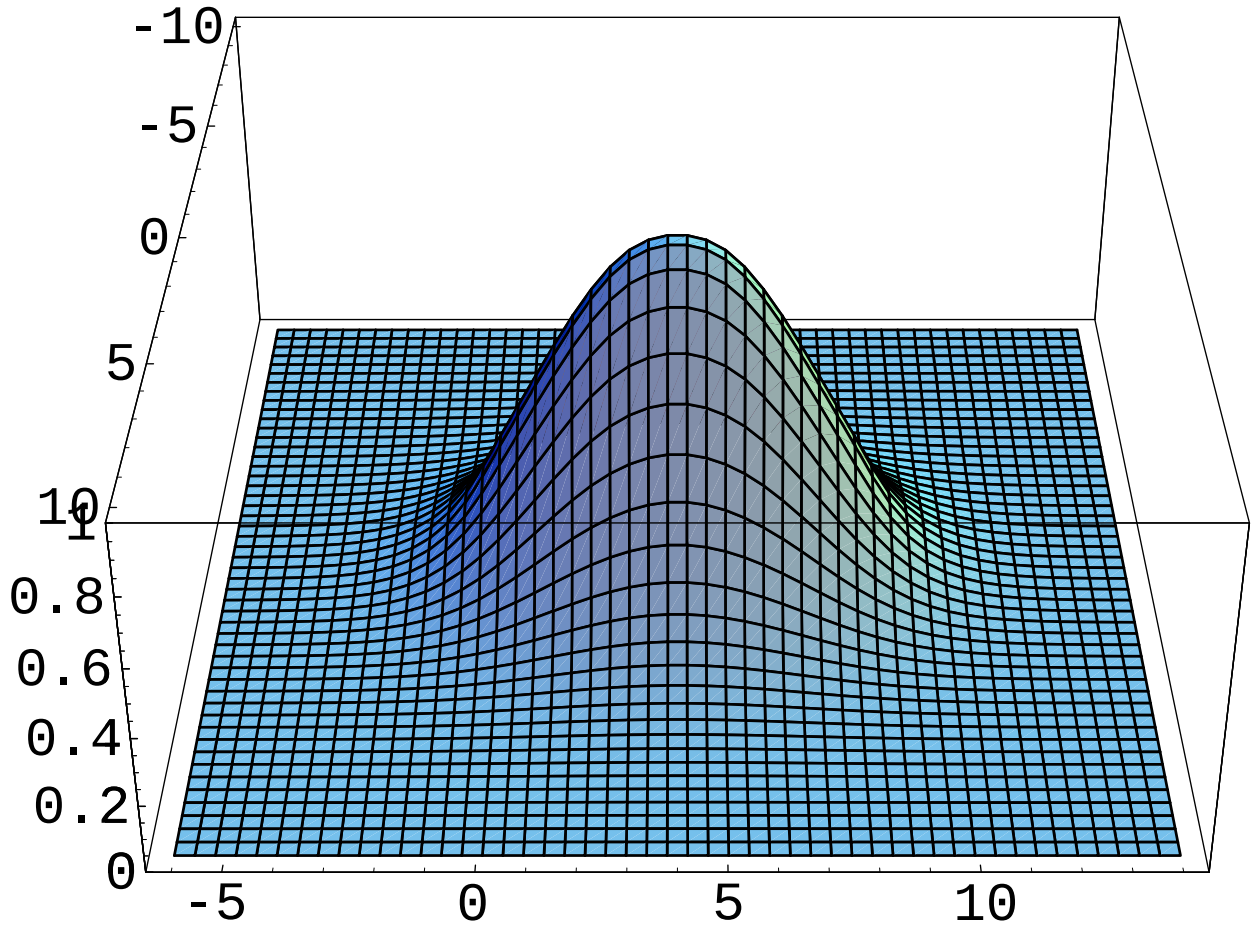
Diquark wavefunction



Λ (red) and Σ^* (green) in the Coulomb gauge, at $R/a = 4.5$ (left) and $R/a = 2.25$ (right).



Correlation function



- Uncorrelated $\rho_2(\mathbf{r}_1, \mathbf{r}_2) = \rho_1(\mathbf{r}_1)\rho_1(\mathbf{r}_2)$ plotted this way can give the appearance of a diquark.
- Want to show only the clustering induced by the diquark interaction.

Correlation function

$$C(\mathbf{r}_1, \mathbf{r}_2) = \frac{\rho_2(\mathbf{r}_1, \mathbf{r}_2) - \rho_1(\mathbf{r}_1)\rho_1(\mathbf{r}_2)}{\rho_1(\mathbf{r}_1)\rho_1(\mathbf{r}_2)}$$

- Is zero if there is no diquark interaction.
- Denominator compensates for presence of static quark at $\mathbf{r} = 0$.

Lattice measurements

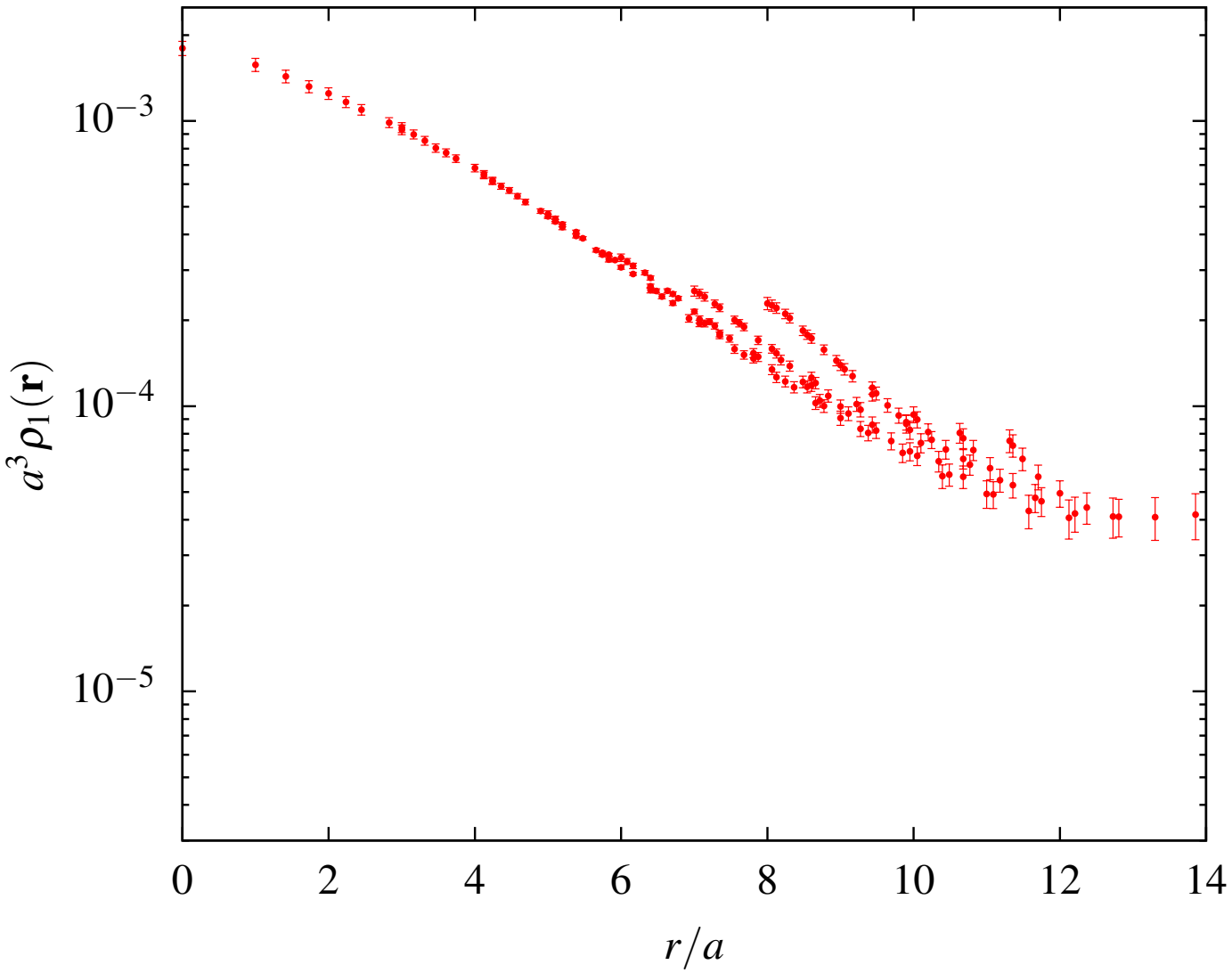
$$\rho_1(\mathbf{r}) \propto \langle 0|B(\mathbf{0}, t_f)J_0^u(\mathbf{r}, t)\overline{B}(\mathbf{0}, t_i)|0\rangle$$

$$\rho_2(\mathbf{r}_1, \mathbf{r}_2) \propto \langle 0|B(\mathbf{0}, t_f)J_0^u(\mathbf{r}_1, t)J_0^d(\mathbf{r}_2, t)\overline{B}(\mathbf{0}, t_i)|0\rangle$$

- Studied both good diquark $B = (u^T C\gamma_5 d)s$ and bad diquark $B = (u^T C\gamma_i d)s$, with degenerate u and d quarks and a static quark s.
- Two lattice ensembles:
  1. $16^3 \times 32$, $a = 0.088$ fm, quenched Wilson quarks with $m_\pi = 940$ MeV.
  2. $20^3 \times 64$, $a = 0.124$ fm, domain wall valence quarks on a staggered sea with $m_\pi = 293$ MeV.

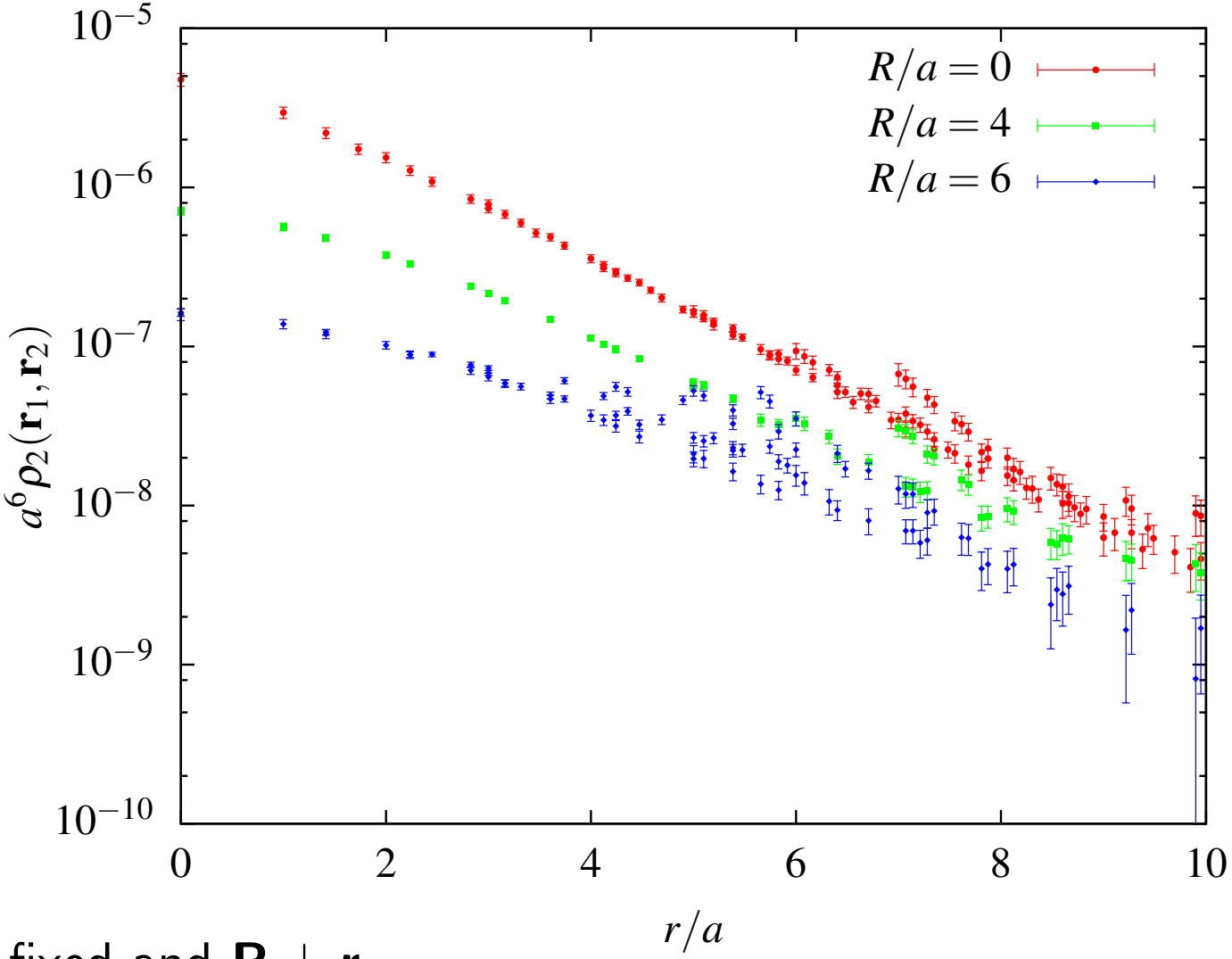
Start by looking at $m_\pi = 940$ MeV good diquark.

Image effects



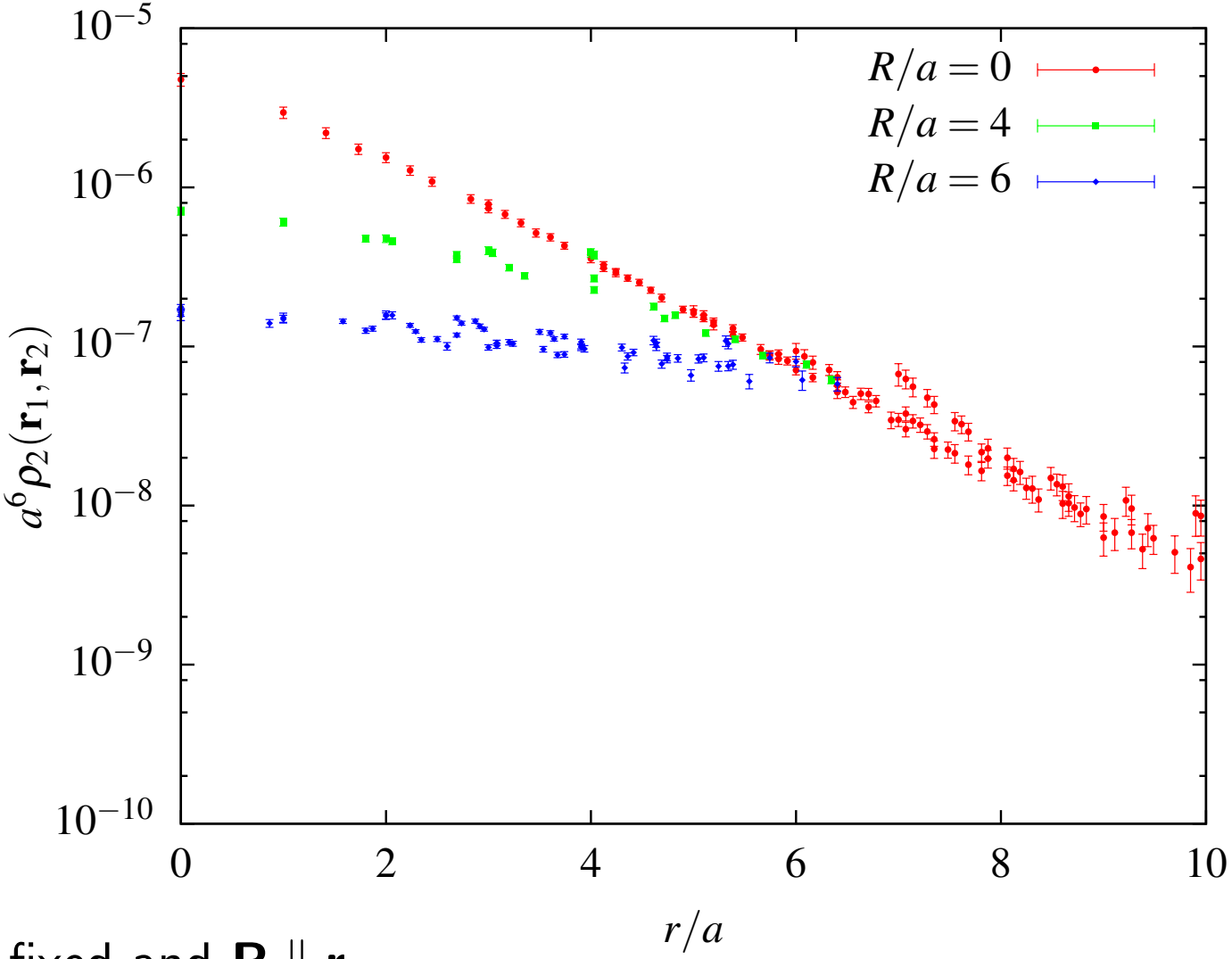
$\rho_1(r)$

Image effects



ρ_2 with R fixed and $\mathbf{R} \perp \mathbf{r}$

Image effects



ρ_2 with R fixed and $\mathbf{R} \parallel \mathbf{r}$

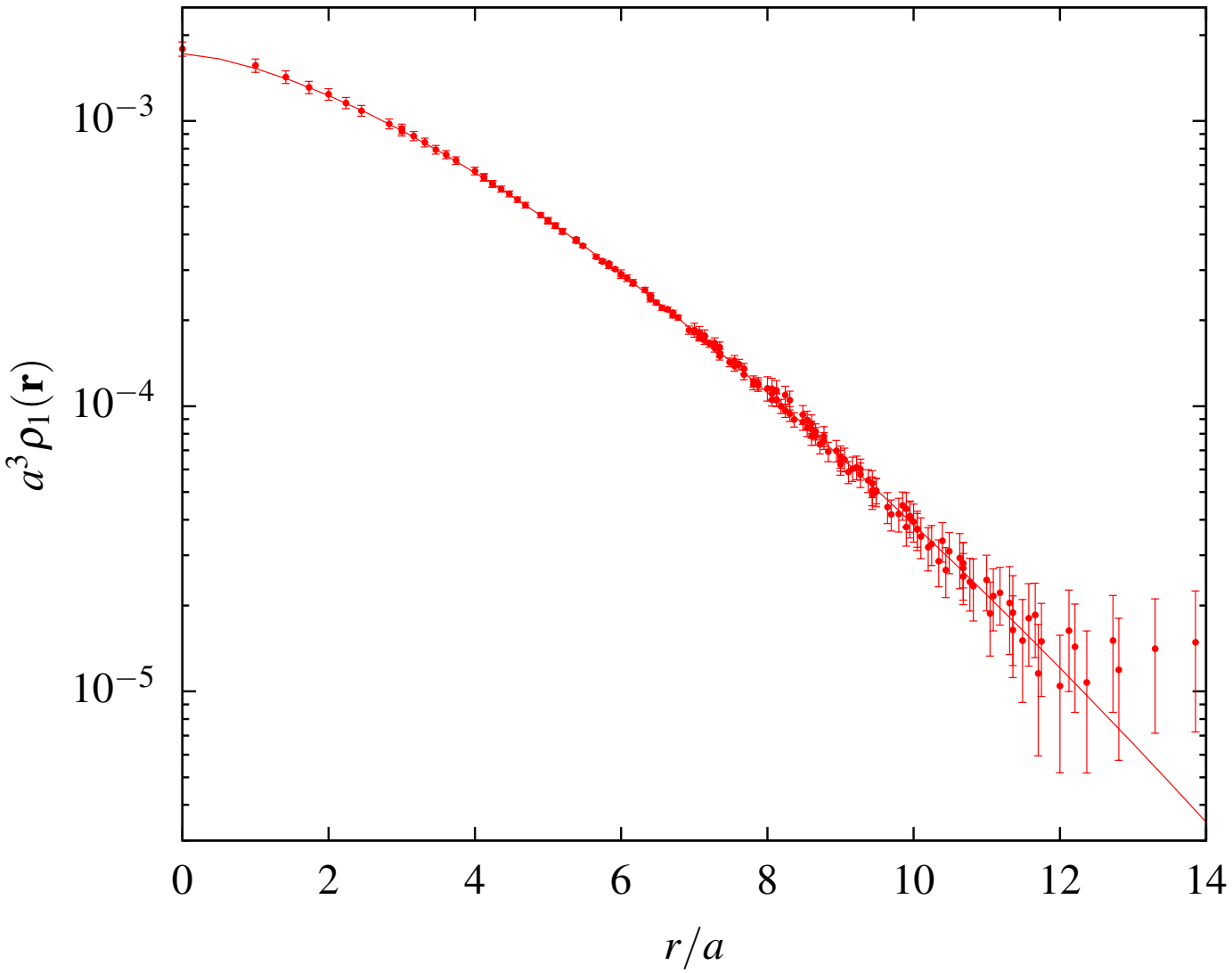
Image correction

- Deal with image effects by fitting a parameterized function to the data.
- Instead of fitting $f(\mathbf{r}_1, \mathbf{r}_2)$, fit

$$f_{\text{img}}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{n_1^i, n_2^i = -1,0,1} f(\mathbf{r}_1 + \mathbf{n}_1 L, \mathbf{r}_2 + \mathbf{n}_2 L)$$

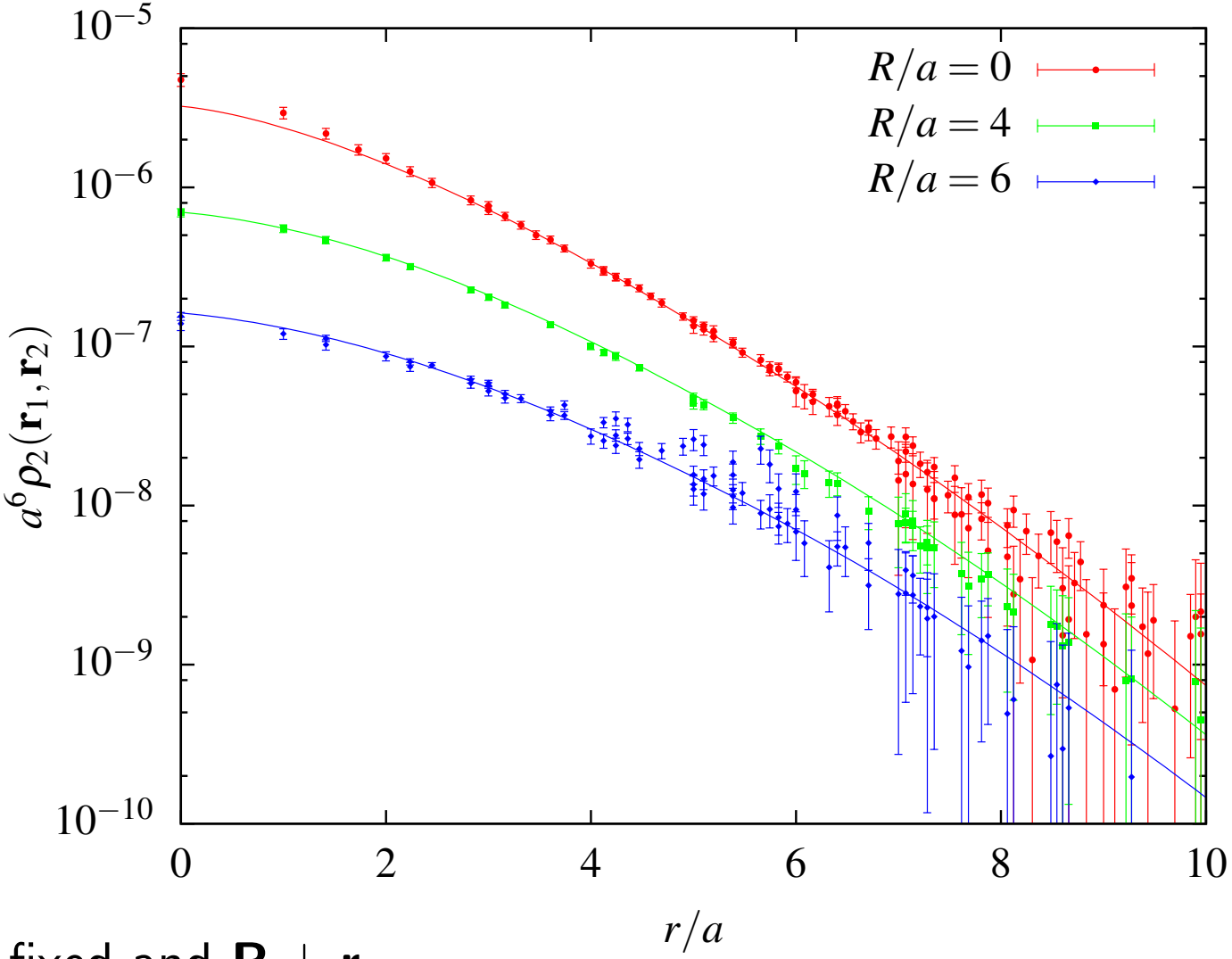
- Given a good fit, the image effects can be subtracted off.
- Data points most affected by images were excluded from the fit shown here.
- 11 parameter fit, $\frac{\chi^2}{\text{dof}} \simeq 0.25$–$1.85$

Image correction



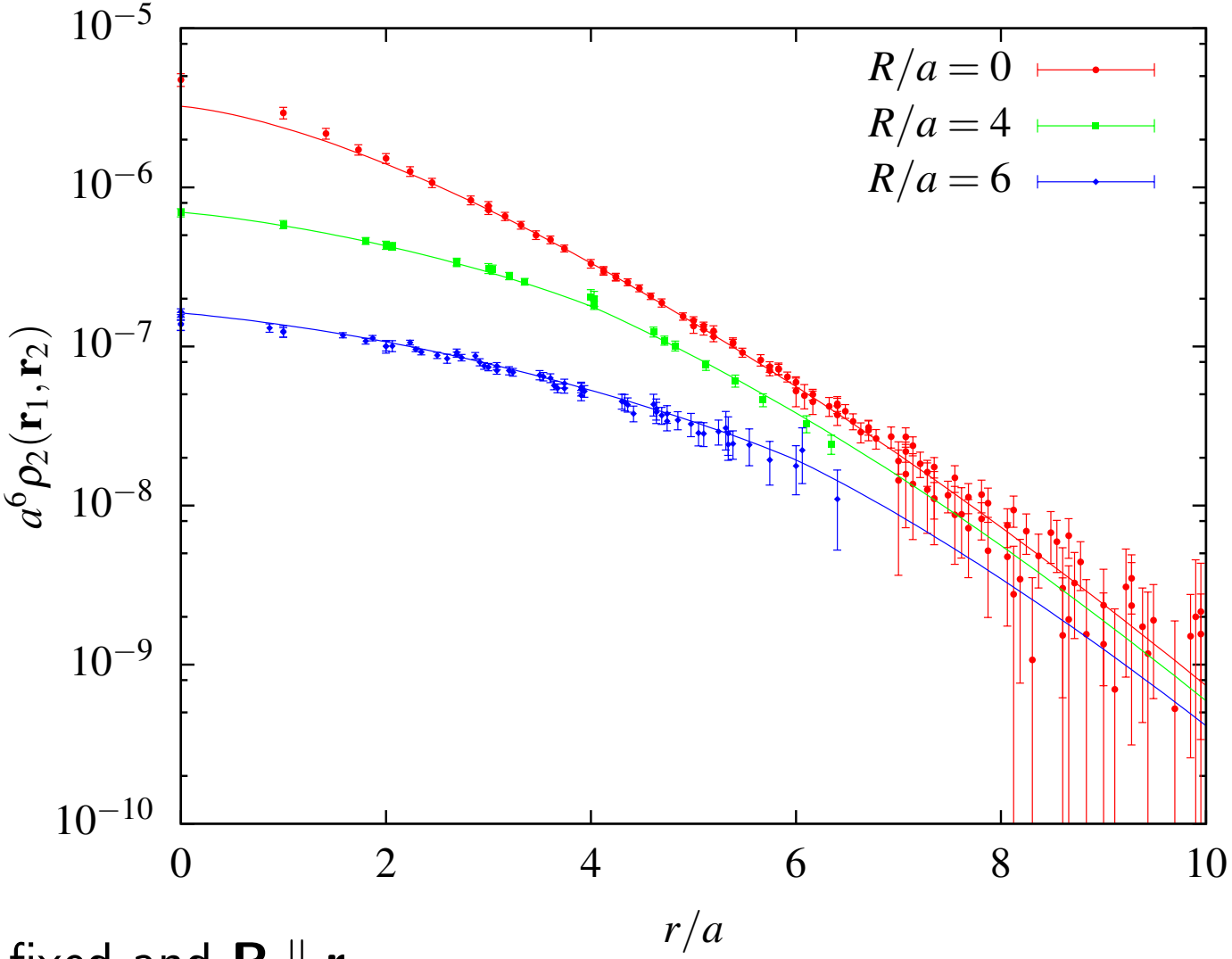
$\rho_1(r)$

Image correction



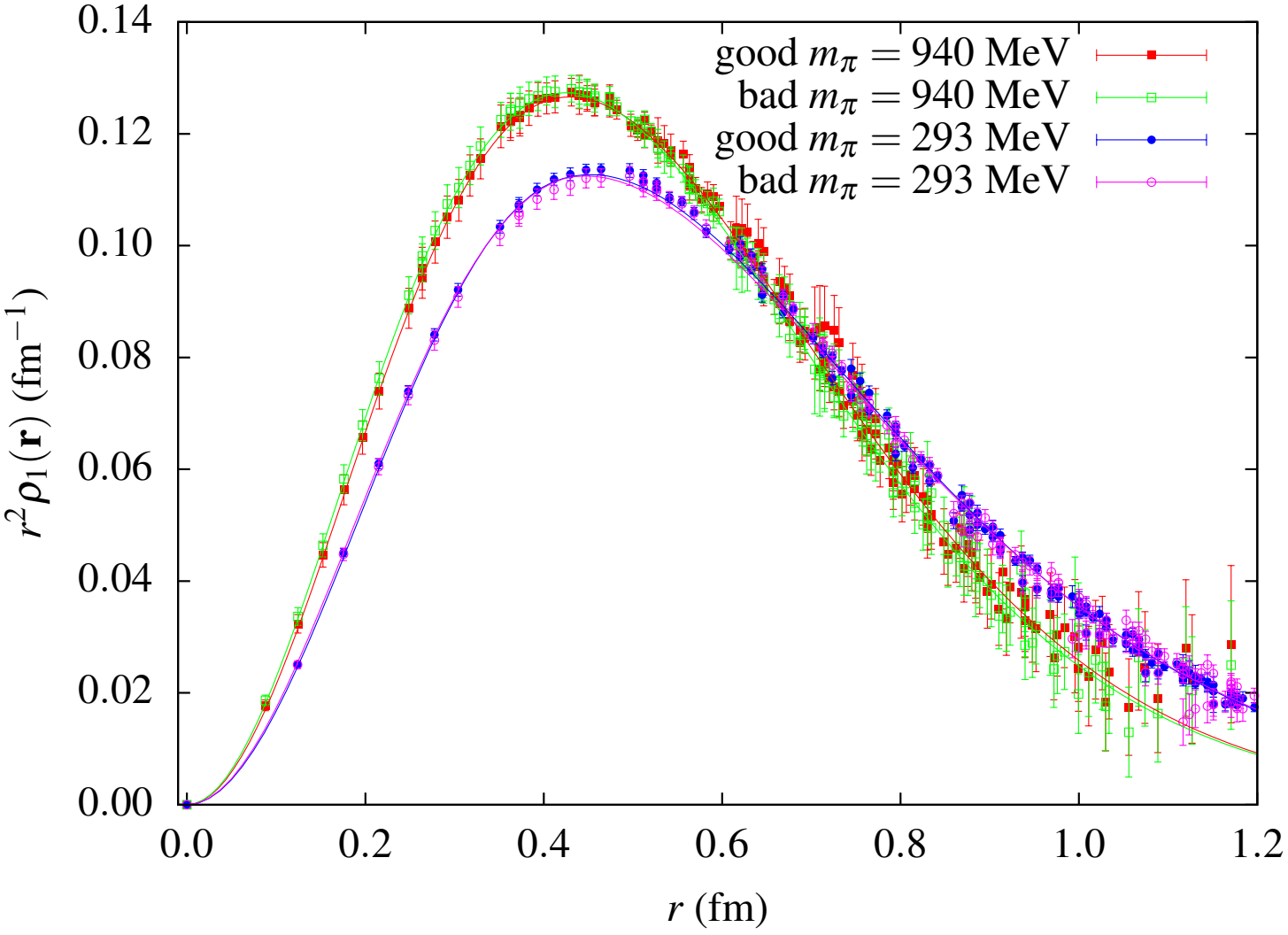
ρ_2 with R fixed and $\mathbf{R} \perp \mathbf{r}$

Image correction



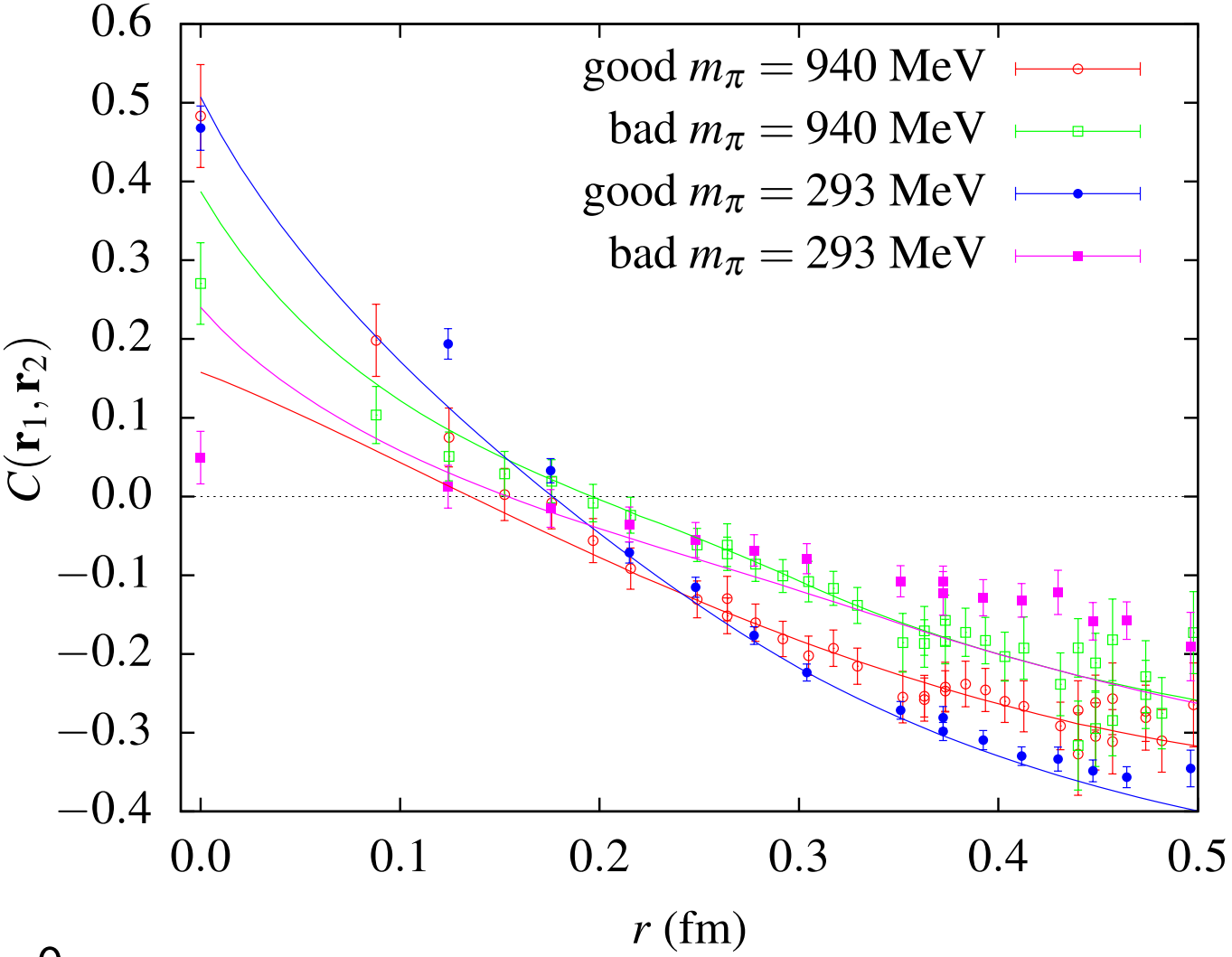
ρ_2 with R fixed and $\mathbf{R} \parallel \mathbf{r}$

Comparison of ρ_1



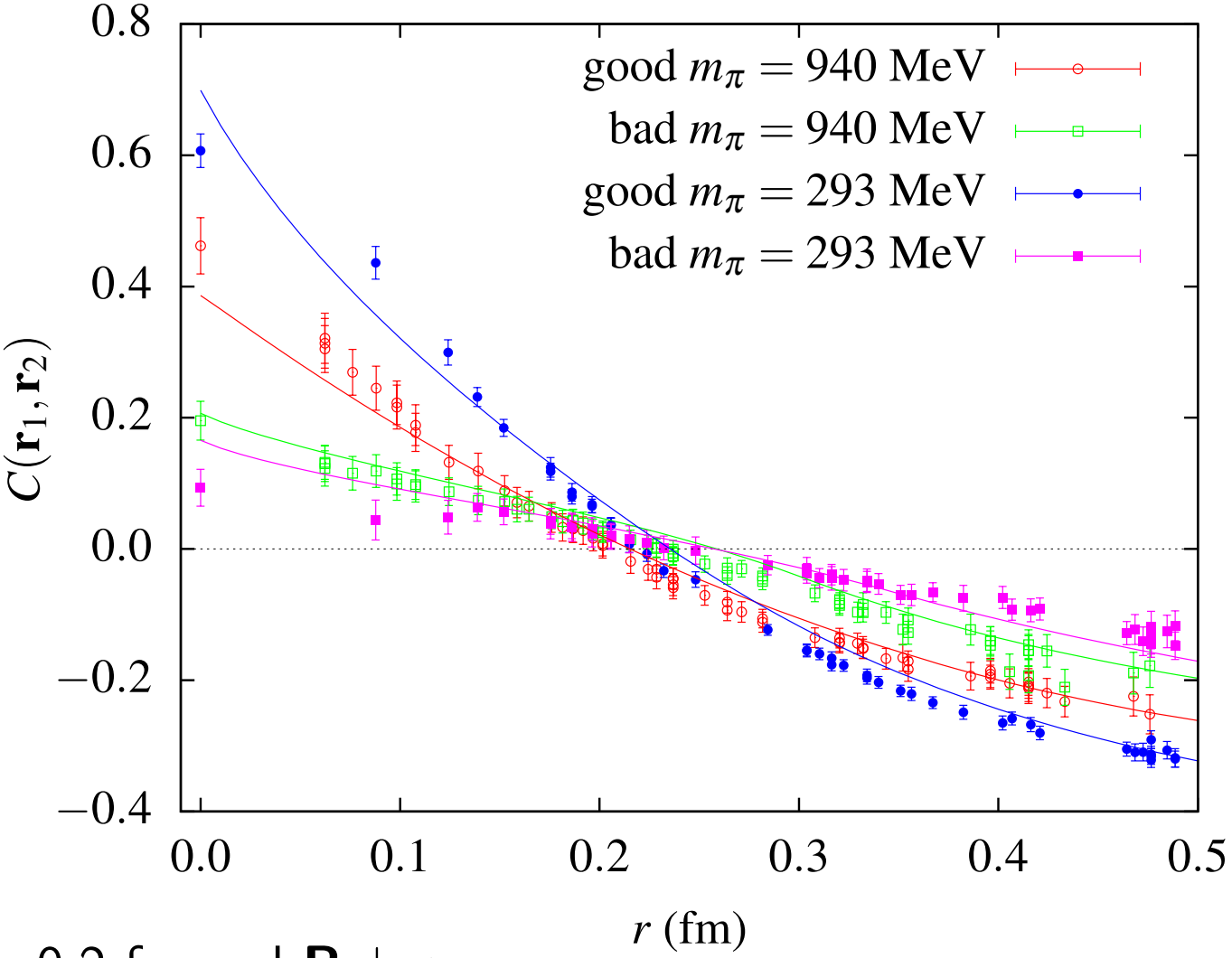
$r^2\rho_1(r)$

Correlation function



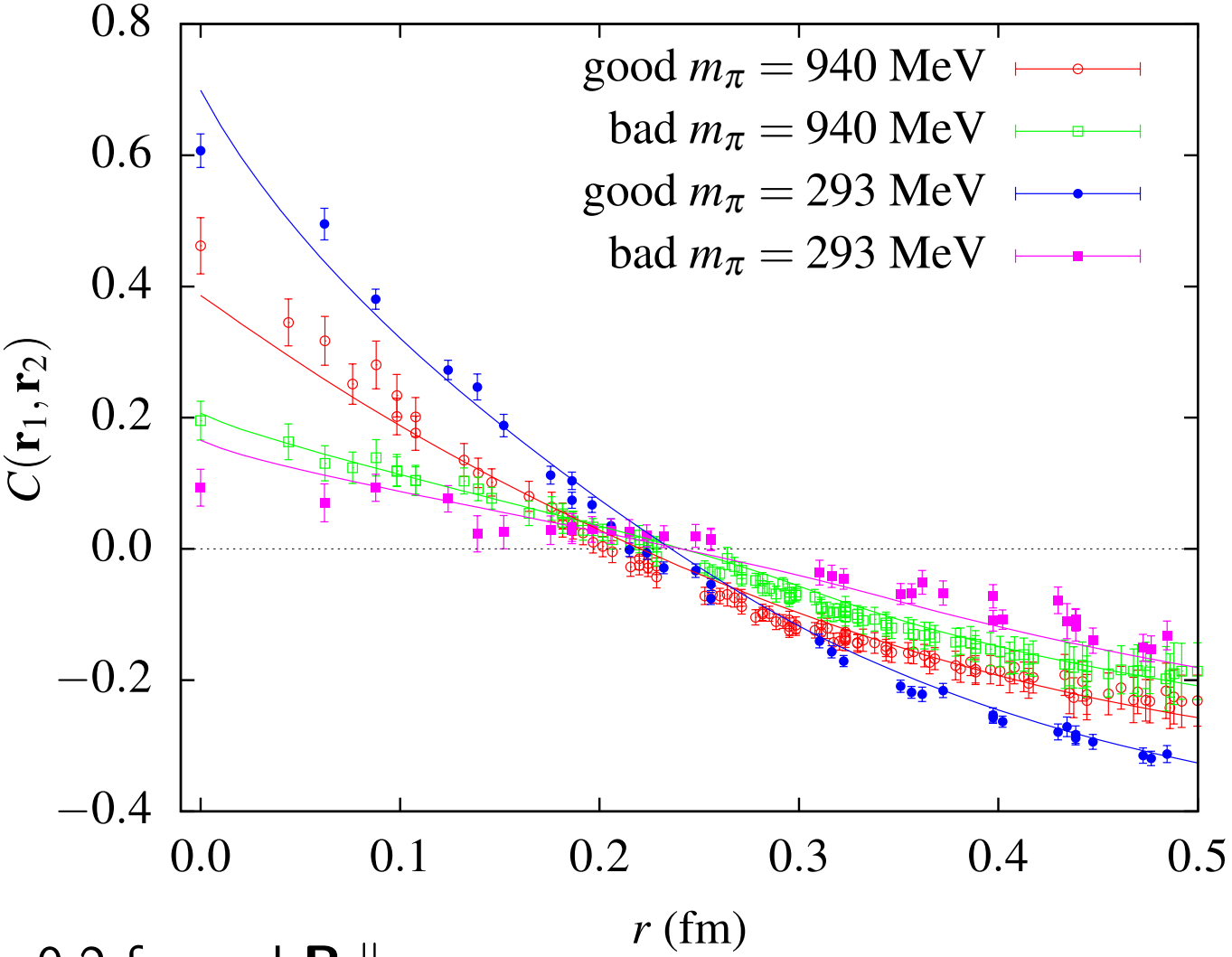
C with $R = 0$

Correlation function



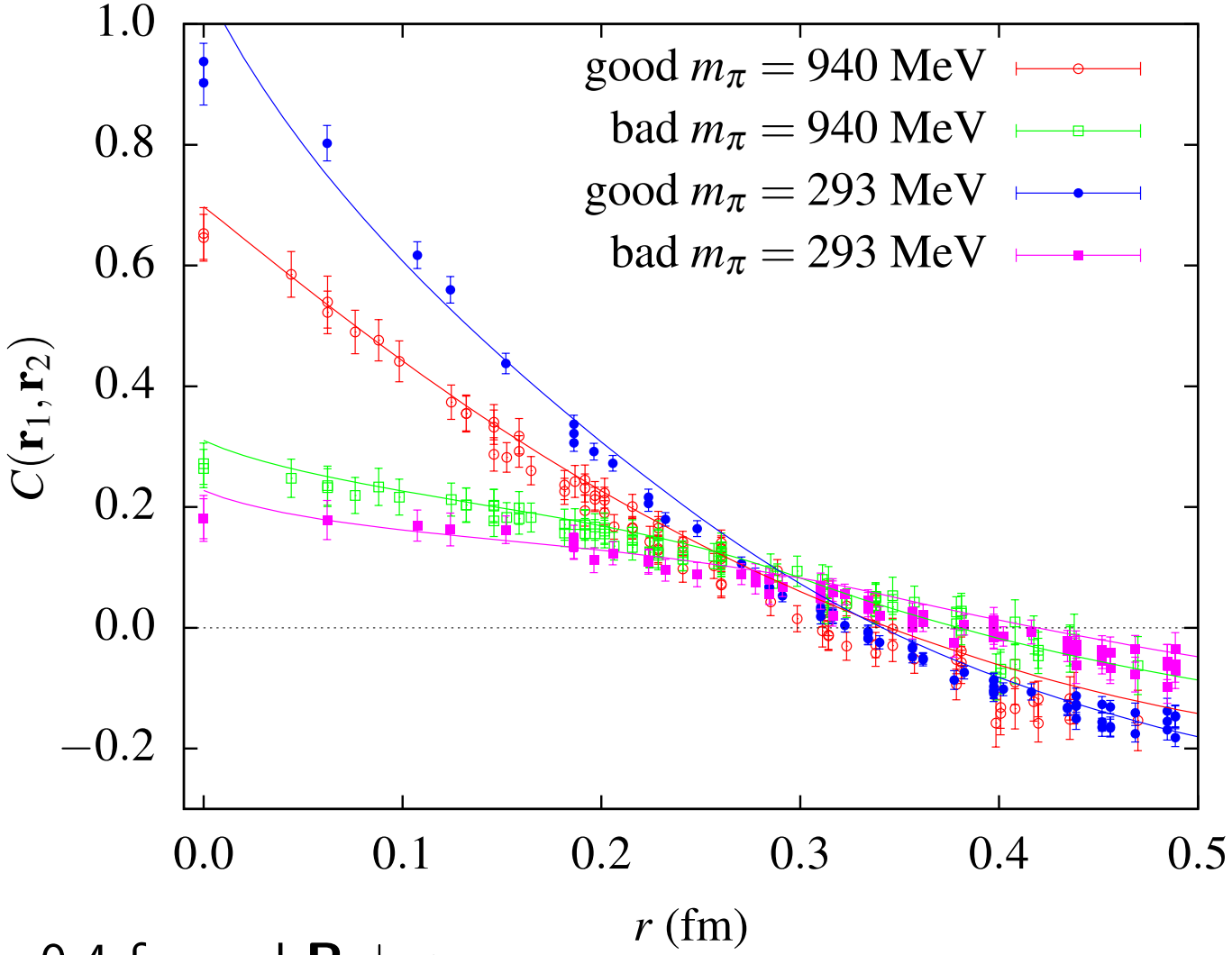
C with $R = 0.2$ fm and $\mathbf{R} \perp \mathbf{r}$

Correlation function



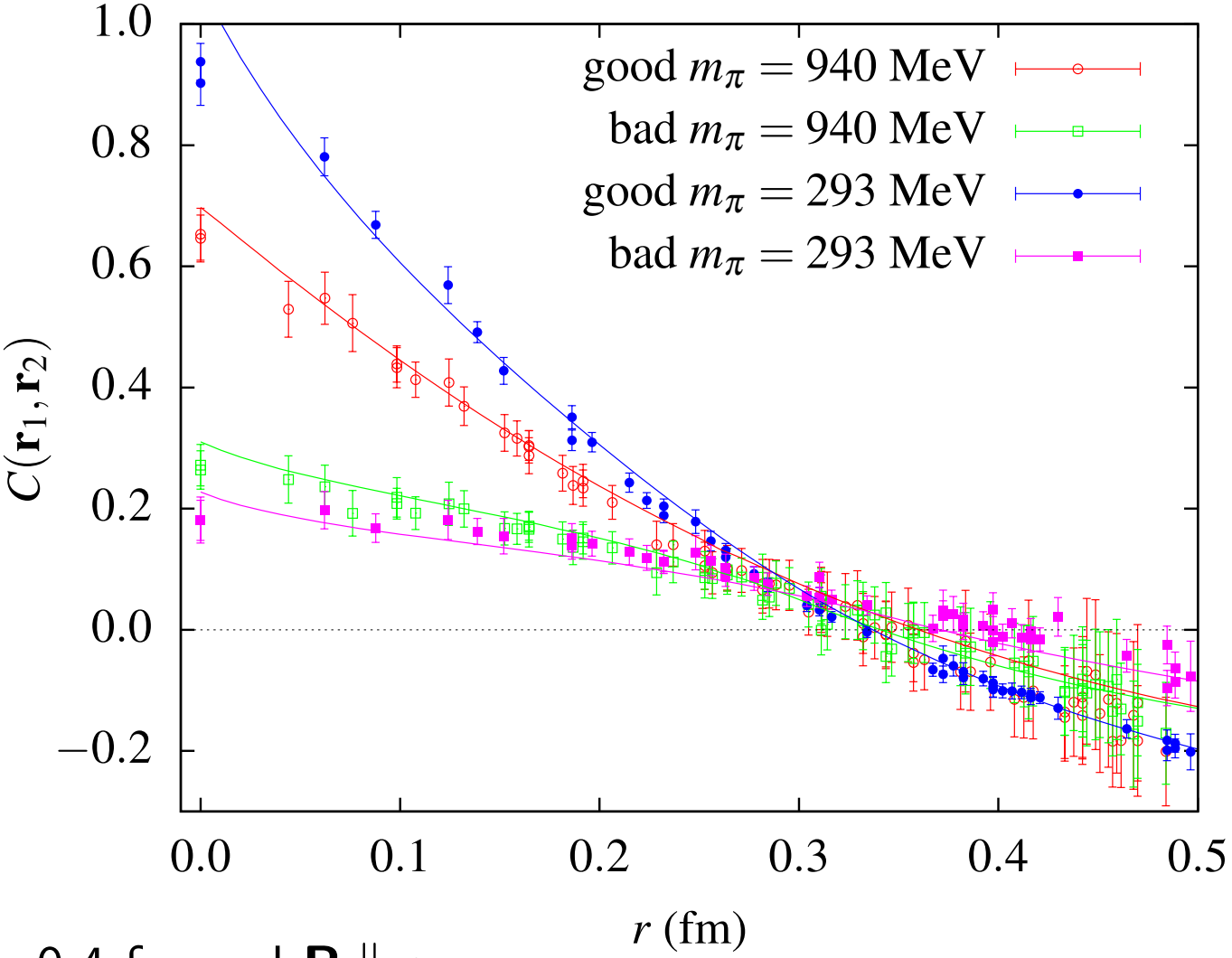
C with $R = 0.2$ fm and $\mathbf{R} \parallel \mathbf{r}$

Correlation function



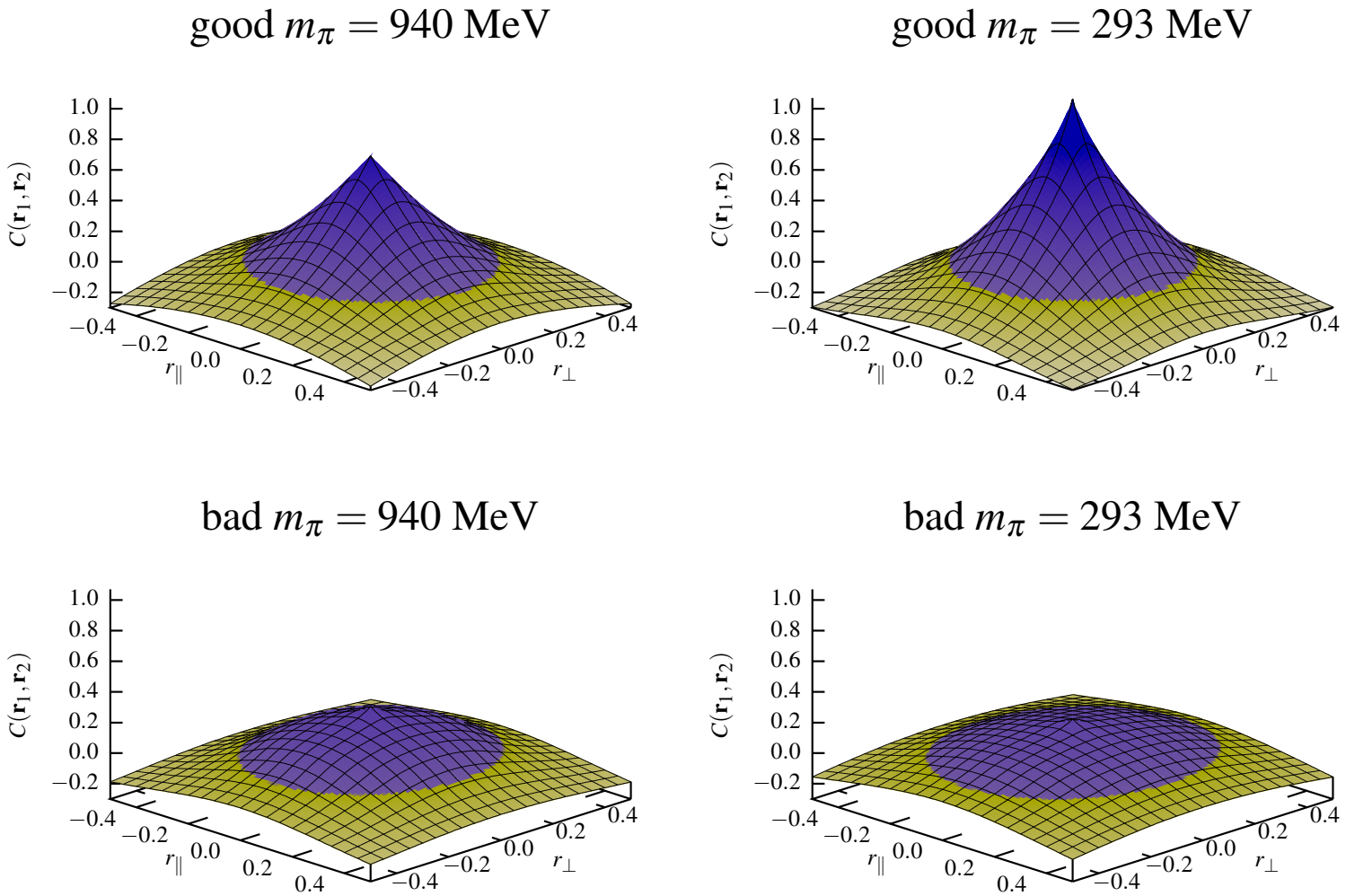
C with $R = 0.4$ fm and $\mathbf{R} \perp \mathbf{r}$

Correlation function



C with $R = 0.4$ fm and $\mathbf{R} \parallel \mathbf{r}$

Correlation function



C from fit versus $\mathbf{r}$ (in fm), with $R = 0.4$ fm.

Summary

- Good and bad diquark compared at $m_\pi = 940$ MeV and $m_\pi = 293$ MeV.
- Good diquark has stronger correlation as expected.
- Difference between good and bad diquark is greater at smaller pion mass.