Event Shape Distributions
and
Precision Results for $\alpha_s(m_Z)$

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arXiv:1006.3080 on Thrust

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work in progress on HJM
Motivation for Precision $\alpha_s$ from Jets

- Sensitivity to $\alpha_s$ in collider cross sections (enhanced by large K factors)
  
  \[ \sigma(gg \rightarrow H) \sim \alpha_s^3(\text{TeV}) \sim \alpha_s^{2.5}(\text{LHC}) \]

- Important to test unification
  (for models predicting GUT threshold corr.)

- Want a Jet based determination that is competitive with other methods (1% level):
  
  eg.
  
  \[ \alpha_s(m_Z) = 0.1184 \pm 0.0006 \quad \text{Lattice QCD/Wilson loops (HPQCD’08)} \]
  
  \[ \alpha_s(m_Z) = 0.1192 \pm 0.0028^{\text{exp}} \pm 0.00009^{\text{thy}} \quad \text{Z-decays/Global EW (Zfitter)} \]
  
  \[ \alpha_s(m_Z) = 0.1141 \pm 0.0021 \quad \text{DIS/ } \mathcal{O}(\alpha_s^3) \quad \text{(BBG’06)} \]
  
  \[ \alpha_s(m_Z) = 0.1180 \pm 0.0008 \quad \text{Tau Decays (FOPT, Beneke & Jamin)} \]
  
  \[ \alpha_s(m_Z) = 0.1212 \pm 0.0011 \quad \text{Tau Decays (CIPT, Davier et al.)} \]
Thrust event shape $e^+ e^- \rightarrow Q$ jets

$$\tau = 1 - \max_i \frac{\sum_i \hat{t} \cdot \vec{p}_i}{\sum_i |\vec{p}_i|}$$

2 jets spherical

$\tau = 0$

$\tau = 1/2$

ALEPH, DELPHI, L3, OPAL, SLD

peak

$Q^2 \gg Q^2 \tau \gg (Q \tau)^2 \sim \Lambda_{QCD}^2$

2 jets, soft radiation

$Q = m_Z$

tail

$Q^2 \gg Q^2 \tau \gg (Q \tau)^2 \gg \Lambda_{QCD}^2$

2 jets, 3 jets

multijet

$Q^2 \sim Q^2 \tau \sim (Q \tau)^2 \gg \Lambda_{QCD}^2$

$> 3$ jets

Thursday, July 28, 2011
Factorization Theorem for Thrust

\[
\frac{d\sigma}{d\tau} = \int dk \left( \frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right) \left( \tau - \frac{k}{Q} \right) S_{\tau}^{\text{mod}}(k - 2\bar{\Delta}) + O\left( \sigma_0 \frac{\alpha_s \Lambda_{\text{QCD}}}{Q} \right)
\]

\[
\frac{d\hat{\sigma}_s}{d\tau} = \sum_n \alpha_s^n \delta(\tau) + \sum_{n,l} \alpha_s^n \left[ \log \frac{\tau}{\tau} \right]_+
\]

\[
= H(\mu_H) \times J(\mu_J) \otimes S(\mu_S)
\]

b-quark mass effects quarks to NNLL

\[
+ O(\alpha_s)
\]

singular partonic cross for massless quarks, QCD+QED final states

\[
\ln \frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s (\alpha_s \ln)^k + \alpha_s^2 (\alpha_s \ln)^k + \ldots
\]

\[
y = FT(\tau) \quad \text{LL} \quad \text{NLL} \quad \text{NNLL} \quad \text{N}^3\text{LL}
\]

resummation for singular partonic

Becher & Schwartz ’08
Factorization Theorem for Thrust

\[
\frac{d\sigma}{d\tau} = \int dk \left( \frac{d\hat{\sigma}}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \Delta d\hat{\sigma}_b \right) \left( \tau - \frac{k}{Q} \right) S_{\tau}^{\text{mod}}(k - 2\bar{\Lambda}) + \mathcal{O}\left( \sigma_0 \frac{\alpha_s \Lambda_{\text{QCD}}}{Q} \right)
\]

nonsingular terms in fixed order expansion

\[= \sigma_0 \sum_n \frac{\alpha_s^n}{(2\pi)^n} f_n(\tau, \mu/Q)\]

\[\mathcal{O}(\alpha_s^2)\]

EVENT2

\[\mathcal{O}(\alpha_s^3)\]

EERAD3

Gehrmann et al. (Weinzierl)

nonsingular terms suppressed except in multijet region
Factorization Theorem for Thrust

\[
\frac{d\sigma}{d\tau} = \int dk \left( \frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \Delta \frac{d\hat{\sigma}_b}{d\tau} \right) \left( \tau - \frac{k}{Q} \right) S_{\tau}^{\text{mod}} (k - 2\Delta) + \mathcal{O} \left( \sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q} \right)
\]

dominant power corrections in peak/tail

Definitions:

- \( \overline{\text{MS}} \): \( \overline{\Omega}_1 = \frac{1}{2N_c} \langle 0 | \text{tr} \ Y_n^T(0) Y_n(0) i\partial \ Y_n(0) \bar{Y}_n(0) | 0 \rangle \)

- Renormalon Free: \( \Omega_1 = \overline{\Omega}_1 - \delta \), \( \delta = \sum_{n=1}^{3} \delta_n \alpha_s^n \), \( \frac{d\hat{\sigma}}{d\tau} \rightarrow e^{-2\delta \frac{\partial}{\partial \tau}} \frac{d\hat{\sigma}}{d\tau} \)

Tail Fits: two parameters \( \alpha_s(m_Z), \Omega_1 \)

Degeneracy broken by doing global fit with \( Q = 35 - 207 \text{ GeV} \)

Power correction needed at 20% accuracy to get \( \alpha_s(m_Z) \) at 1% level

(L3, ALEPH, DELPHI, OPAL, SLD, TASSO, JADE, AMY)
**Global tail fit for** \( \alpha_s(m_Z) \)

\[ \alpha_s(m_Z) \text{ from global thrust fits} \]

\[ \begin{align*}
    \alpha_s(m_Z) & \approx 0.1300 \pm 0.0047 \\
    \frac{\chi^2}{\text{dof}} & = \frac{550}{486} = 1.11
\end{align*} \]

\( \pm \rightarrow \text{perturbative error from scale variation (dominates)} \)

**Pure Fixed Order**
Global tail fit for $\alpha_s(m_Z)$

$\alpha_s(m_Z)$ from global thrust fits

$\alpha_s$ from scale variation

$\pm \rightarrow$ perturbative error from scale variation

Pure Fixed Order

Global tail fit for $\alpha_s(m_Z)$

$0.1300 \pm 0.0047$

$\chi^2_{dof} = \frac{550}{486} = 1.11$

$\mathcal{O}(\alpha_s^3)$

ALEPH $Q = m_Z$ thrust

$0.1274 \pm 0.0042_{\text{pert}}$

Dissertori et al. ‘07

$\mathcal{O}(\alpha_s^2)$

$\mathcal{O}(\alpha_s)$$

(\text{ALEPH all } Q \text{ and all event shapes } 0.1240 \pm 0.0029_{\text{pert}})$

$\frac{1}{\sigma} \frac{d\sigma}{d\tau}$

$Q = m_Z$

$\tau$

Fixed Order

$\mathcal{O}(\alpha_s^3)$

$\mathcal{O}(\alpha_s^2)$

$\mathcal{O}(\alpha_s)$

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Global tail fit for $\alpha_s(m_Z)$

$\alpha_s(m_Z)$ from global thrust fits

$\alpha_s$ from global thrust fits

$\pm \rightarrow$ perturbative error from scan of theory parameters

$O(\alpha_s^3)$

$0.1300 \pm 0.0047$

$\chi^2_{dof} = \frac{603}{486} = 1.24$

$0.1194 \pm 0.0028$

$\mathrm{N}^3\mathrm{LL}$ summation

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Global tail fit for $\alpha_s(m_Z)$

$\alpha_s(m_Z)$ from global thrust fits

$0.1300 \pm 0.0047$

$\pm \rightarrow$ perturbative error from scan of theory parameters

$0.1194 \pm 0.0028$

$\chi^2$/dof = $603/486 = 1.24$

Fit to ALEPH and OPAL
$0.1172 \pm 0.0012_{\text{pert}}$
Becher Schwartz '08

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Global tail fit for \( \alpha_s(m_Z) \)

\( \alpha_s(m_Z) \) from global thrust fits

\[ \frac{\chi^2}{\text{dof}} = \frac{603}{486} = 1.24 \]

Fit to ALEPH and OPAL

0.1172 \( \pm \) 0.0012\text{pert}

Becher Schwartz ’08

\( \pm \rightarrow \) perturbative error from scan of theory parameters

Becher  Schwartz '08

\( \tau \)

\( Q=m_Z \)
Global tail fit for $\alpha_s(m_Z)$

$\alpha_s(m_Z)$ from global thrust fits

$\chi^2$/dof $= \frac{593.6}{486} = 1.22$

$O(\alpha_s^3)$

$0.1300 \pm 0.0047$

$\pm$ → perturbative error from scan of theory parameters

+ multijet boundary

$0.1245 \pm 0.0034$

+ $N^3$LL summation

$0.1194 \pm 0.0028$
Global tail fit for $\alpha_s(m_Z)$

$\alpha_s(m_Z)$ from global thrust fits

$\pm$ → perturbative error from scan of theory parameters

In multijet region, resummation must be switched off

$\chi^2_{dof} = \frac{593.6}{486} = 1.22$

$\alpha_s(m_Z)$ from global thrust fits

$O(\alpha_3^3)$

$0.1300 \pm 0.0047$

$\mu^2 + N^3LL$

$\mu_H$

$\mu_i$

$\mu_J(\tau) \propto \tau^{1/2}$

$\mu_S(\tau) \propto \tau$

w.o. multijet boundaries

$\mu_J = \mu_H \mu_S + \epsilon_J$ terms

$\frac{1}{\sigma} \frac{d\sigma}{d\tau}$

$N^3LL'$ results

$Q=m_Z$

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Global tail fit for $\alpha_s(m_Z)$ and $\Omega_1$

$\alpha_s(m_Z)$ from global thrust fits

- $O(\alpha_s^3)$: $0.1300 \pm 0.0047$
- $N^3LL$ summation: $0.1194 \pm 0.0028$
- Multijet boundary: $0.1245 \pm 0.0034$
- Power Corrections: $0.1152 \pm 0.0021$

$\chi^2$ \(\text{dof} = \frac{490}{485} = 1.00\)

± → perturbative error from scan of theory parameters

Power corrections give a $-7.5\%$ shift
Global tail fit for $\alpha_s(m_Z)$ and $\Omega_1$

$\alpha_s(m_Z)$ from global thrust fits

+$\Lambda^3$LL summation

$0.1194 \pm 0.0028$

$O(\alpha_s^3)$

$0.1300 \pm 0.0047$

$\pm \to$ perturbative error from scan of theory parameters

Power corrections give a $-7.5\%$ shift

$\chi^2_{dof} = \frac{490}{485} = 1.00$

$\frac{1}{\sigma} \frac{d\sigma}{d\tau}$

Sum Logs, no power corrections
Global tail fit for $\alpha_s(m_Z)$ and $\Omega_1$

$\alpha_s(m_Z)$ from global thrust fits

powers corrections give a $-7.5\%$ shift

agree with naive estimate using data:

$$\frac{\delta \alpha_s}{\alpha_s} = \frac{2 \Lambda}{Q} \left[ \frac{h'(\tau)}{h(\tau)} \right]^{\text{exp}} = -(9 \pm 3)\%$$
Global tail fit for $\alpha_s(m_Z)$ and $\Omega_1$

$\alpha_s(m_Z)$ from global thrust fits

$O(\alpha_s^3)$
0.1300 ± 0.0047

± → perturbative error from scan of theory parameters

use Renormalon free $\Omega_1$

$1152 \pm 0.0021$ + R–scheme
$0.1140 \pm 0.0009$

$\chi^2_{dof} = \frac{440}{485} = 0.91$
Global tail fit for $\alpha_s(m_Z)$ and $\Omega_1$

$\alpha_s(m_Z)$ from global thrust fits

$O(\alpha_s^3)\quad 0.1300 \pm 0.0047$

$\pm \rightarrow$ perturbative error from scan of theory parameters

use Renormalon free $\Omega_1$

Power Corrections

$1.152 \pm 0.0021$ + R-scheme

$0.1140 \pm 0.0009$

$\chi^2 / \text{dof} = \frac{440}{485} = 0.91$
Global tail fit for $\alpha_s(m_Z)$ and $\Omega_1$

$\alpha_s(m_Z)$ from global thrust fits

- $O(\alpha_s^3)$: $0.1300 \pm 0.0047$
- $N^3$LL summation: $0.1194 \pm 0.0028$
- Power Corrections: $0.1152 \pm 0.0021$
- $R$-scheme: $0.1140 \pm 0.0009$
- Multijet boundary: $0.1245 \pm 0.0034$
- Perturbative error from scan of theory parameters:

$\chi^2 / \text{dof} = \frac{440}{485} = 0.91$

$\Delta \rightarrow$ b-mass & QED

$0.1135 \pm 0.0009$
Convergence of results

$\Omega_1$ determined to 16% accuracy

Largest contribution to perturbative uncertainty comes from variation of profile parameters.
$\Omega_2$ effects increase

$\frac{\Omega_1}{50.2 \text{GeV}} = 0.1200 - \alpha_s(m_Z)$

statistical errors decrease
Global tail fit for $\alpha_s(m_Z)$ and $\Omega_1$

$\alpha_s(m_Z)$ from global thrust fits

$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}}$ GeV

$\Omega_1 = 0.323 \pm 0.009_{\text{exp}} \pm 0.013 \Omega_2 \pm 0.020 \alpha_s(m_Z) \pm 0.045_{\text{pert}}$ GeV
Global tail fit for $\alpha_s(m_Z)$ and $\Omega_1$

$\alpha_s(m_Z)$ from global thrust fits

$$Q = m_Z$$

Fit at N$^3$LL for $\alpha_s(m_Z)$ & $\Omega_1$

$\chi^2_{\text{dof}} = \frac{440}{485} = 0.91$

$\Omega_1 = 0.1135 \pm 0.0009$

Thursday, July 28, 2011
Analysis of Heavy Jet Mass

goal is same precision (N³LL with power corrections)

N³LL results  Chien & Schwartz (arXiv:1005.1644)

global fit  AHMS & Schwartz work in progress
Heavy Jet Mass Event Shape

\[ \rho = \max \left\{ \frac{M_1^2}{Q^2}, \frac{M_2^2}{Q^2} \right\} \]

hemisphere invariant masses

\[ M_i^2 = \left( \sum_{a \in i} p_{\mu a} \right)^2 \]
Comparison with Heavy Jet Mass

\[ \alpha_s(m_Z) \text{ from global thrust fits} \]

- \( O(\alpha_s^3) \): \( 0.1300 \pm 0.0047 \)
- \(+ N^3LL \) summation: \( 0.1194 \pm 0.0028 \)
- \(+ \text{multijet boundary} \): \( 0.1245 \pm 0.0034 \)

\[ \alpha_s(m_Z) \]

**Fit only to ALEPH data**

- **Thrust**: 0.1169, 0.1223
- **Heavy Jet Mass**: 0.1177, 0.1221

**ALEPH data**

- \( \alpha_s^\tau(m_Z) = 0.1175 \pm 0.0026 \)  
  Becher&Schwartz '08
- \( \alpha_s^\rho(m_Z) = 0.1220 \pm 0.0031 \)  
  Chien&Schwartz '10

**Central values agree**

??
There are two ways to calculate the theoretical value for a bin

**Difference of Cumulants**

\[
\Sigma(\tau_2, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1))
\]

\[
\Sigma(\tau, \mu_i(\tau)) = \int_0^\tau dt \frac{d\sigma}{dt}(t, \mu_i(\tau))
\]

Classical Resummation analyses
Becher&Schwartz, Chien&Schwartz

**Integral of Differential Distribution**

\[
\int_{\tau_1}^{\tau_2} dt \frac{d\sigma}{dt}(t, \mu_i(t))
\]

AFHMS, AHMS&Schwartz

- These two agree on all terms at the order in resummation one is working
- They give different estimates for higher order terms (as they should)

- The difference of cumulants always includes a source of uncertainty from the peak region.

For high precision results for the tail region we must use the integral of diff.distn. (or cumulant difference with common scale choice)
\[
\Sigma(\tau_2, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1)) = \\
\int_{\tau_1}^{\tau_2} d\tau' \frac{d\sigma}{d\tau'}(\tau', \mu_i(\tau_2)) + \Sigma(\tau_1, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1)) \\
\approx \int_{\tau_1}^{\tau_2} d\tau' \frac{d\sigma}{d\tau'}(\tau', \mu_i(\tau')) + (\tau_2 - \tau_1) \frac{d\mu_i(\tau_1)}{d\tau_1} \int_0^{\tau_1} d\tau' \frac{d\sigma}{d\tau'}(\tau', \mu_i(\tau_1)) \\
\text{Enhanced uncertainty from the peak!}
\]

- Fixing \( \Omega_1 \) to thrust value (pure QCD) \( \Omega_1 = 0.332 \text{ GeV} \)

\[
\alpha_s^\rho(m_Z) = 0.1158, \quad \frac{\chi^2}{dof} = 1.08 \quad \rho \in [8/Q, 0.2] \\
\alpha_s^\rho(m_Z) = 0.1136, \quad \frac{\chi^2}{dof} = 1.06 \quad \rho \in [12/Q, 0.2] \\
\]

(\( \alpha_s^\tau(m_Z) = 0.1140 \pm 0.0011 \))
Towards a Heavy Jet Mass global fit

- Global data set
- Power corrections in the peak region differ from thrust.
  
  \[ M_1^\tau = \int d\tau \frac{1}{\sigma} \frac{d\sigma}{d\tau} = \frac{2\Omega_1}{Q} + \ldots \]
  
  \[ M_1^\rho = \frac{\Omega_1 + \chi_1}{Q} + \ldots \]
  
  require two dimensional soft function.

- \( O(\alpha_s^2) \) Soft Function has non-global structure

- Power corrections in tail region are universal for massless factorization.
  Treatment of hadron masses may spoil universality of \( \Omega_1 \)

- Theory error analysis
- b-mass, QED effects

- Dataset subsets

* some cross-checking still required
The End
Backup
Convergence of results

\( \alpha_s(m_Z) \) from global thrust fits

- \( \mathcal{O}(\alpha_s) + \text{NLL} \)
- \( \mathcal{O}(\alpha_s) + \text{NNLL} \)
- \( \mathcal{O}(\alpha_s^2) + \text{NNLL} \)
- \( \mathcal{O}(\alpha_s^2) + \text{N}^3\text{LL} \)
- \( \mathcal{O}(\alpha_s^3) + \text{N}^3\text{LL} \)

\( \alpha_s(m_Z) \)

NLL', NNLL, NNLL', N^3LL, N^3LL'

perturbative errors only
## Estimate of perturbative uncertainties

<table>
<thead>
<tr>
<th>parameter</th>
<th>default value</th>
<th>range of values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>2 GeV</td>
<td>1.5 to 2.5 GeV</td>
</tr>
<tr>
<td>$n_1$</td>
<td>5</td>
<td>2 to 8</td>
</tr>
<tr>
<td>$t_2$</td>
<td>0.25</td>
<td>0.20 to 0.30</td>
</tr>
<tr>
<td>$e_J$</td>
<td>0</td>
<td>-1,0,1</td>
</tr>
<tr>
<td>$e_H$</td>
<td>1</td>
<td>0.5 to 2.0</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0</td>
<td>-1,0,1</td>
</tr>
</tbody>
</table>

$s_2$ = -39.1, $-36.6$ to $-41.6$

$\Gamma_3^{cusp}$ = 1553.06, $-1553.06$ to $+4569.18$

$j_3$ = 0, $-3000$ to $+3000$

$s_3$ = 0, $-500$ to $+500$

$\epsilon_2$ = 0, $-1,0,1$

$\epsilon_3$ = 0, $-1,0,1$

Profile functions

$\begin{aligned} h_3 &= 8998.05 \\ Baikov et al \end{aligned}$

Padè approximants for range

Non-singular statistical error
Effect of the various scan parameters
assuming that $h \sim \alpha_s$

\[ \frac{\delta \alpha_s}{\alpha_s} \approx \frac{2 \Lambda}{Q} \frac{h'(\tau)}{h(\tau)} \]

\[ \frac{h'(\tau)}{h(\tau)} \approx -14 \pm 4 \]

assuming $\Lambda \sim 0.3 \text{GeV}$

\[ \frac{\delta \alpha_s}{\alpha_s} \approx -(9 \pm 3) \% \]
Theory uncertainty is from a flat scan

Renormalon-free results have smaller theory errors and better fits

$\Omega_1$ determined to 16% accuracy

500 points random scan per order

Adding individual errors in quadrature gives similar (but smaller) error
Experimental error

\[ \chi^2 \text{ dof} = \frac{440}{485} = 0.91 \]

1σ (39% confidence level)

\[ \alpha_s (m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}} \]

mostly \( \Omega_1 \), and includes \( \Omega_2 \)
\( \alpha_s(m_Z) \) from global thrust fits

\[ \pm \rightarrow \text{perturbative error} \]

- Resummation at N\(^3\)LL
- Multijet boundary condition
- Power correction, in a scheme free of the O(\(\Lambda_{QCD}\)) renormalon
- QED & bottom mass corrections