Event Shape Distributions and Precision Results for $\alpha_s(m_Z)$

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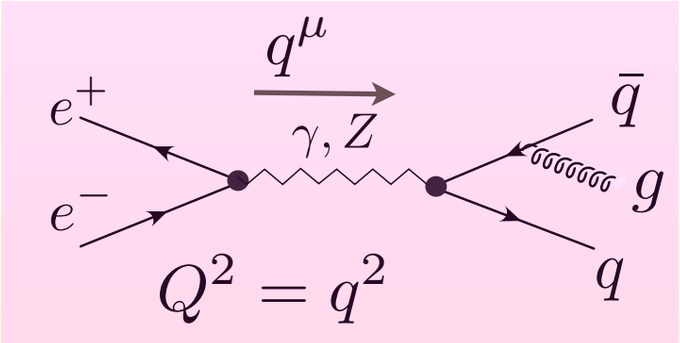
R.Abbate, M. Fickinger, A. Hoang, V. Mateu, IS — arXiv:1006.3080 on Thrust

R.Abbate, A. Hoang, V. Mateu, M. Schwartz, IS — work in progress on HJM

Motivation for Precision α_s from Jets

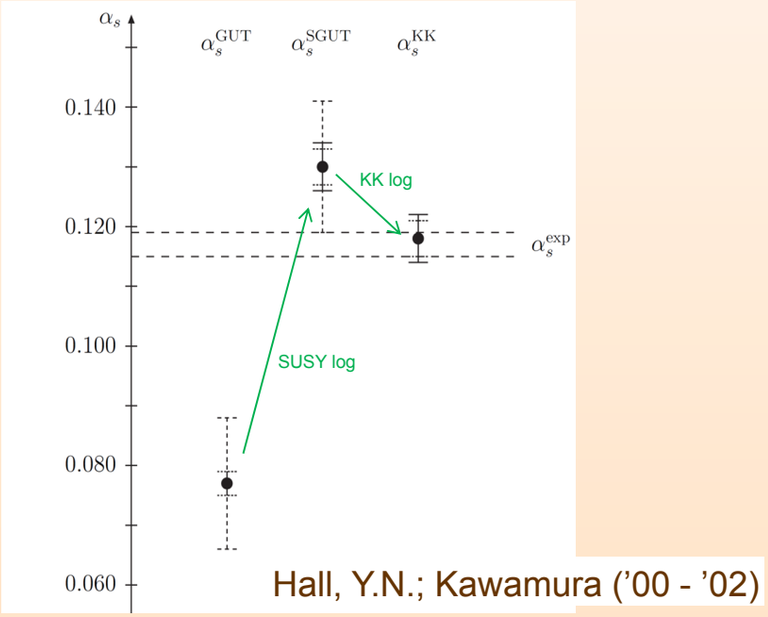


- Sensitivity to α_s in collider cross sections (enhanced by large K factors)

  eg. $\sigma(gg \to H) \sim \alpha_s^3(TeV)$

  $\sim \alpha_s^{2.5}(LHC)$

- Important to test unification (for models predicting GUT threshold corr.)



- Want a Jet based determination that is competitive with other methods (1% level):

eg.

- $\alpha_s(m_Z) = 0.1184 \pm 0.0006$ Lattice QCD/Wilson loops (HPQCD'08)
- $\alpha_s(m_Z) = 0.1192 \pm 0.0028^{\rm expt} \pm 0.00009^{\rm thy}$ Z-decays/Global EW (Zfitter)
- $\alpha_s(m_Z) = 0.1141 \pm 0.0021$ DIS/ $\mathcal{O}(\alpha_s^3)$ (BBG'06)
- $\alpha_s(m_Z) = 0.1180 \pm 0.0008$ Tau Decays (FOPT, Beneke & Jamin)

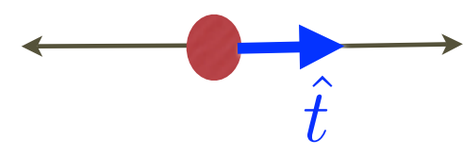
  $\alpha_s(m_Z) = 0.1212 \pm 0.0011$ Tau Decays (CIPT, Davier et al.)

Thrust event shape

$e^+ e^- \xrightarrow{Q}$ jets

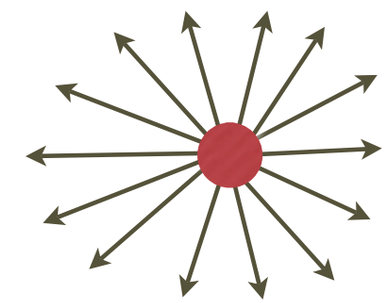
$$\tau = 1 - \max_{\hat{t}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$

2 jets



$\tau = 0$

spherical



$\tau = 1/2$

ALEPH, DELPHI, L3, OPAL, SLD

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$$

20
15
10
5
0

0.0 0.1 0.2 0.3 0.4

τ

peak $\quad Q^2 \gg Q^2\tau \gg (Q\tau)^2 \sim \Lambda^2_{\rm QCD}$

2 jets, soft radiation

$Q = m_Z$

$Q^2 \gg Q^2\tau \gg (Q\tau)^2 \gg \Lambda^2_{\rm QCD}$

tail $\quad$ 2 jets, 3 jets

$Q^2 \sim Q^2\tau \sim (Q\tau)^2 \gg \Lambda^2_{\rm QCD}$

multijet

> 3 jets

Factorization Theorem for Thrust

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}_{\mathrm{s}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\Delta \mathrm{d}\hat{\sigma}_b}{\mathrm{d}\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\mathrm{mod}}(k - 2\bar{\Delta}) + \mathcal{O}\left(\sigma_0 \frac{\alpha_s \Lambda_{\mathrm{QCD}}}{Q} \right)$$

$$\begin{aligned} \frac{d\hat{\sigma}_s}{d\tau} &= \sum_n \alpha_s^n \, \delta(\tau) + \sum_{n,l} \alpha_s^n \left[\frac{\log^l \tau}{\tau} \right]_+ \\ &= H(\mu_H) \times J(\mu_J) \otimes S(\mu_S) \end{aligned}$$

singular partonic cross for massless quarks, QCD+QED final states

b-quark mass effects quarks to NNLL $+\mathcal{O}(\alpha_s)$

$$\ln \frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s (\alpha_s \ln)^k + \alpha_s^2 (\alpha_s \ln)^k + \ldots$$

$y = FT(\tau)$ LL NLL NNLL $\mathrm{N^3LL}$

resummation for singular partonic

Becher & Schwartz '08

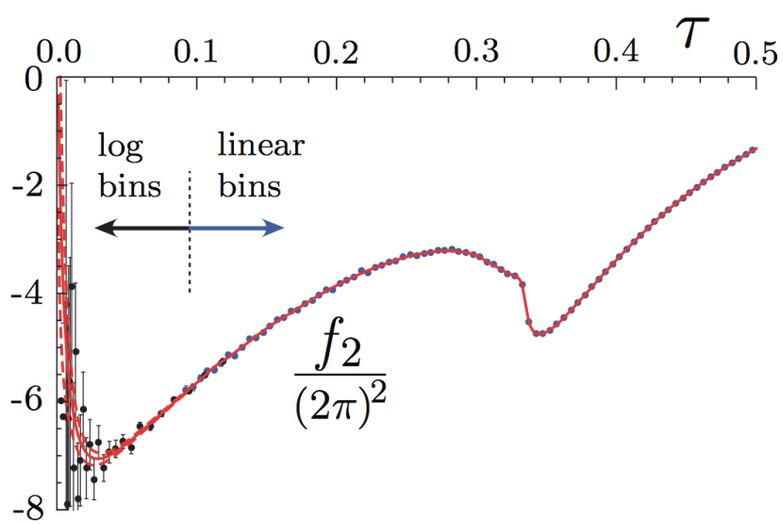
Factorization Theorem for Thrust

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\rm mod}(k - 2\bar{\Delta}) + \mathcal{O}\left(\sigma_0 \frac{\alpha_s \Lambda_{\rm QCD}}{Q} \right)$$

nonsingular terms in fixed order expansion $= \sigma_0 \sum_n \frac{\alpha_s^n}{(2\pi)^n} f_n(\tau, \mu/Q)$

$\mathcal{O}(\alpha_s^2)$

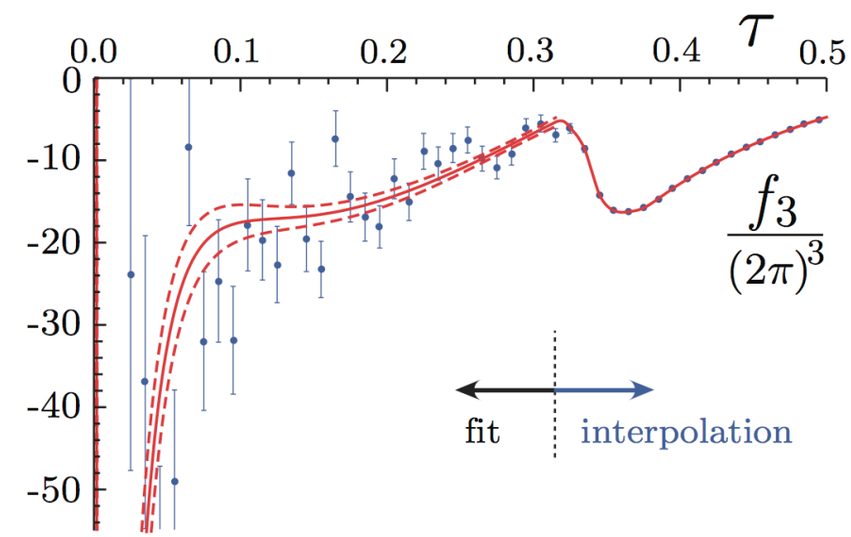
EVENT2



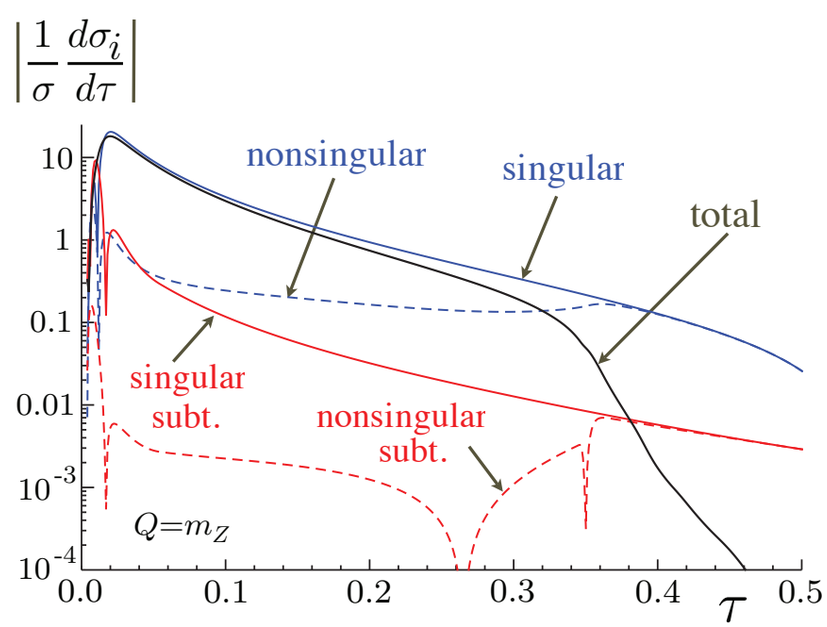
$\mathcal{O}(\alpha_s^3)$

EERAD3

Gehrmann et al. (Weinzierl)



nonsingular terms suppressed except in multijet region



Factorization Theorem for Thrust

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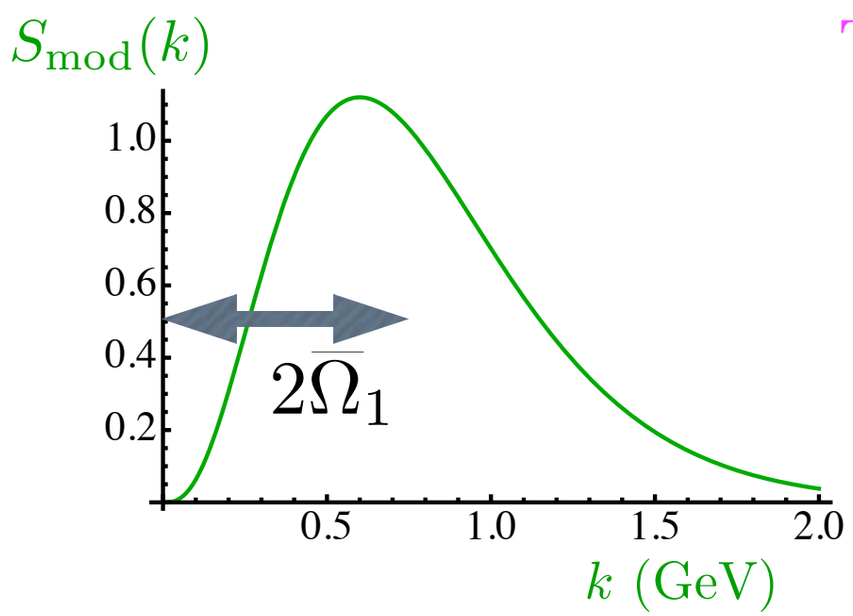
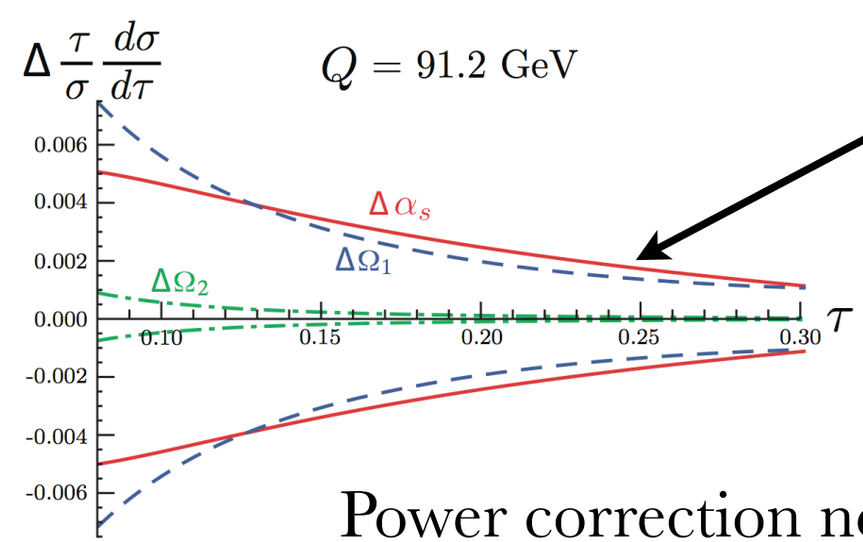
dominant power corrections in peak/tail

$$\frac{\delta\alpha_s}{\alpha_s} \sim 0.5\%$$



Tail Fits: two parameters $\alpha_s(m_Z), \Omega_1$



Degeneracy broken by doing global fit with Q=35 - 207 GeV

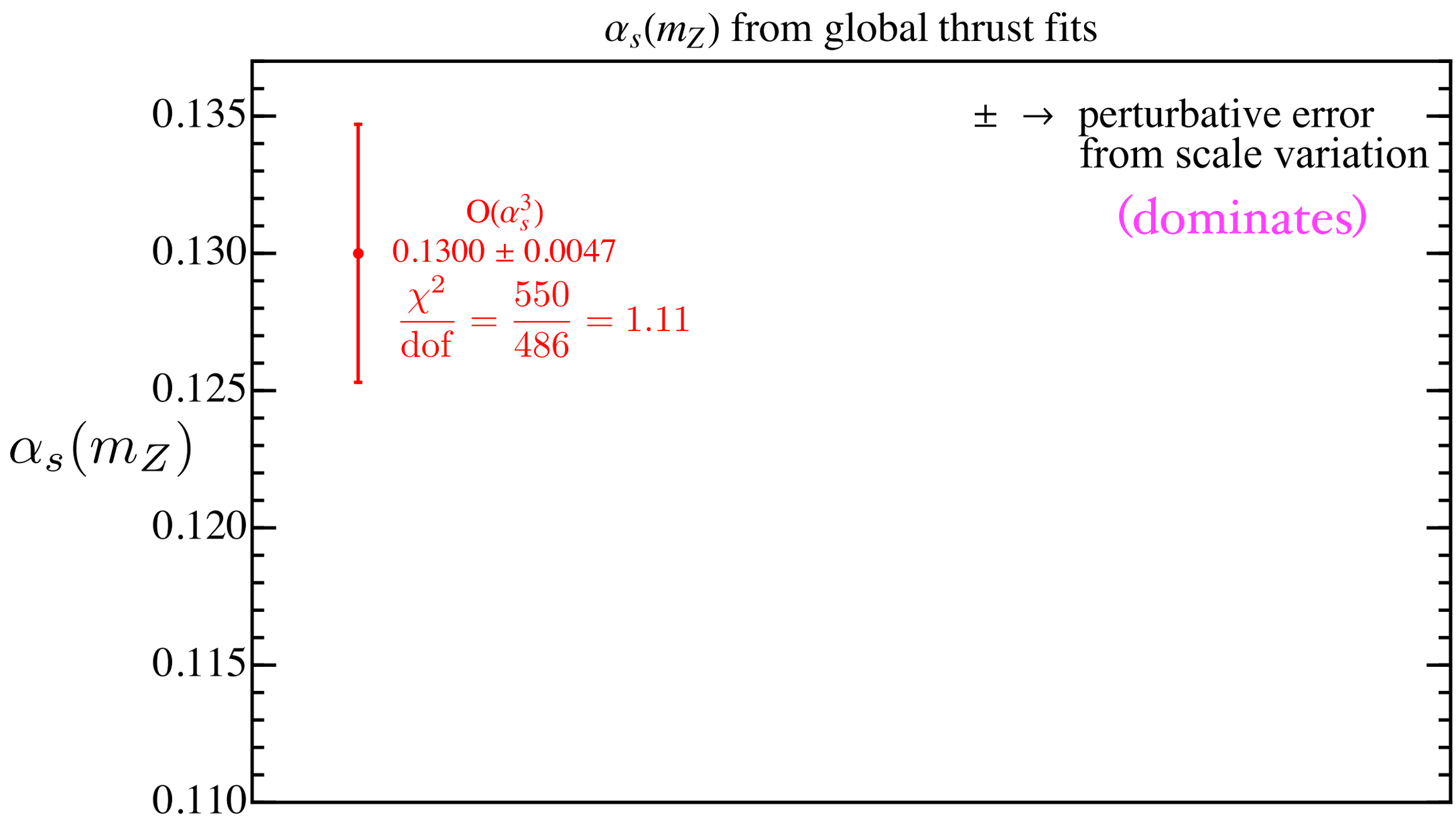
(L3, ALEPH, DELPHI, OPAL, SLD, TASSO, JADE, AMY)

Power correction needed at 20% accuracy to get $\alpha_s(m_Z)$ at 1% level

Definitions:

- $\overline{\rm MS}$: $\bar{\Omega}_1 = \frac{1}{2N_c} \langle 0 | {\rm tr}\, \overline{Y}^T_{\bar{n}}(0) Y_n(0)\, i\hat{\partial}\, Y^\dagger_n(0) \overline{Y}^*_{\bar{n}}(0) | 0 \rangle$
- Renormalon Free: $\Omega_1 = \bar{\Omega}_1 - \delta$, $\delta = \sum_{n=1}^{3} \delta_n\, \alpha_s^n$, $\frac{d\hat{\sigma}}{d\tau} \to e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}} \frac{d\hat{\sigma}}{d\tau}$

Global tail fit for $\alpha_s(m_Z)$



Pure Fixed Order

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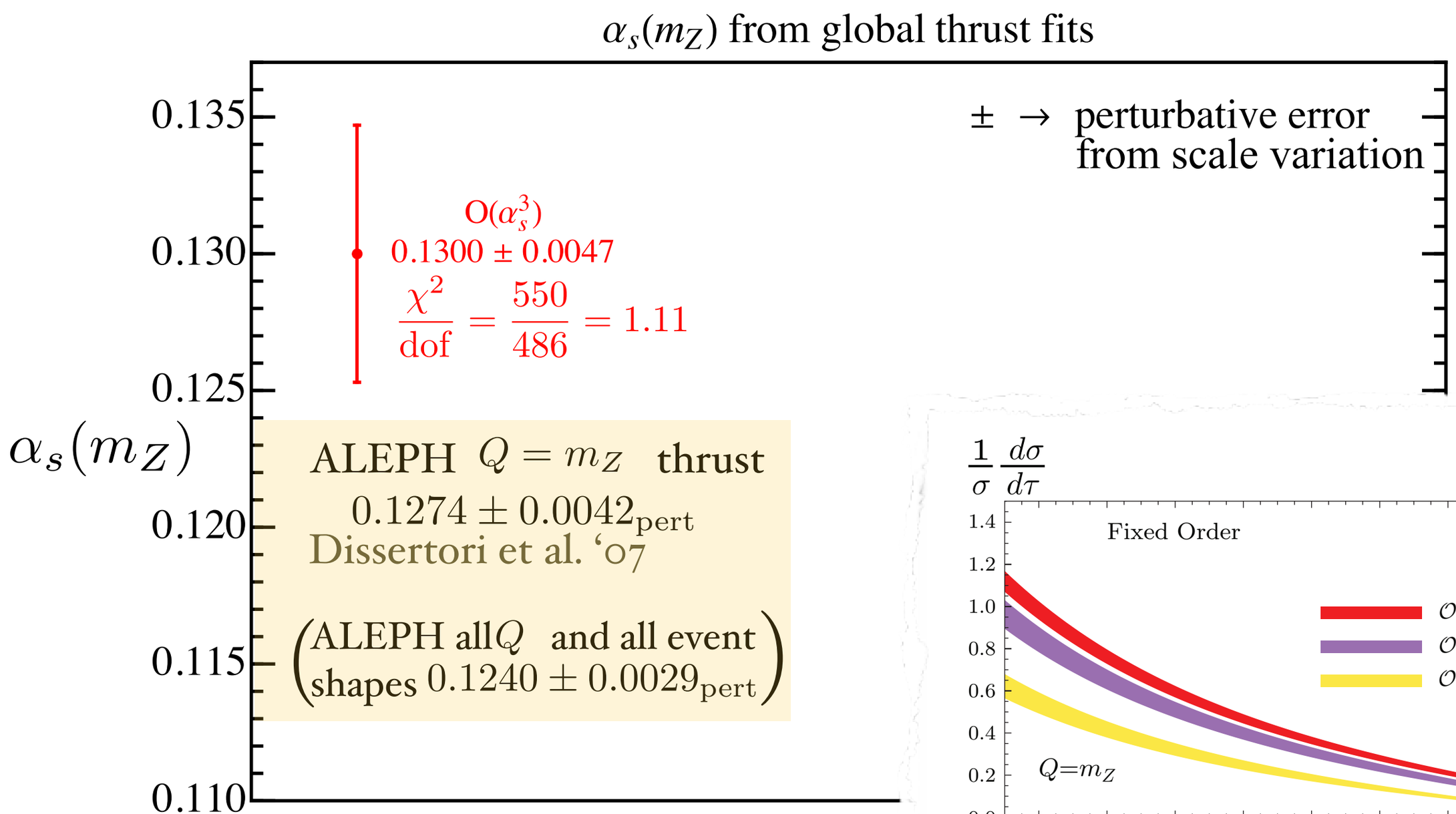

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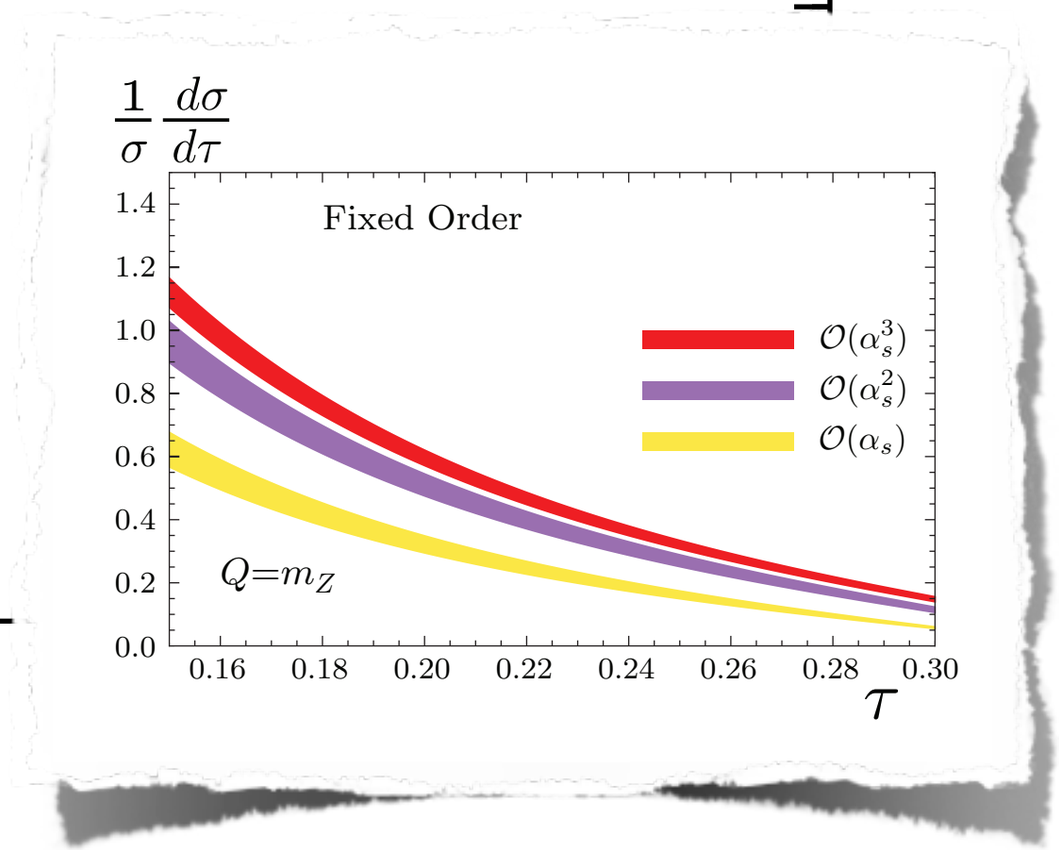

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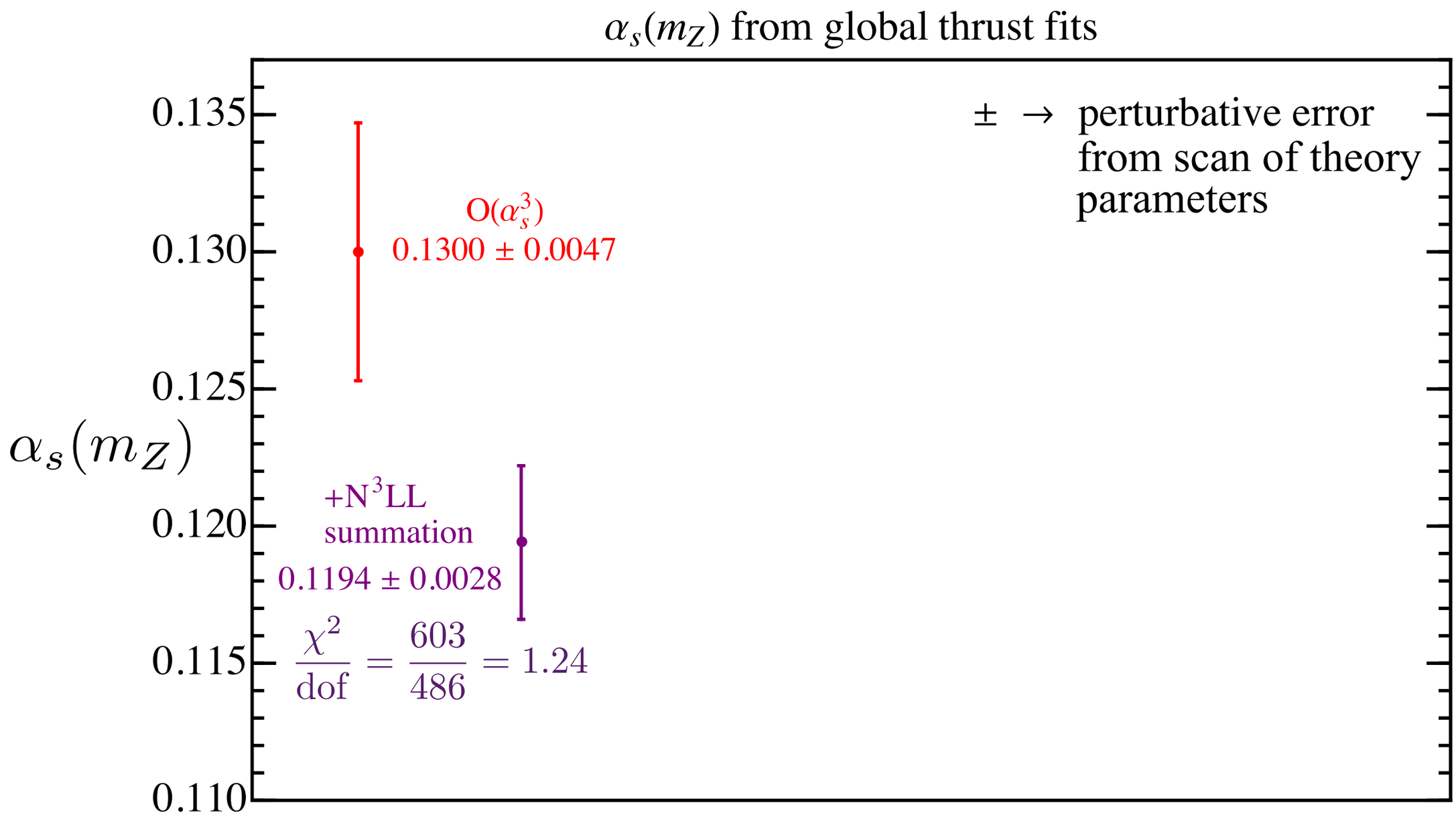

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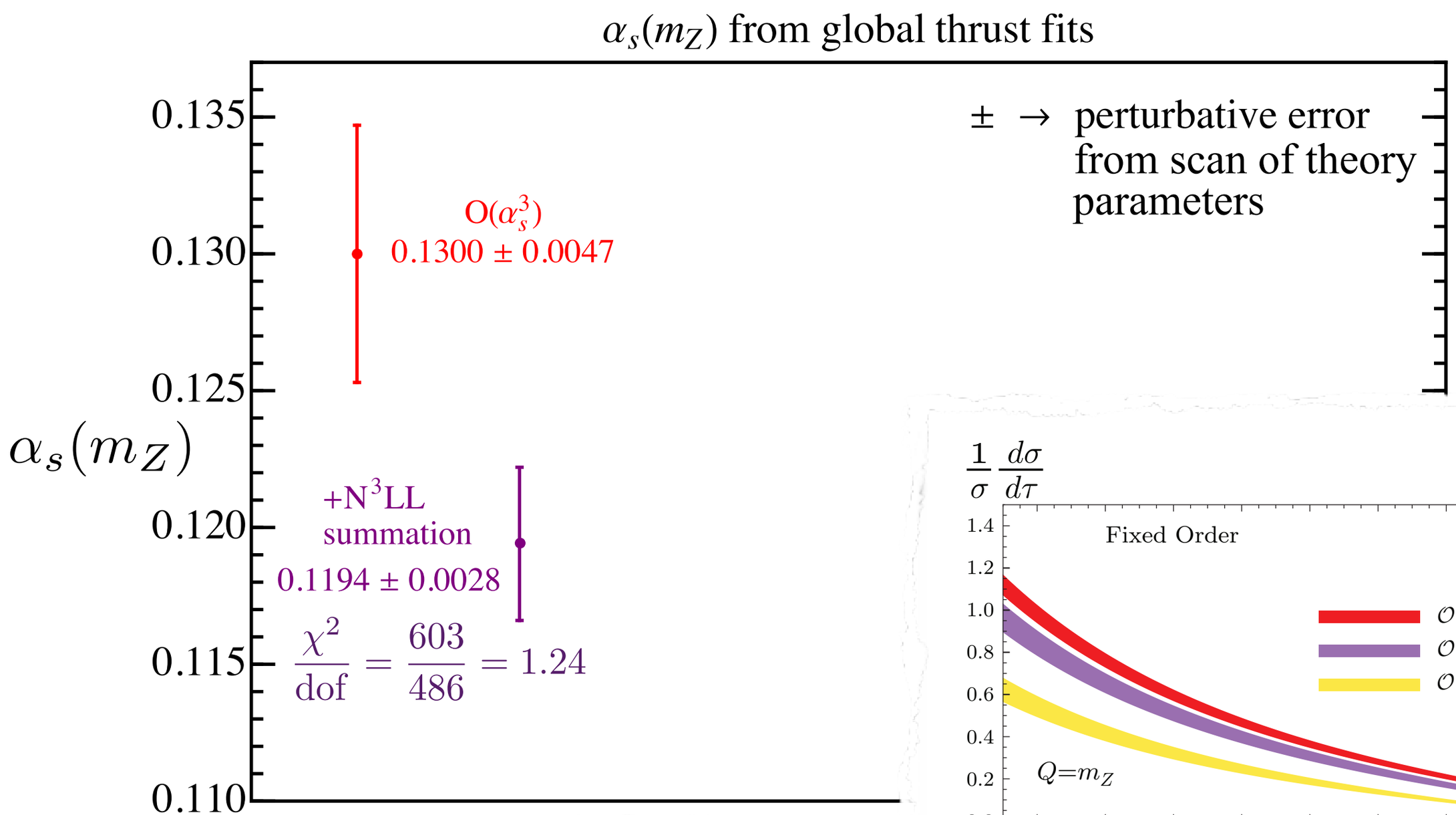

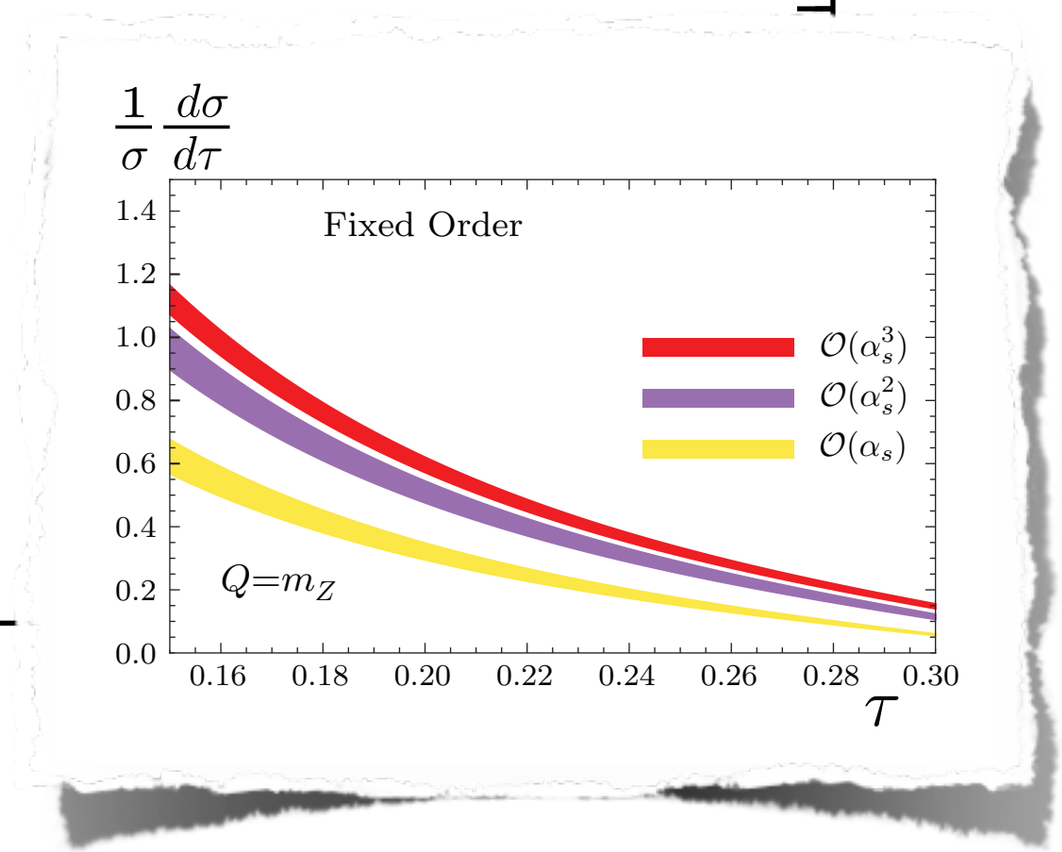

Fit to ALEPH and OPAL
$0.1172 \pm 0.0012_{\text{pert}}$
Becher Schwartz '08

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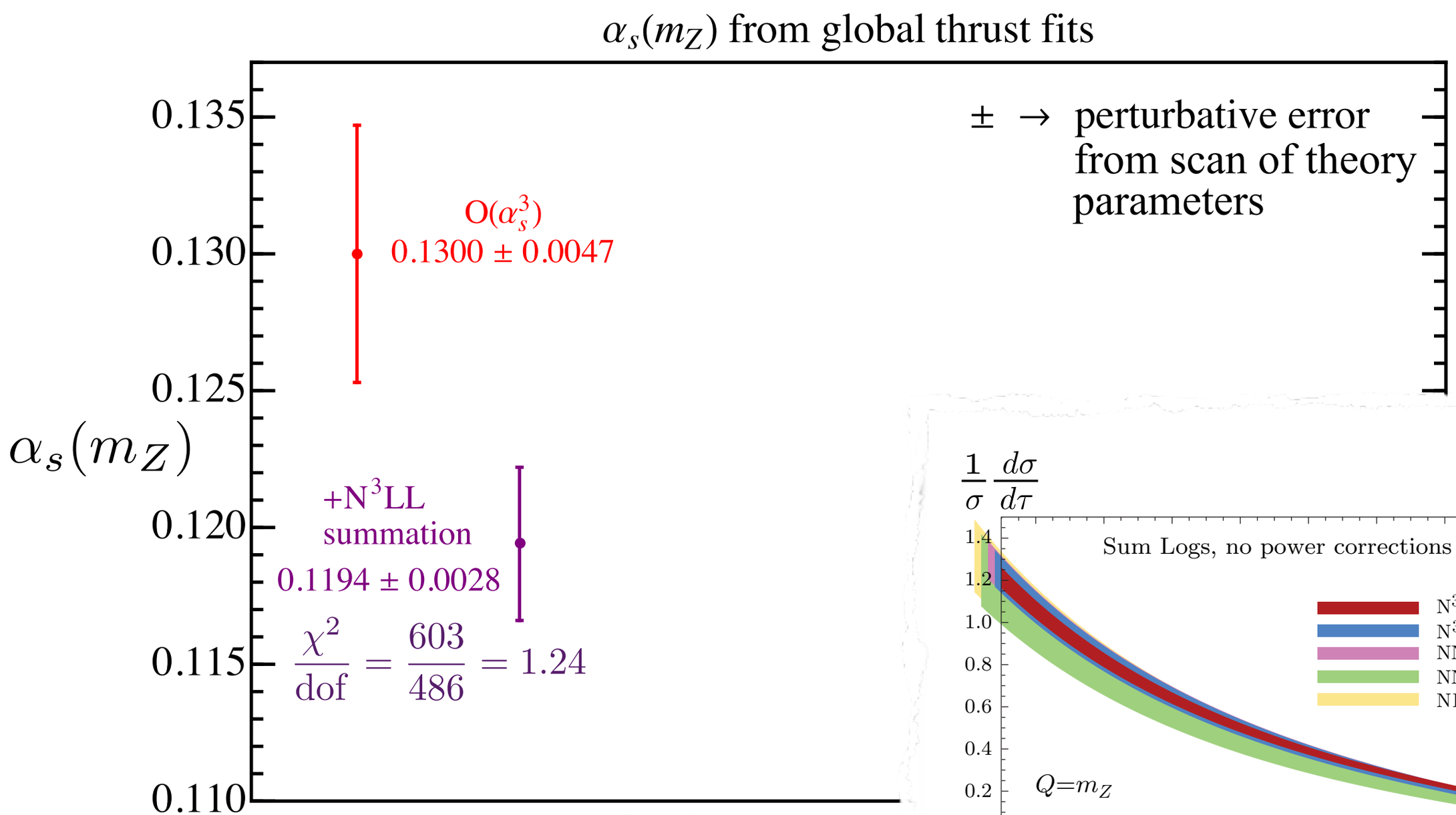

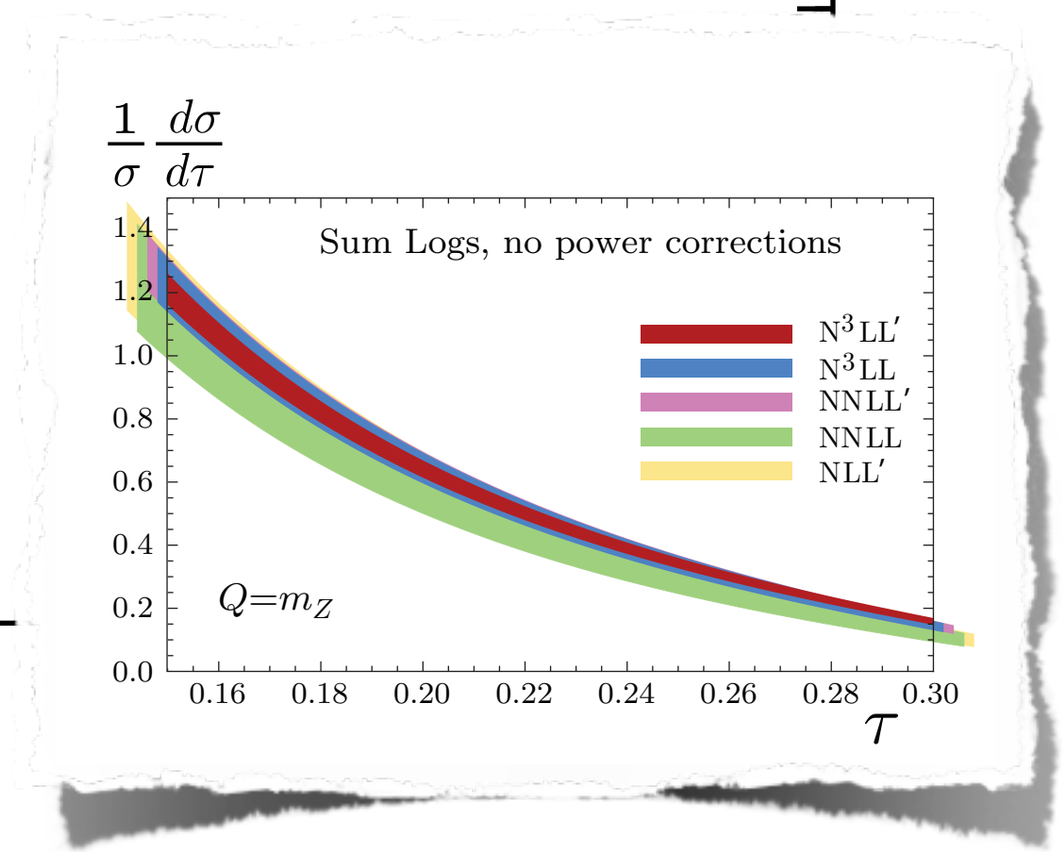

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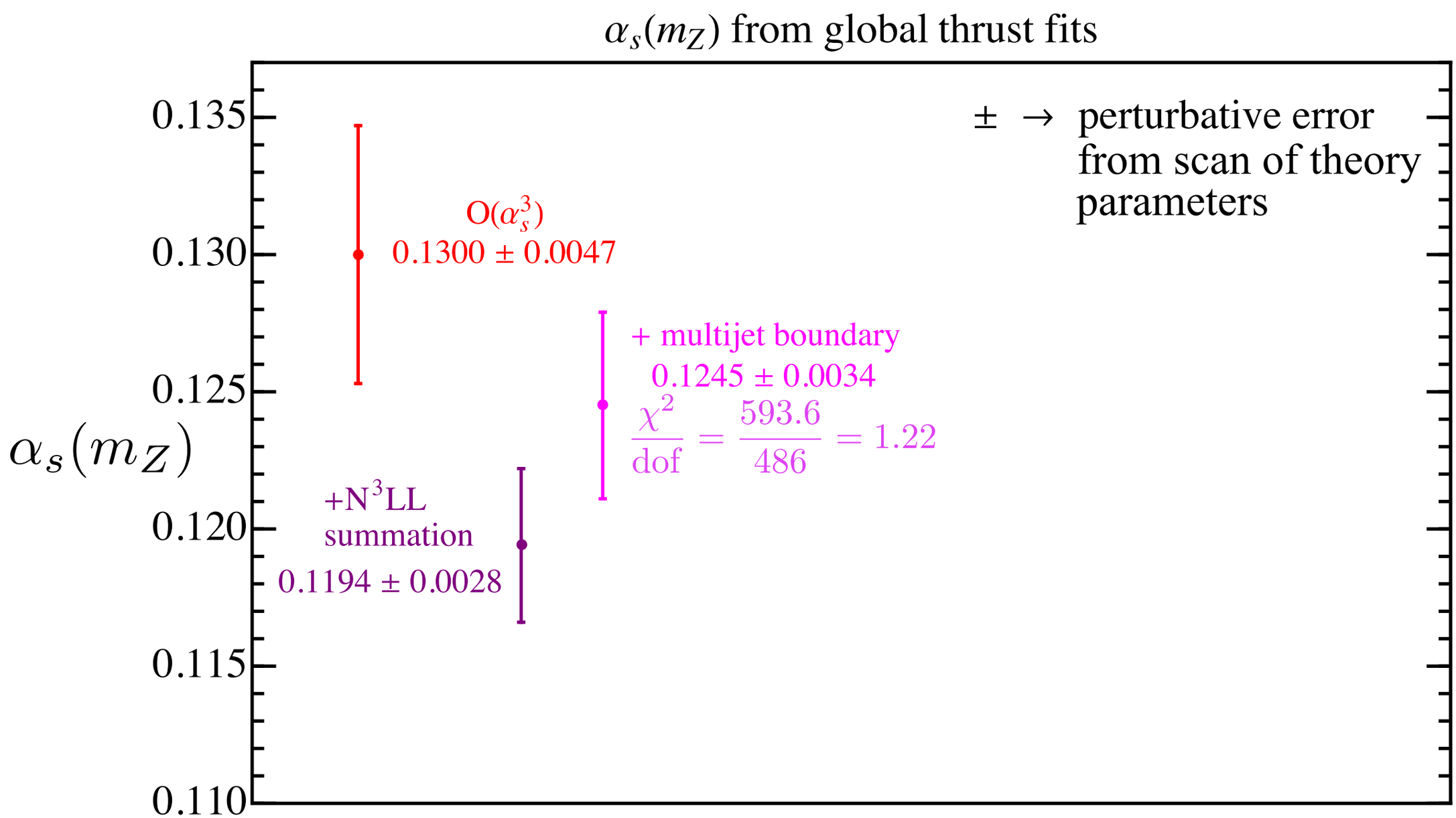

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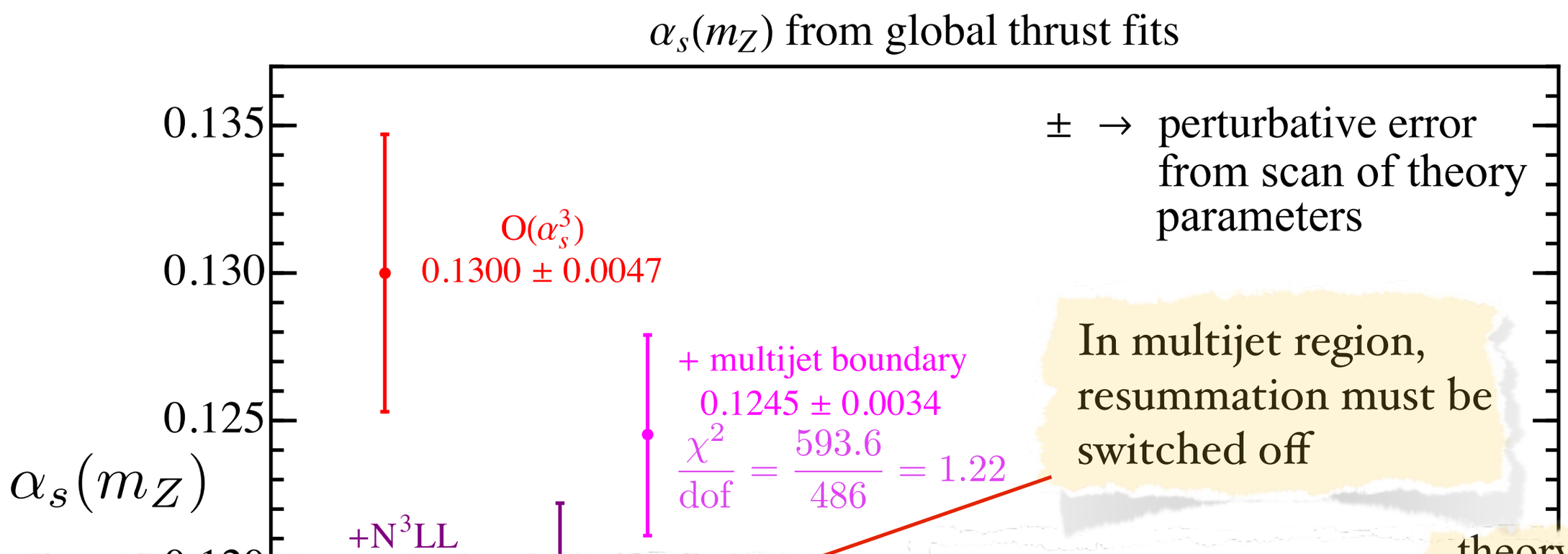
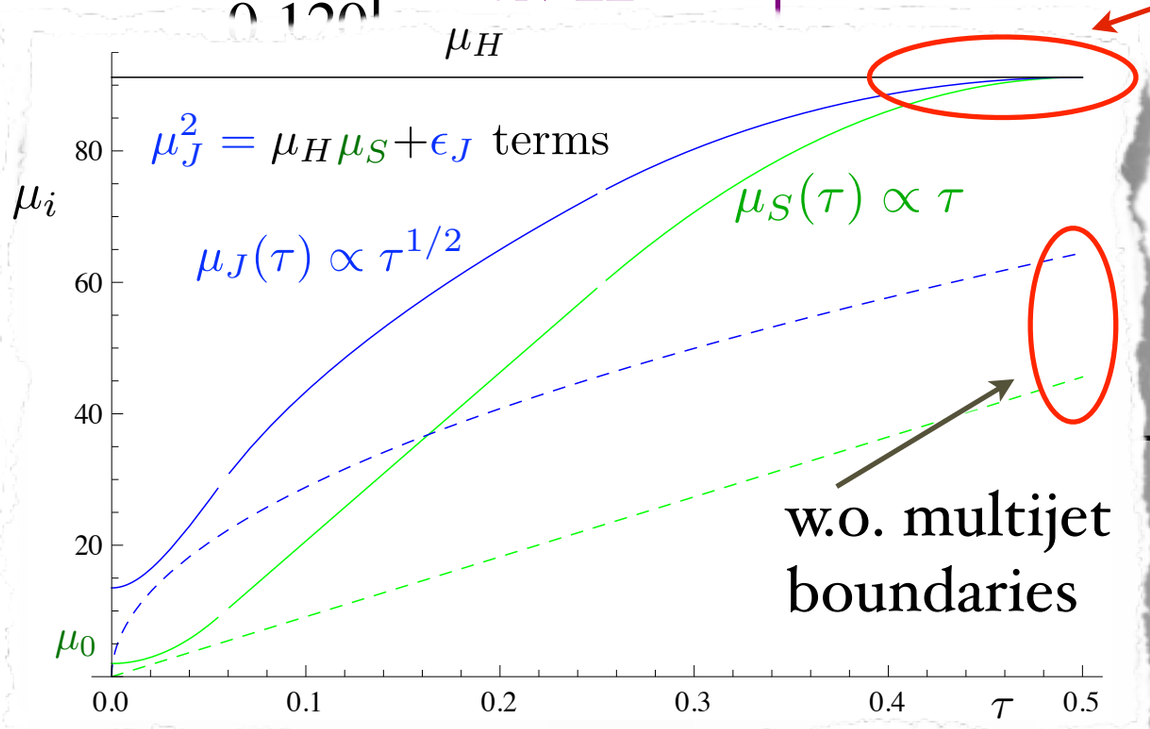
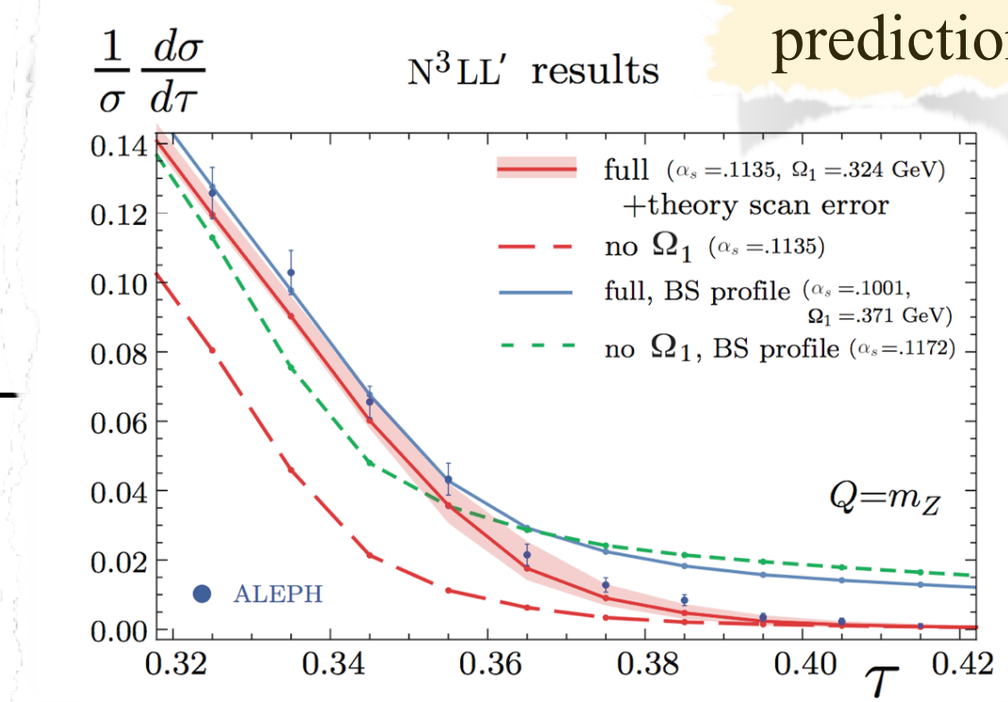


In multijet region, resummation must be switched off

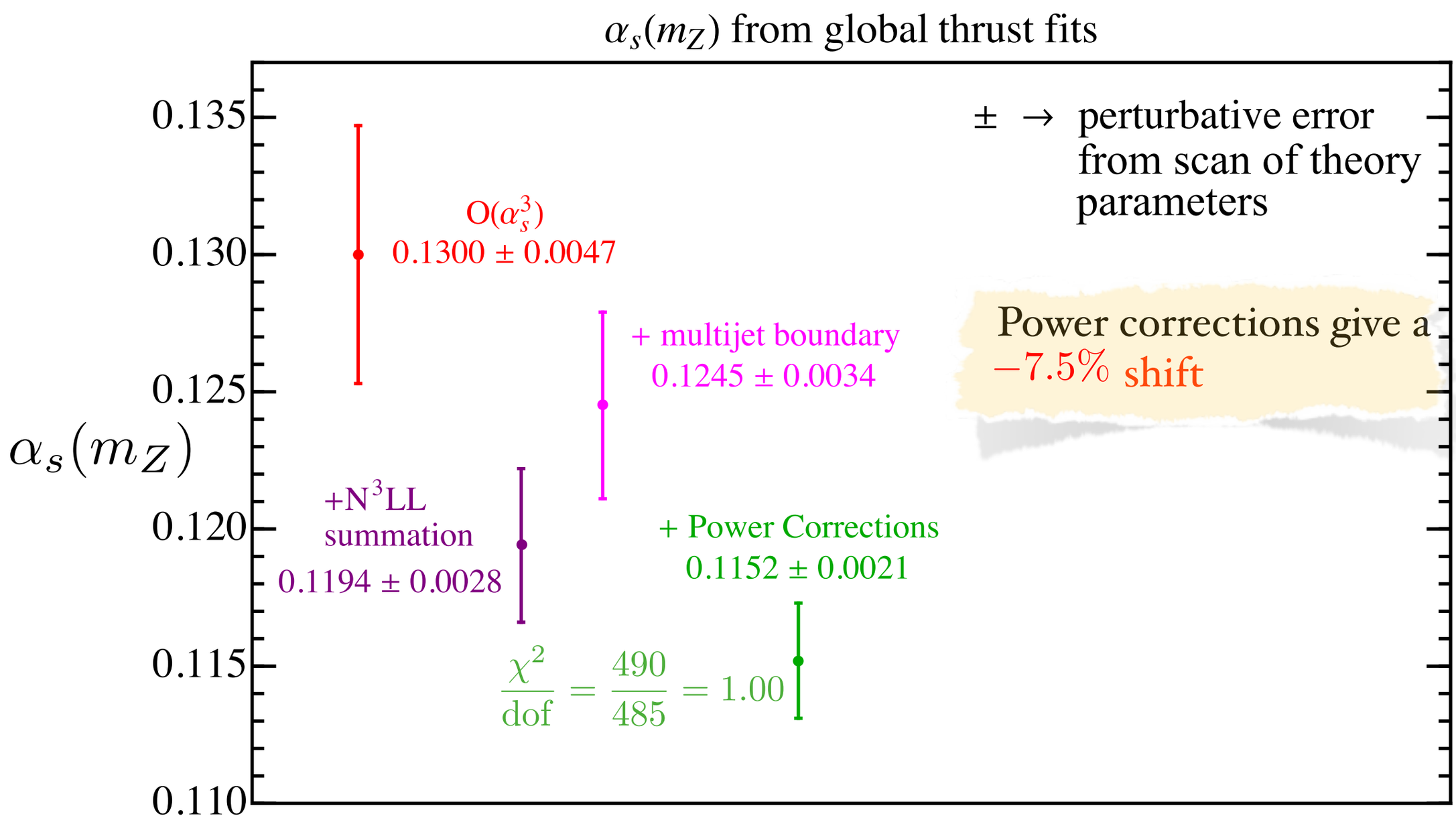


w.o. multijet boundaries

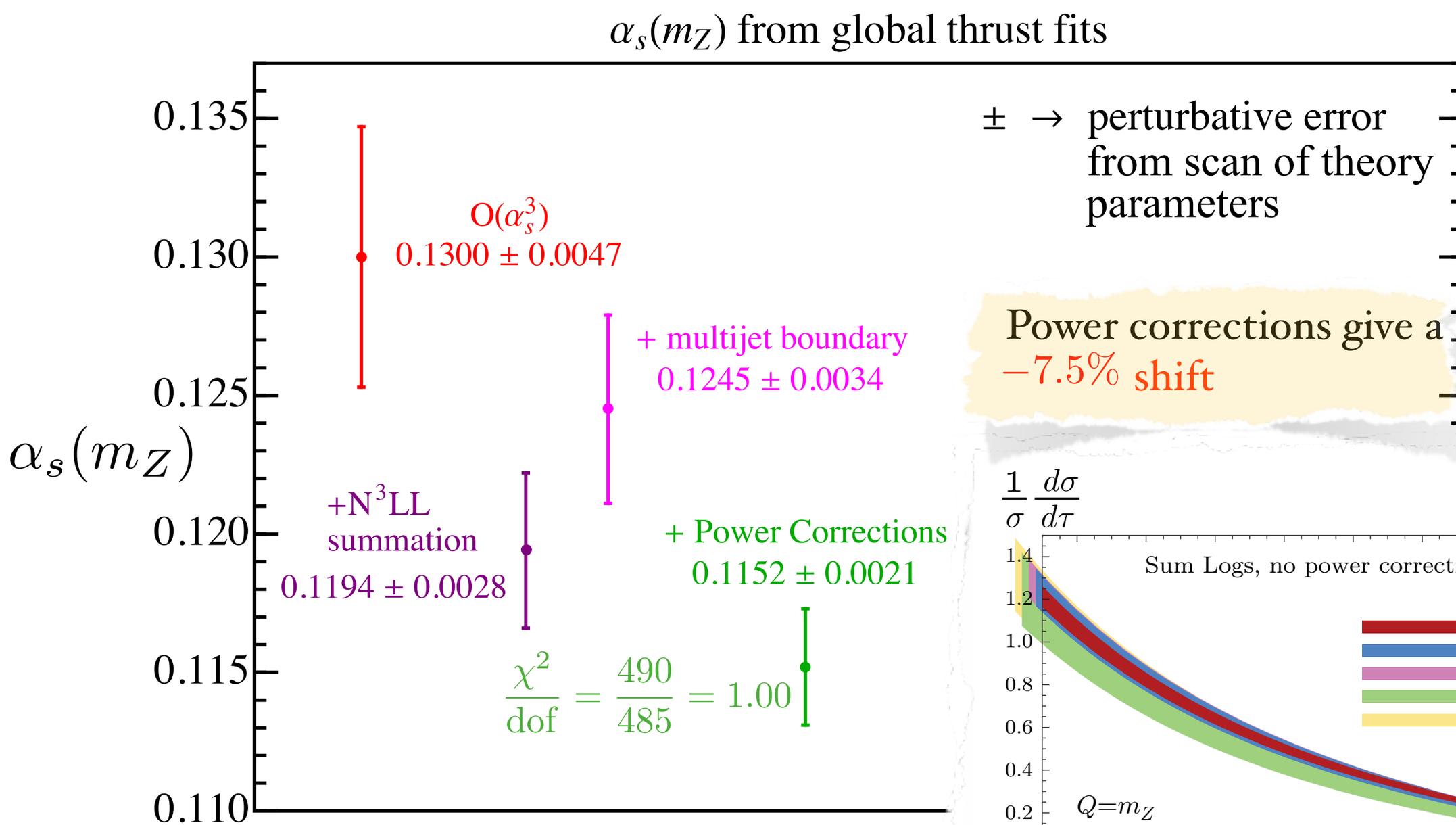
theory prediction



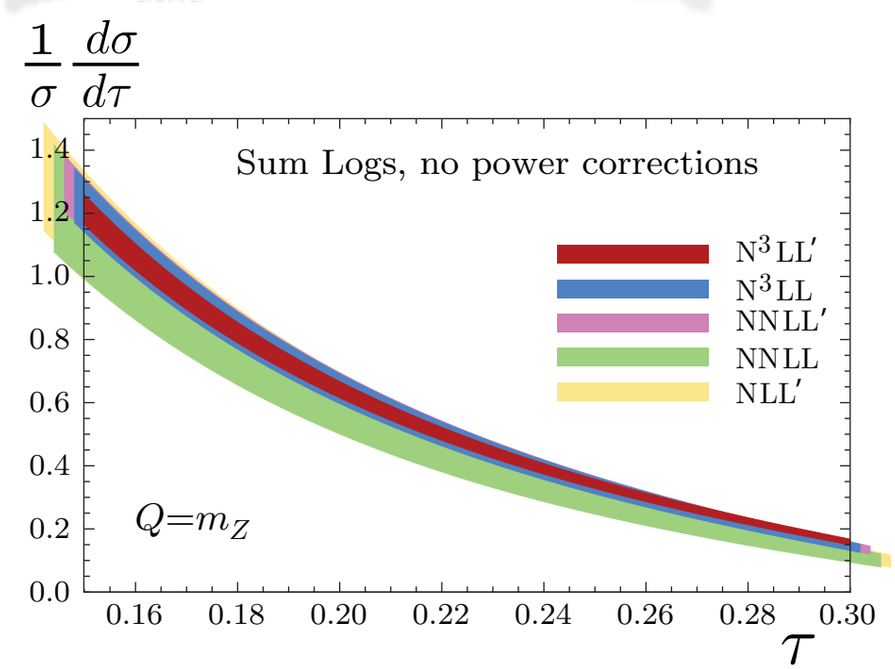
Global tail fit for $\alpha_s(m_Z)$ and Ω_1



Global tail fit for $\alpha_s(m_Z)$ and Ω_1



Power corrections give a -7.5% shift



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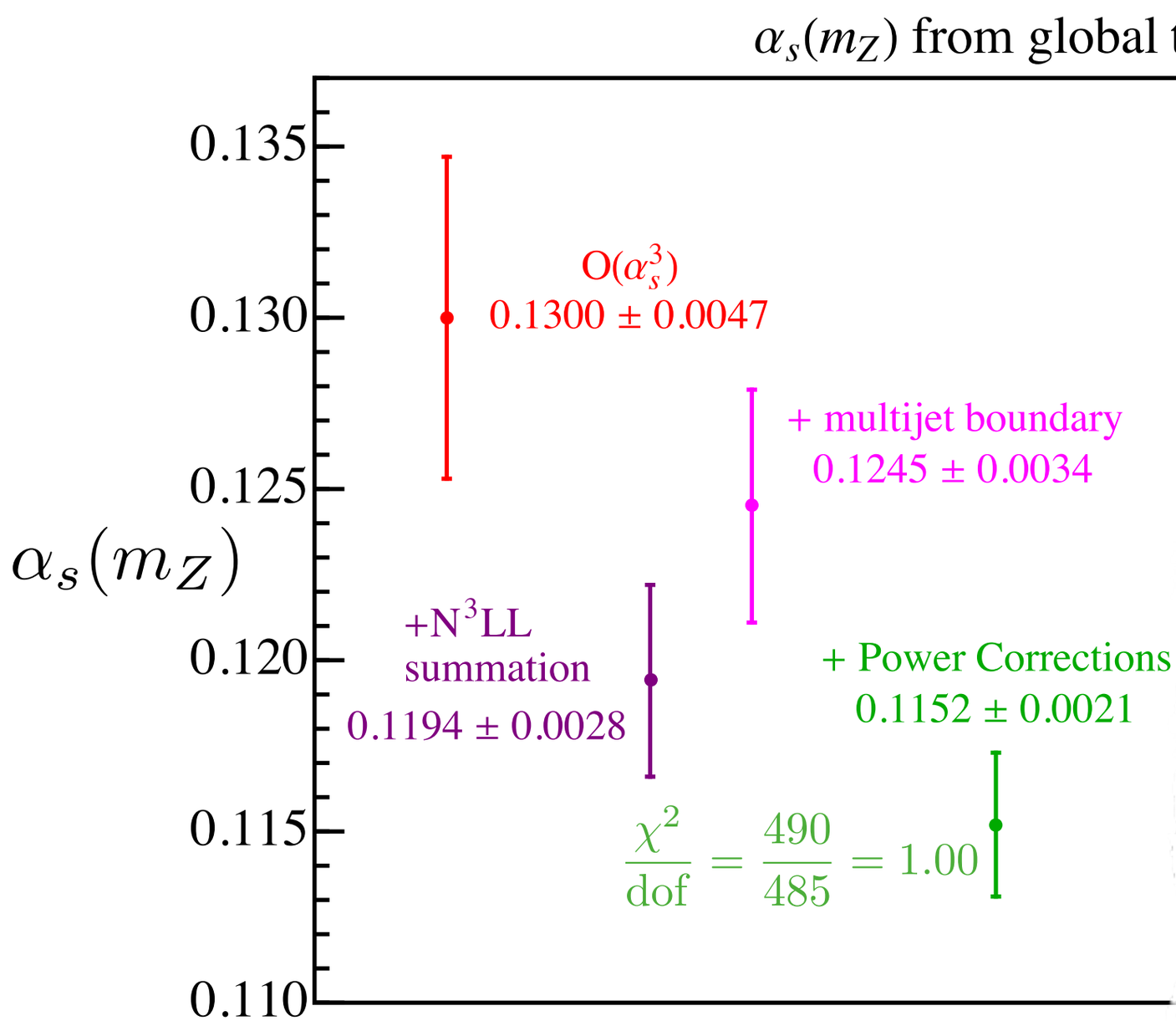

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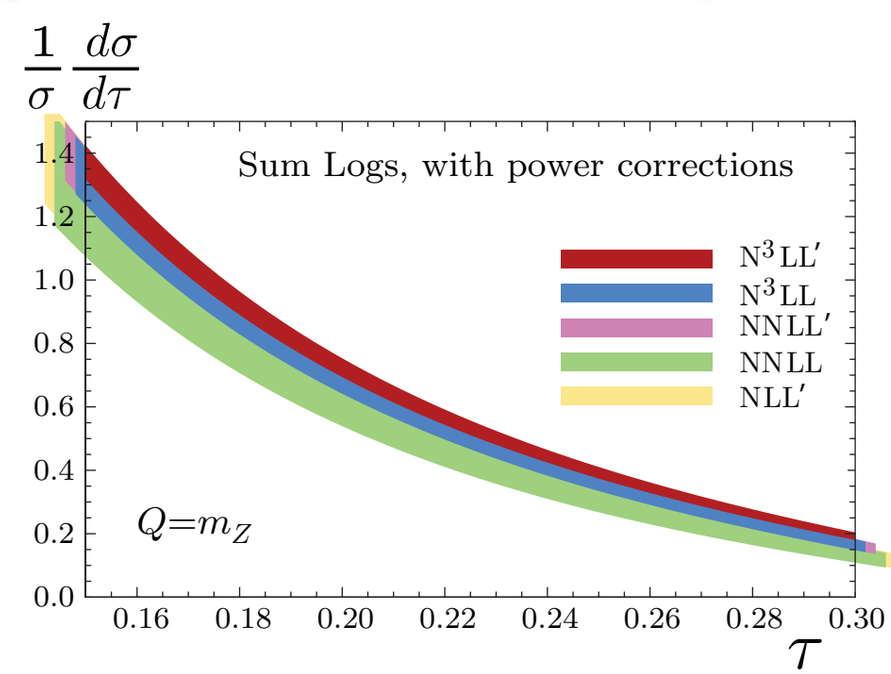
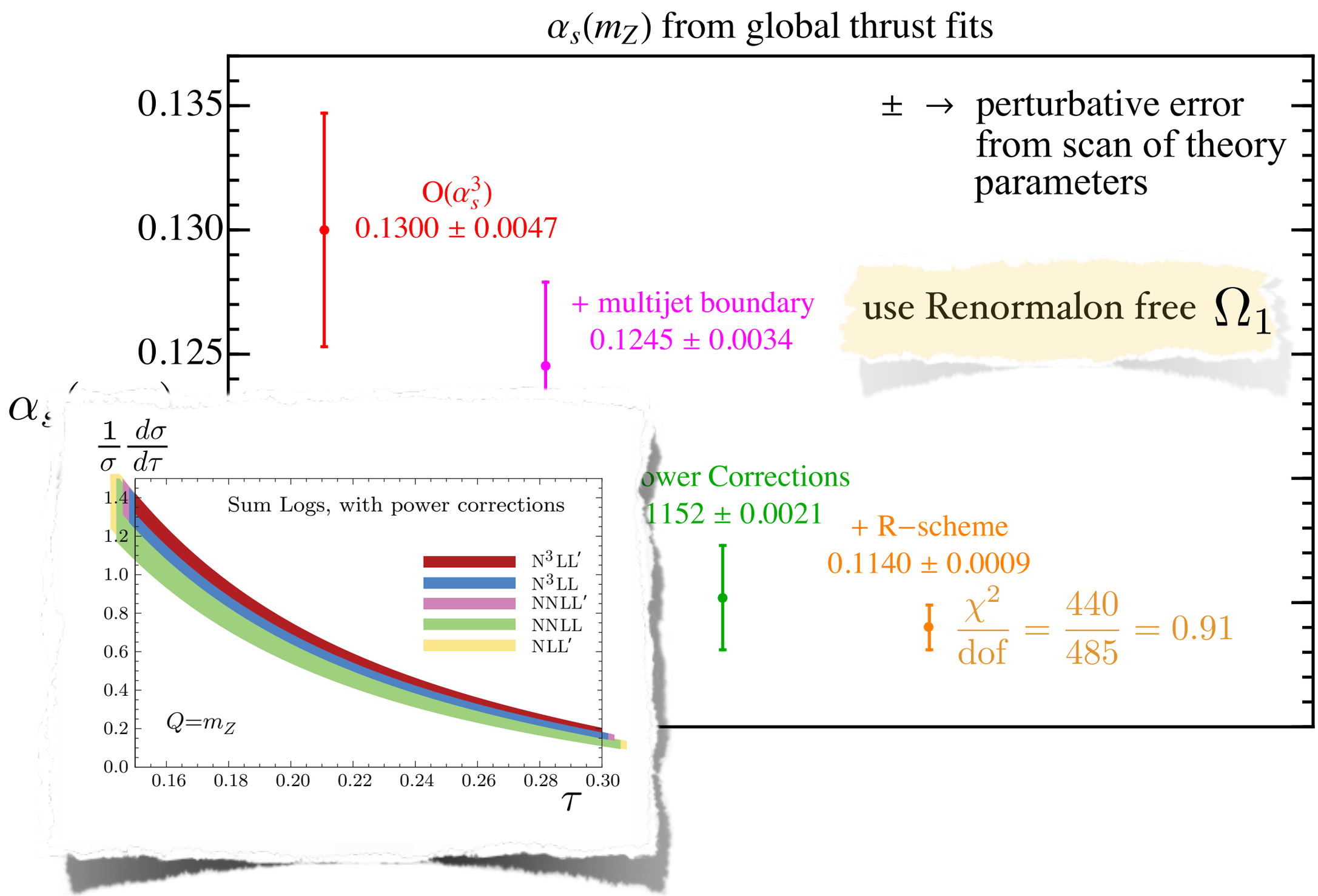


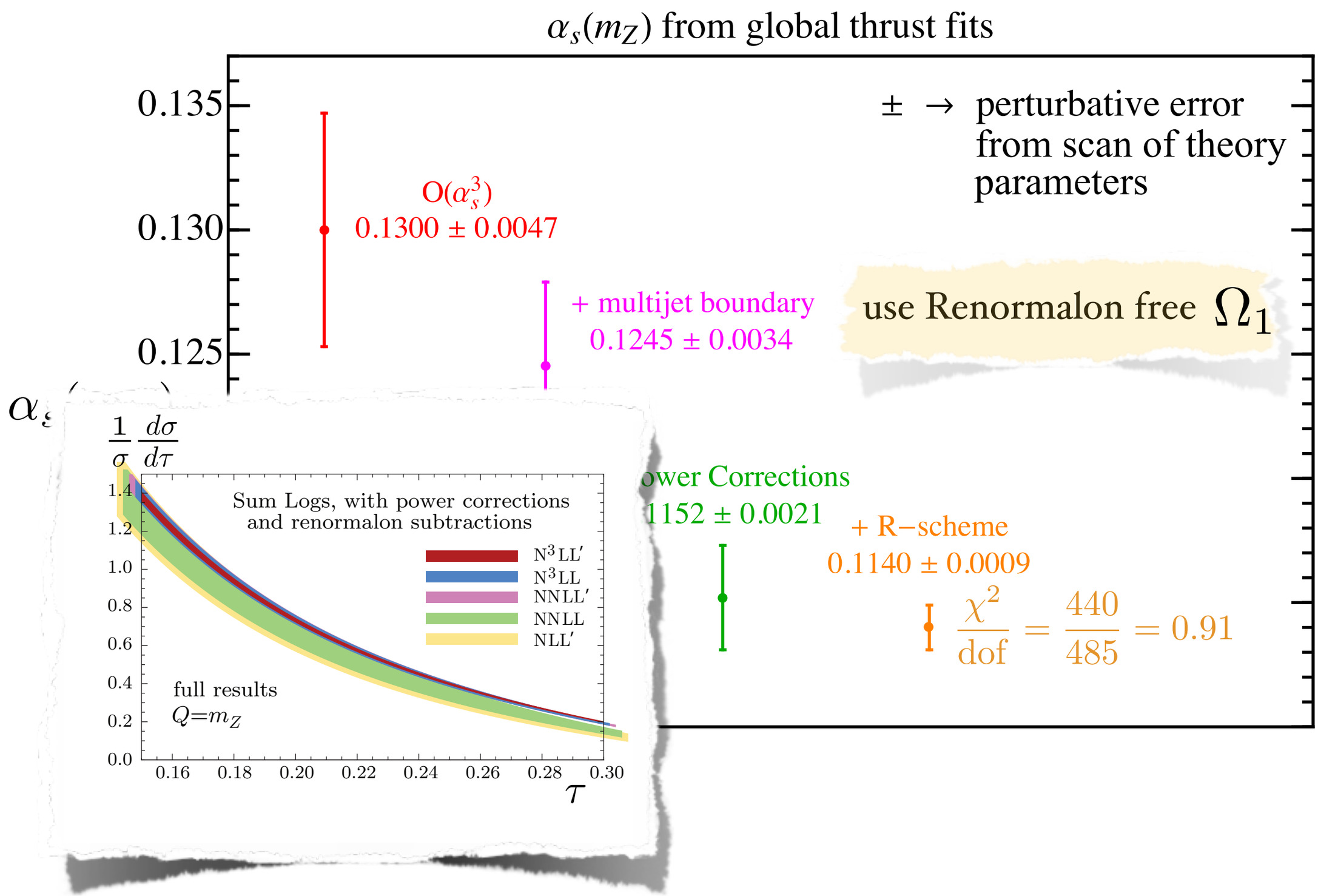
agrees with naive estimate using data:

$$\frac{\delta\alpha_s}{\alpha_s} = \frac{2\Lambda}{Q}\left[\frac{h'(\tau)}{h(\tau)}\right]^{expt} = -(9 \pm 3)\%$$

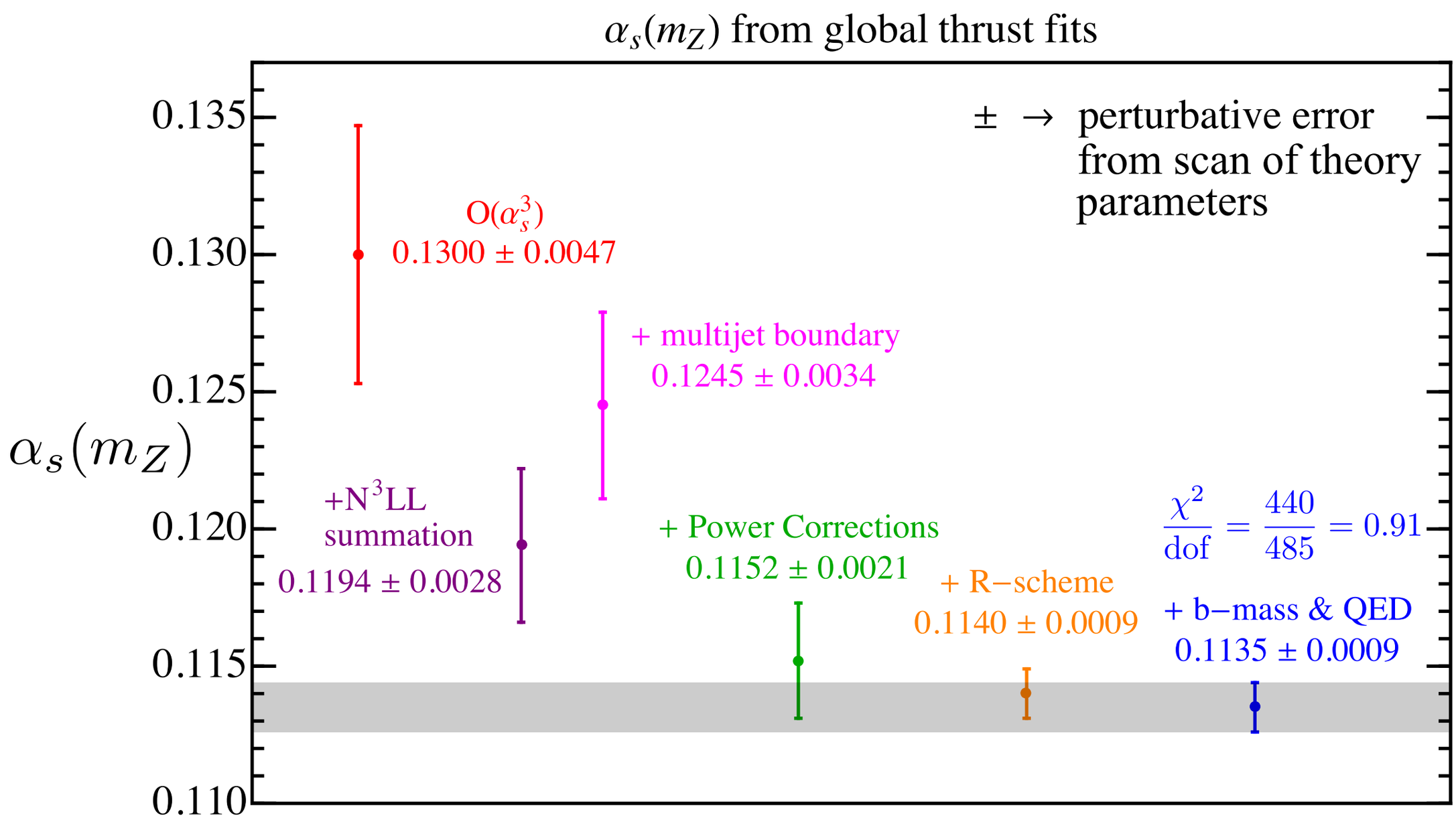
Global tail fit for $\alpha_s(m_Z)$ and Ω_1



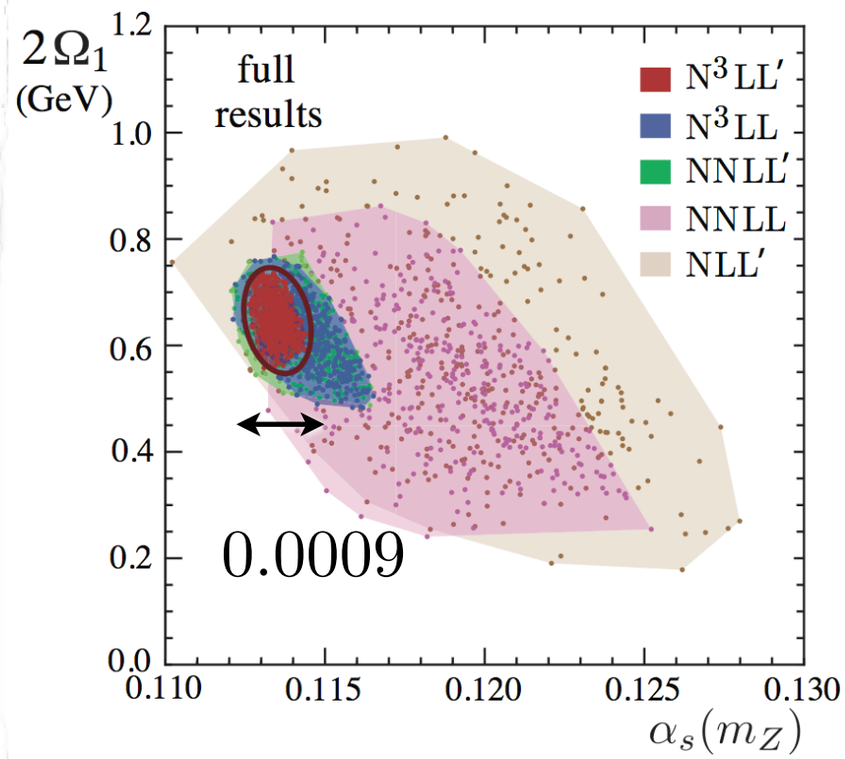
Global tail fit for $\alpha_s(m_Z)$ and Ω_1

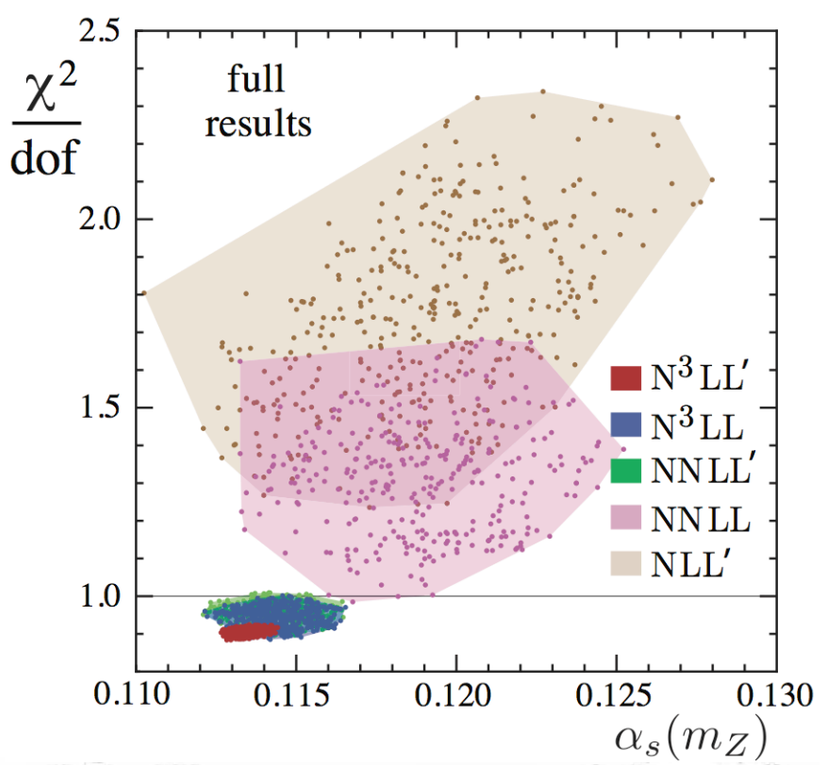


Global tail fit for $\alpha_s(m_Z)$ and Ω_1



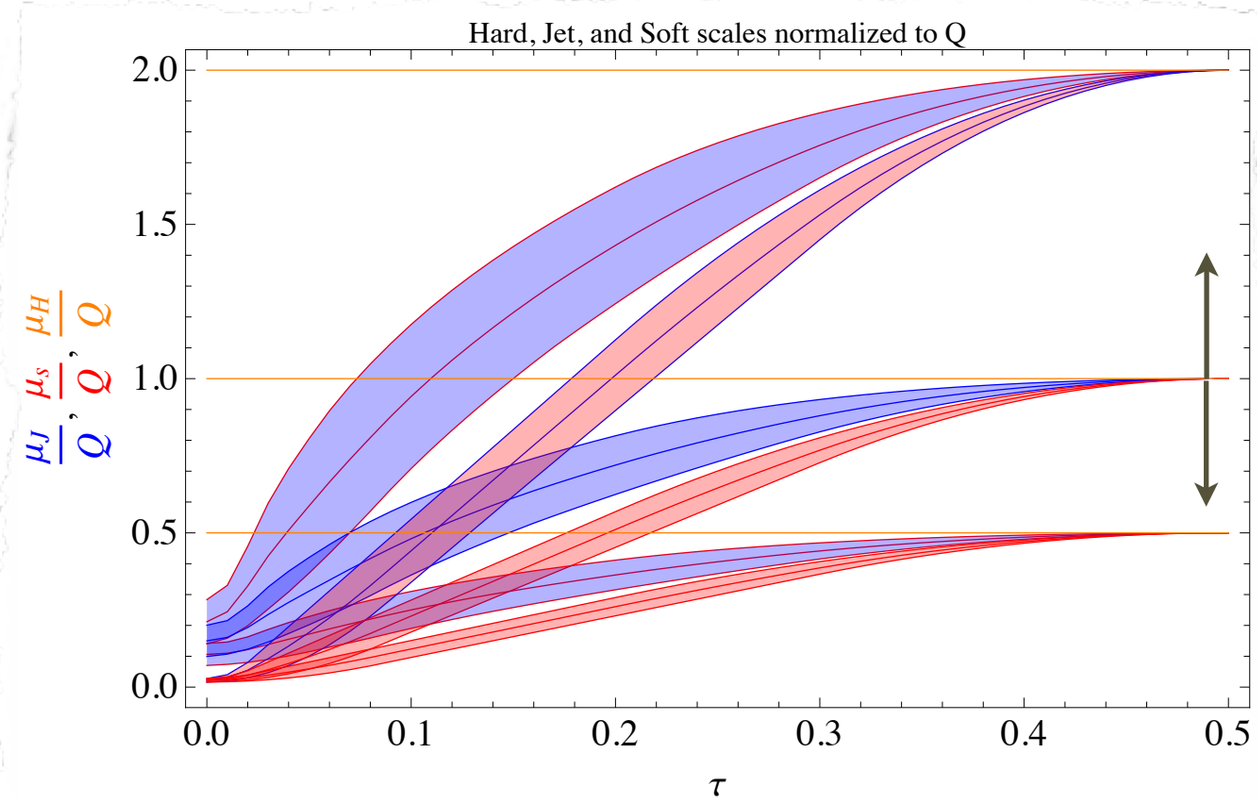
Convergence of results





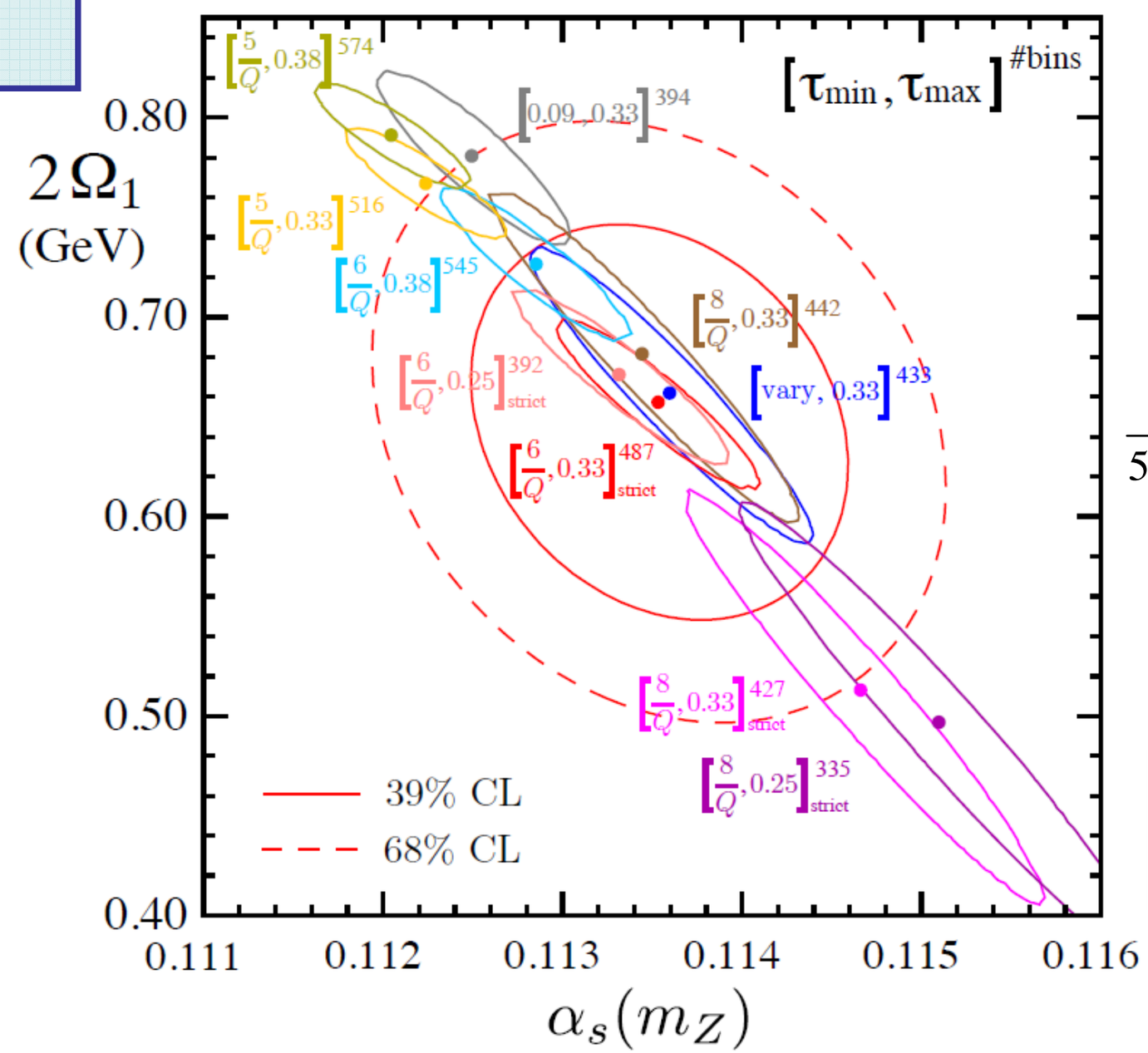
Ω_1 determined to 16% accuracy

Largest contribution to perturbative uncertainty comes from variation of profile parameters



Fit for bins: different data sets

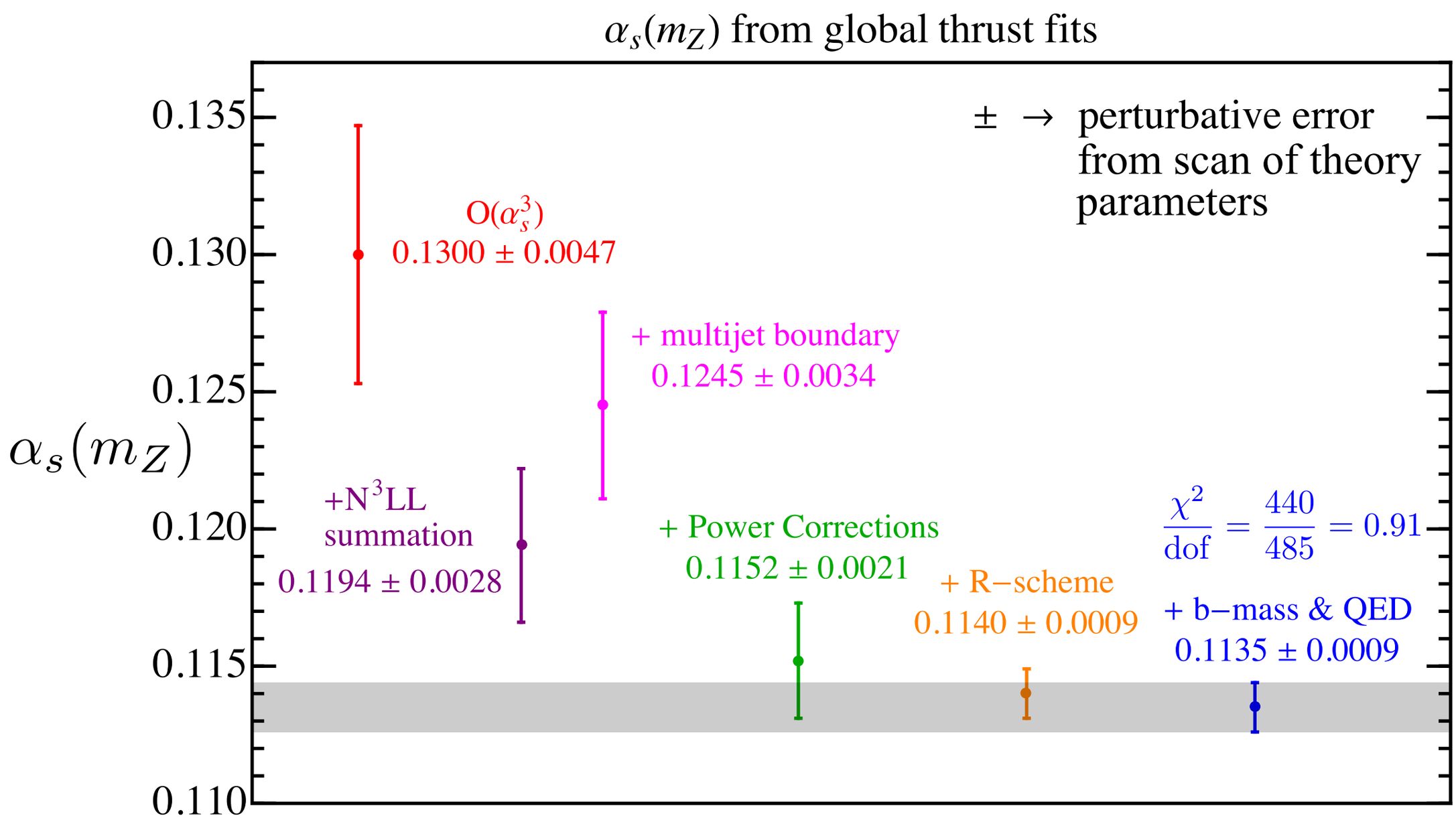
Ω_2 effects increase



$$\frac{\Omega_1}{50.2\text{GeV}} = 0.1200 - \alpha_s(m_Z)$$

statistical errors decrease

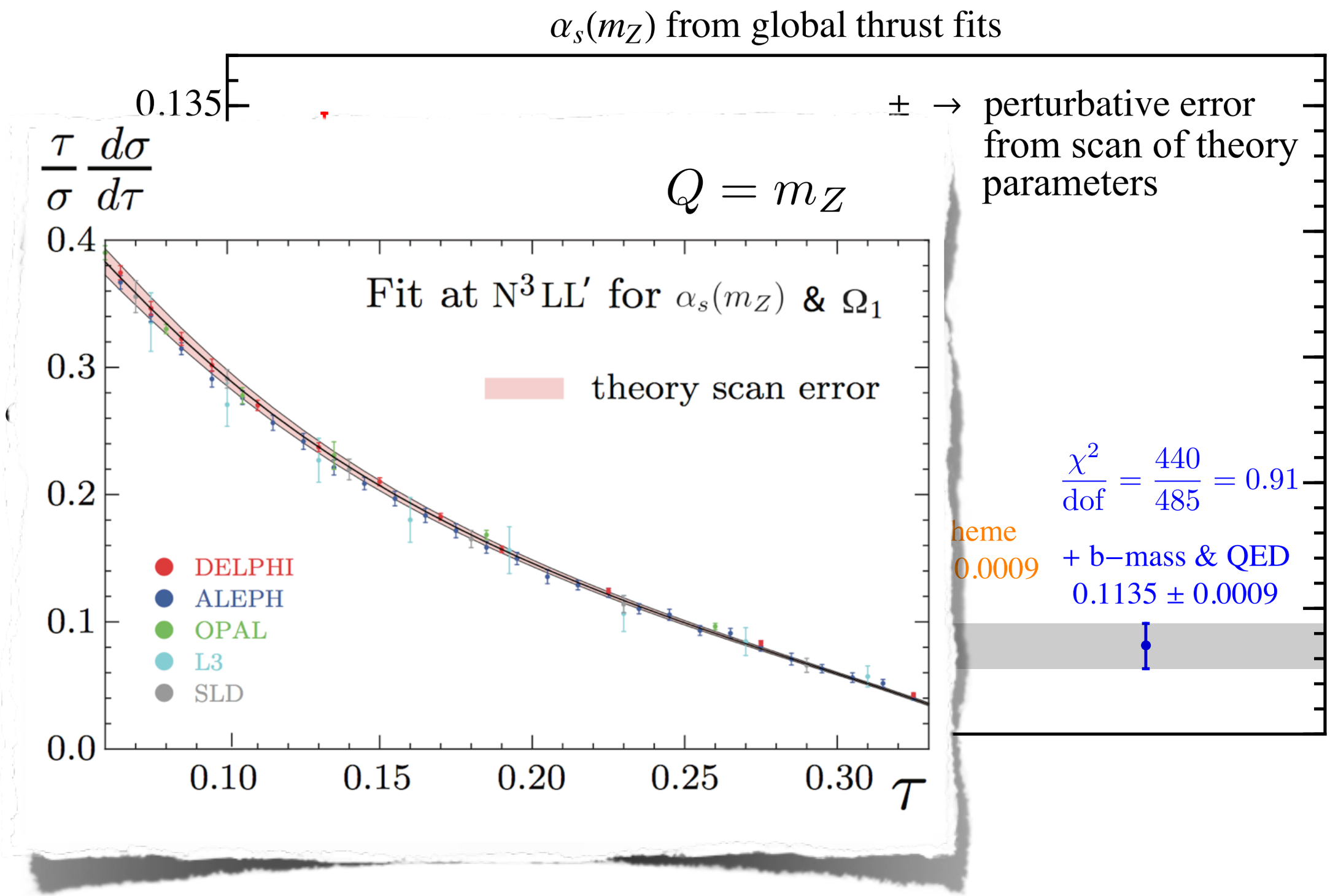
Global tail fit for $\alpha_s(m_Z)$ and Ω_1



$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}}$$

$$\Omega_1 = 0.323 \pm 0.009_{\text{exp}} \pm 0.013_{\Omega_2} \pm 0.020_{\alpha_s(m_Z)} \pm 0.045_{\text{pert}} \text{ GeV}$$

Global tail fit for $\alpha_s(m_Z)$ and Ω_1

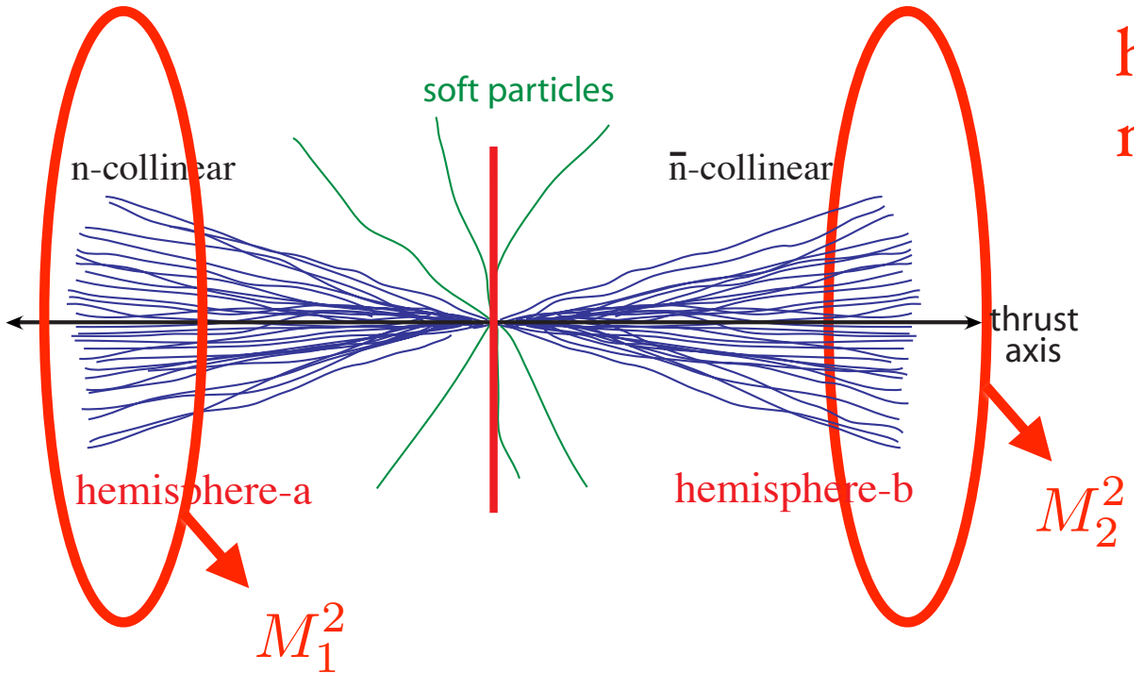


Analysis of Heavy Jet Mass

goal is same precision (N^3LL with power corrections)

N^3LL results	Chien & Schwartz (arXiv:1005.1644)
global fit	AHMS & Schwartz work in progress

Heavy Jet Mass Event Shape

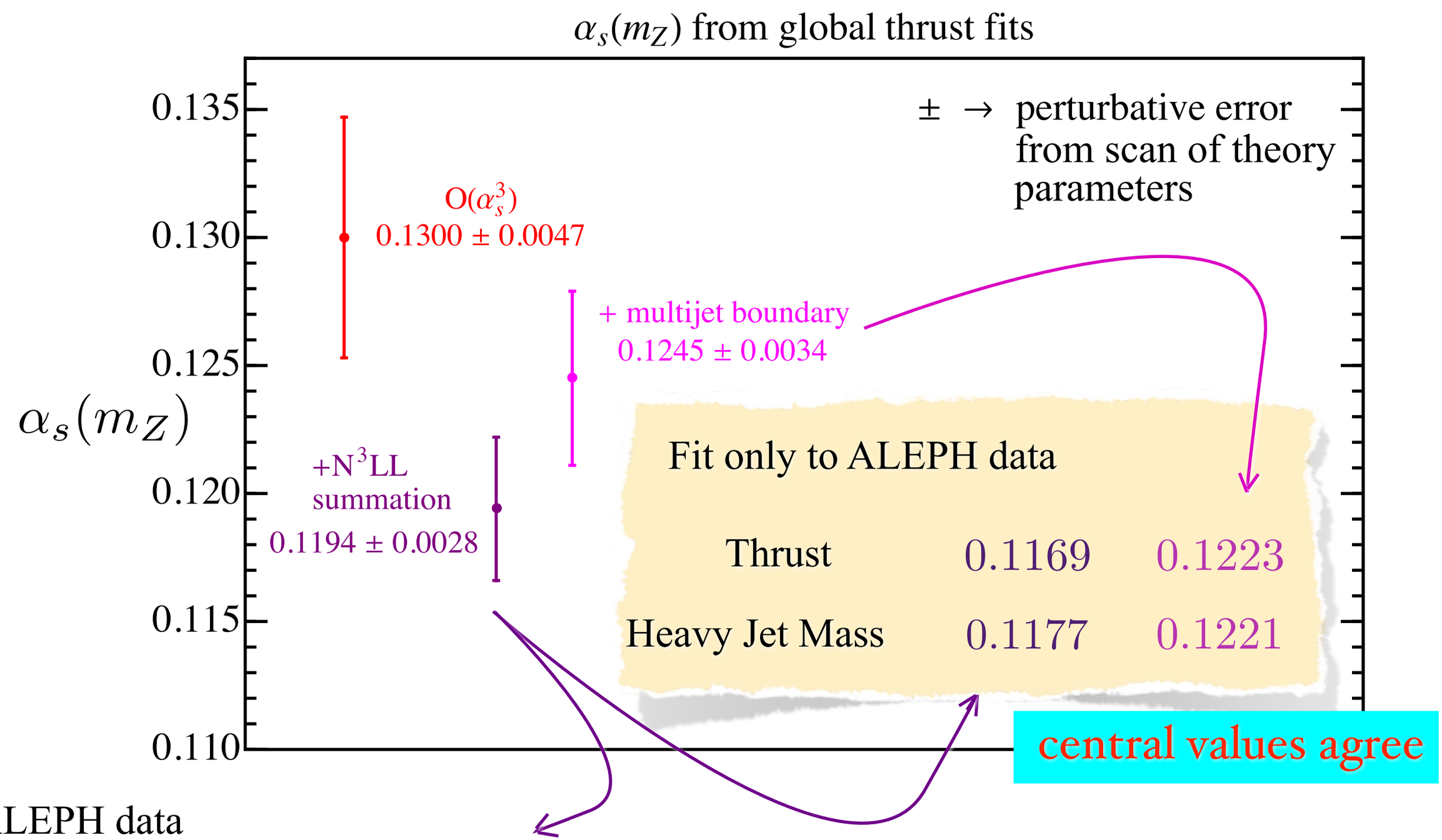


hemisphere invariant masses

$$M_i^2 = \left(\sum_{a \in i} p_a^\mu \right)^2$$

$$\rho = \max \left\{ \frac{M_1^2}{Q^2}, \frac{M_2^2}{Q^2} \right\}$$

Comparison with Heavy Jet Mass



ALEPH data

$\alpha_s^\tau(m_Z) = 0.1175 \pm 0.0026$ Becher&Schwartz '08

$\alpha_s^\rho(m_Z) = 0.1220 \pm 0.0031$ Chien&Schwartz '10

central values differ

?

There are two ways to calculate the theoretical value for a bin

Difference of Cumulants

$$\Sigma(\tau_2, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1))$$

$$\Sigma(\tau, \mu_i(\tau)) = \int_0^\tau dt \, \frac{d\sigma}{dt}(t, \mu_i(\tau))$$

Classical Resummation analyses

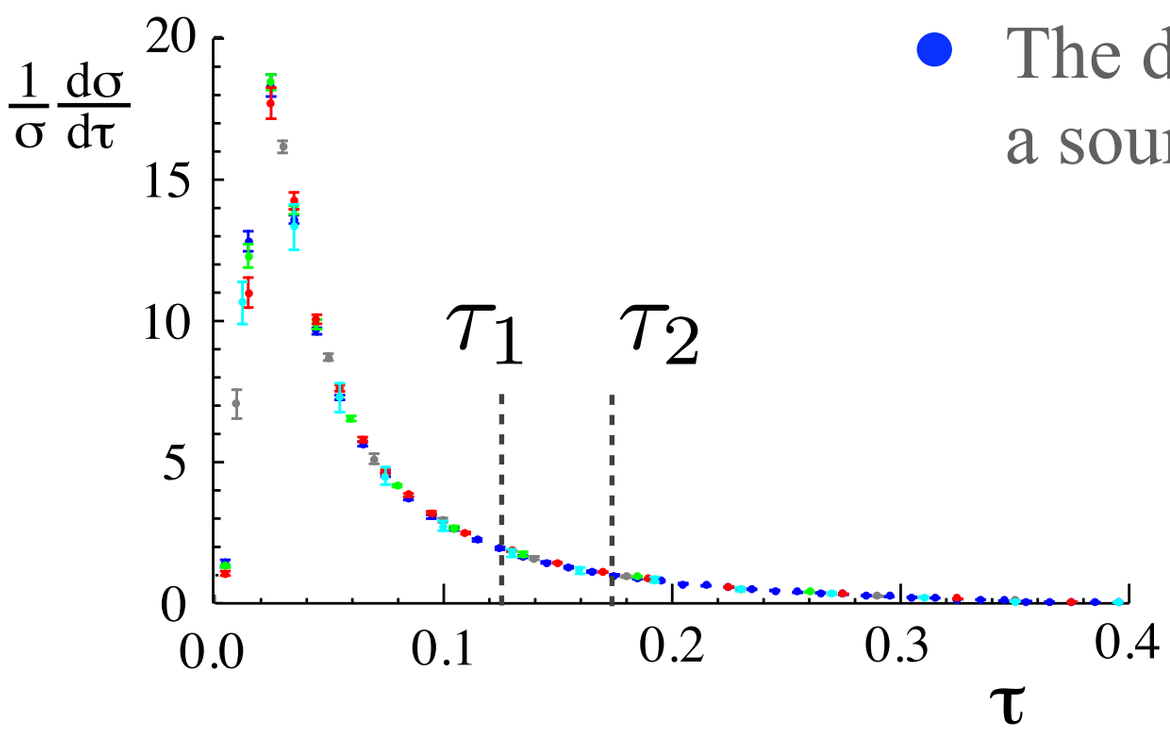
Becher&Schwartz, Chien&Schwartz

Integral of Differential Distribution

$$\int_{\tau_1}^{\tau_2} dt \, \frac{d\sigma}{dt}(t, \mu_i(t))$$

AFHMS, AHMS&Schwartz

- These two agree on all terms at the order in resummation one is working
- They give different estimates for higher order terms (as they should)



- The difference of cumulants always includes a source of uncertainty from the peak region.

For high precision results for the tail region we must use the integral of diff.distn. (or cumulant difference with common scale choice)

$$\Sigma(\tau_2,\mu_i(\tau_2))-\Sigma(\tau_1,\mu_i(\tau_1))=$$

$$\underbrace{\int_{\tau_1}^{\tau_2}\mathrm{d}\tau'\frac{\mathrm{d}\sigma}{\mathrm{d}\tau'}(\tau',\mu_i(\tau_2))}+\underbrace{\Sigma(\tau_1,\mu_i(\tau_2))-\Sigma(\tau_1,\mu_i(\tau_1))}$$

$$\simeq\int_{\tau_1}^{\tau_2}\mathrm{d}\tau'\frac{\mathrm{d}\sigma}{\mathrm{d}\tau'}(\tau',\mu_i(\tau'))+(\tau_2-\tau_1)\frac{\mathrm{d}\mu_i(\tau_1)}{\mathrm{d}\tau_1}\int_0^{\tau_1}\mathrm{d}\tau'\frac{\mathrm{d}\sigma}{\mathrm{d}\tau'}(\tau',\mu_i(\tau_1))$$

Enhanced uncertainty from the peak !

- Fixing Ω_1 to thrust value (pure QCD) $\Omega_1 = 0.332\,\mathrm{GeV}$

$$\alpha_s^\rho(m_Z)=0.1158,\quad \frac{\chi^2}{dof}=1.08 \qquad \rho\in[8/Q,0.2]$$

preliminary

$$\alpha_s^\rho(m_Z)=0.1136,\quad \frac{\chi^2}{dof}=1.06 \qquad \rho\in[12/Q,0.2]$$

$$(\alpha_s^\tau(m_Z)=0.1140\pm0.0011)$$

Towards a Heavy Jet Mass global fit

- Global data set ✓*
- Power corrections in the peak region differ from thrust. ✓*

  eg. $M_1^\tau = \int d\tau\, \tau \frac{1}{\sigma}\frac{d\sigma}{d\tau} = \frac{2\Omega_1}{Q} + \ldots$ $\qquad M_1^\rho = \frac{\Omega_1 + \Upsilon_1}{Q} + \ldots$

  require two dimensional soft function.

- $\mathcal{O}(\alpha_s^2)$ Soft Function has non-global structure ✓*

Kelley, Schabinger, Schwartz, Zhu

Hornig, Lee, IS, Walsh, Zuberi

Monni, Gehrmann, Luisoni

- Power corrections in tail region are universal for massless factorization.

  Treatment of hadron masses may spoil universality of Ω_1

  D-scheme
  E-scheme
  P-scheme

  Salam & Wicke

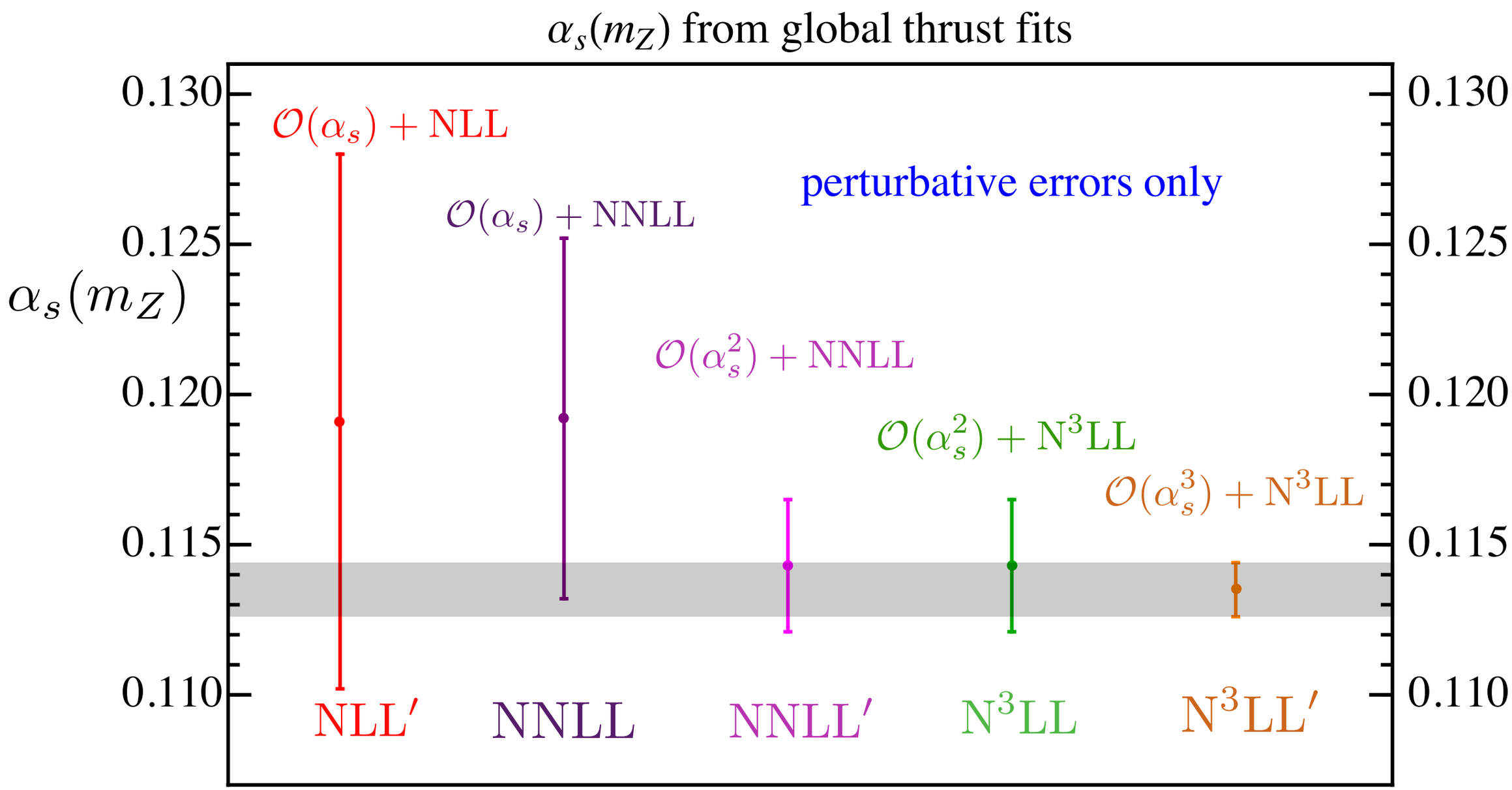
- Theory error analysis
- b-mass, QED effects
- Dataset subsets

* some cross-checking still required

The End

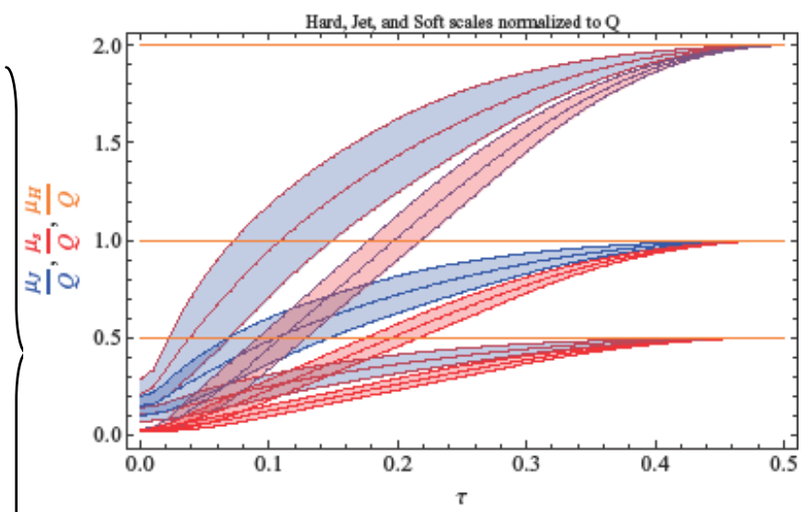
Backup

Convergence of results



Estimate of perturbative uncertainties

parameter	default value	range of values
μ_0	2 GeV	1.5 to 2.5 GeV
n_1	5	2 to 8
t_2	0.25	0.20 to 0.30
e_J	0	-1,0,1
e_H	1	0.5 to 2.0
n_s	0	-1,0,1
s_2	-39.1	-36.6 to -41.6
$\Gamma_3^{\rm cusp}$	1553.06	-1553.06 to $+4569.18$
j_3	0	-3000 to $+3000$
s_3	0	-500 to $+500$
ϵ_2	0	-1,0,1
ϵ_3	0	-1,0,1



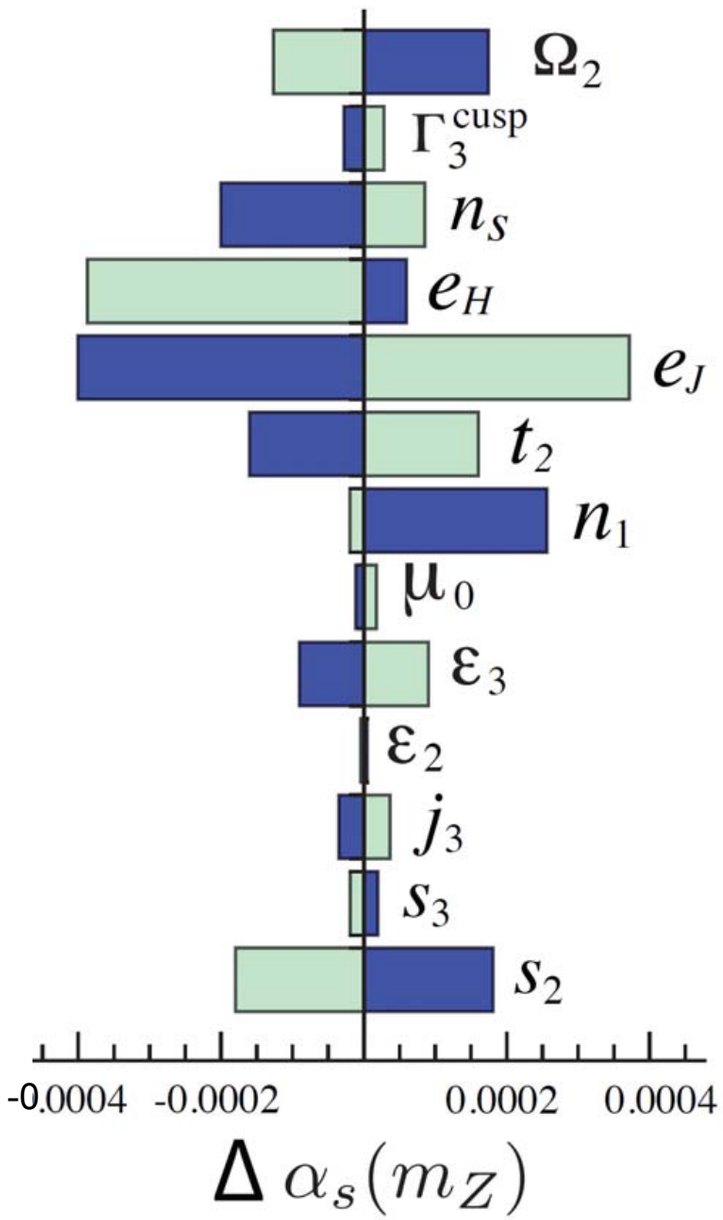
Profile functions

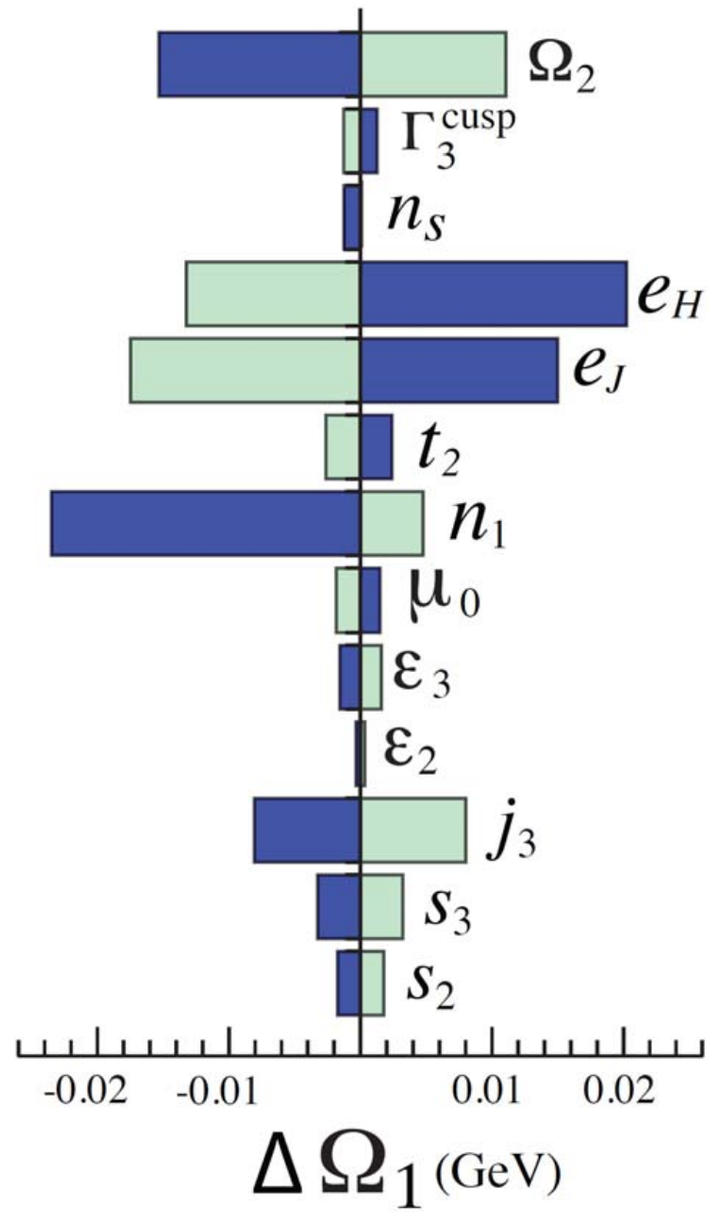
$h_3 = 8998.05$
Baikov et al

Padè approximants for range

Nonsingular statistical error

Effect of the various scan parameters





Consistency check

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} = h\left(\tau - \frac{2\Lambda}{Q}\right)$$

Perturbative expression

Power correction

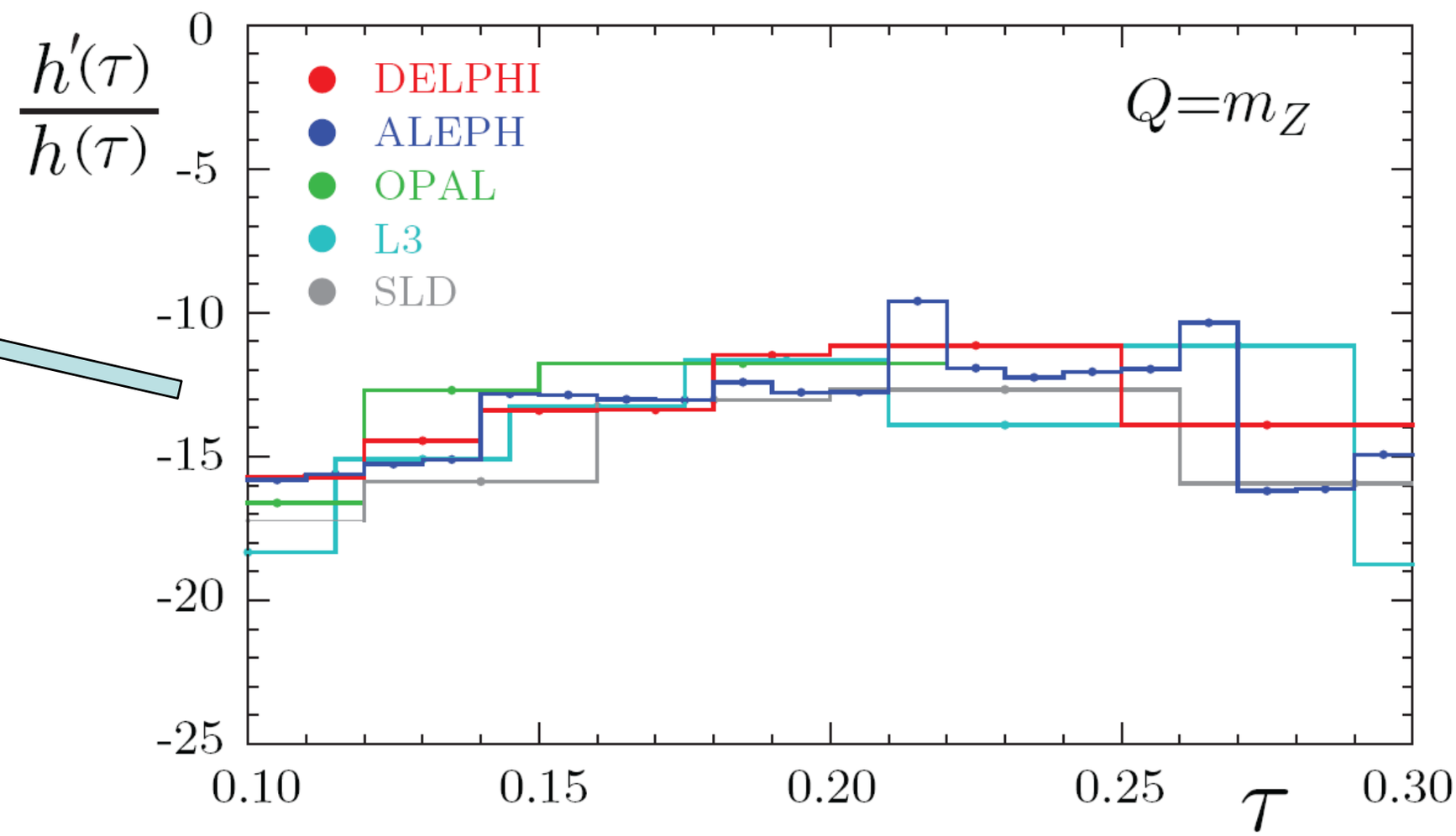
assuming that $h \sim \alpha_s$

$$\frac{\delta\alpha_s}{\alpha_s} \approx \frac{2\Lambda}{Q}\frac{h'(\tau)}{h(\tau)}$$

$$\frac{h'(\tau)}{h(\tau)} \approx -14 \pm 4$$

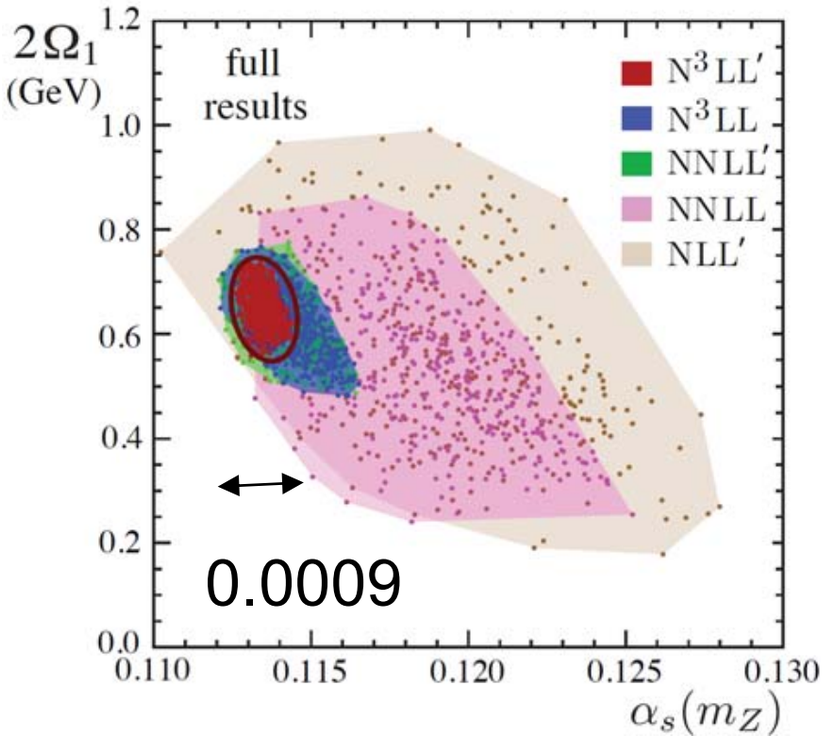
assuming $\Lambda \sim 0.3\,\text{GeV}$

$$\frac{\delta\alpha_s}{\alpha_s} \approx -(9 \pm 3)\ \%$$

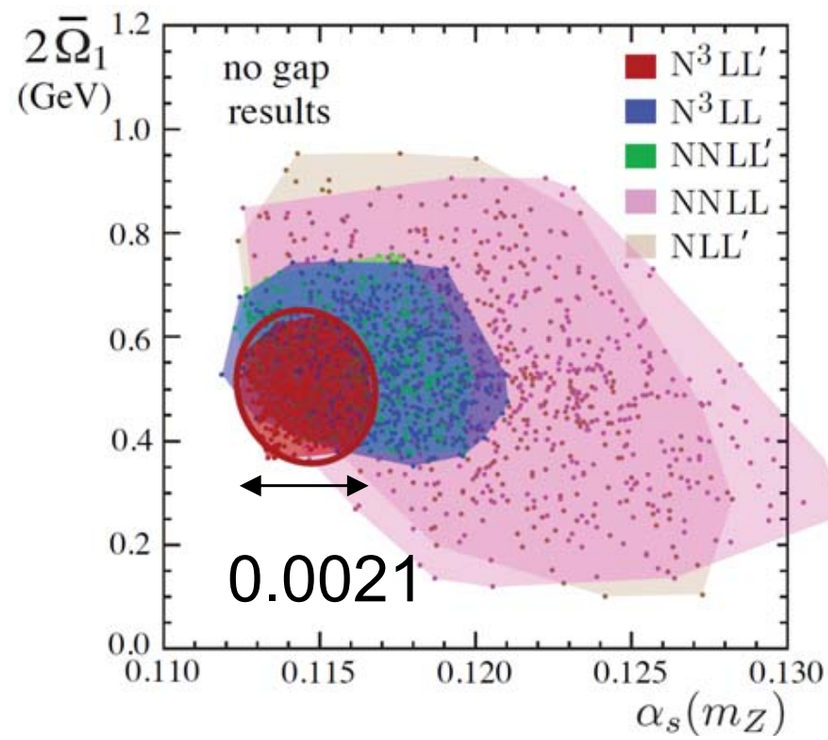


Theory uncertainty is from a flat scan

Renormalon free

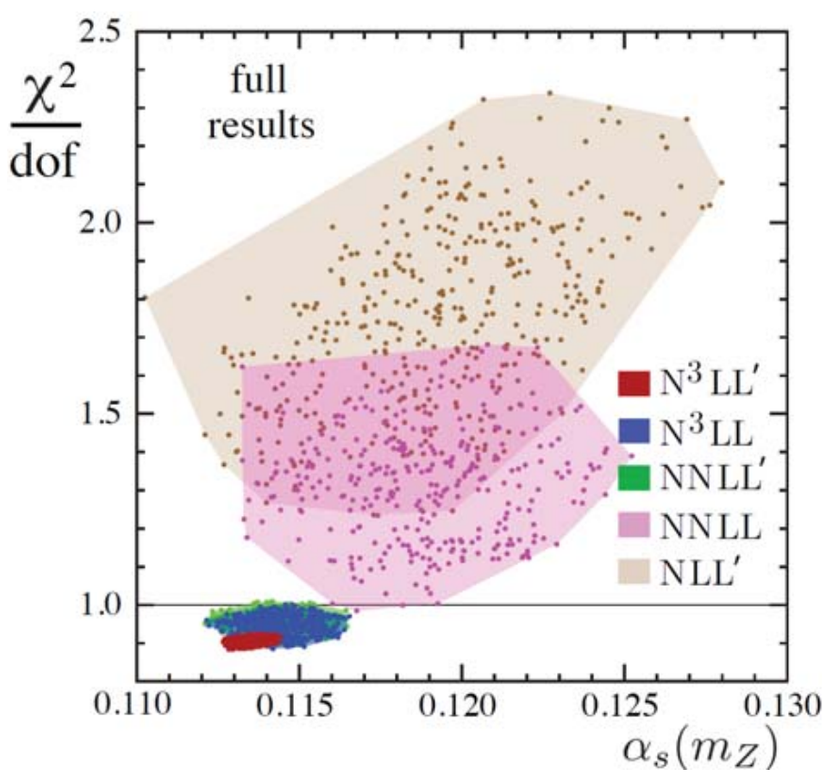


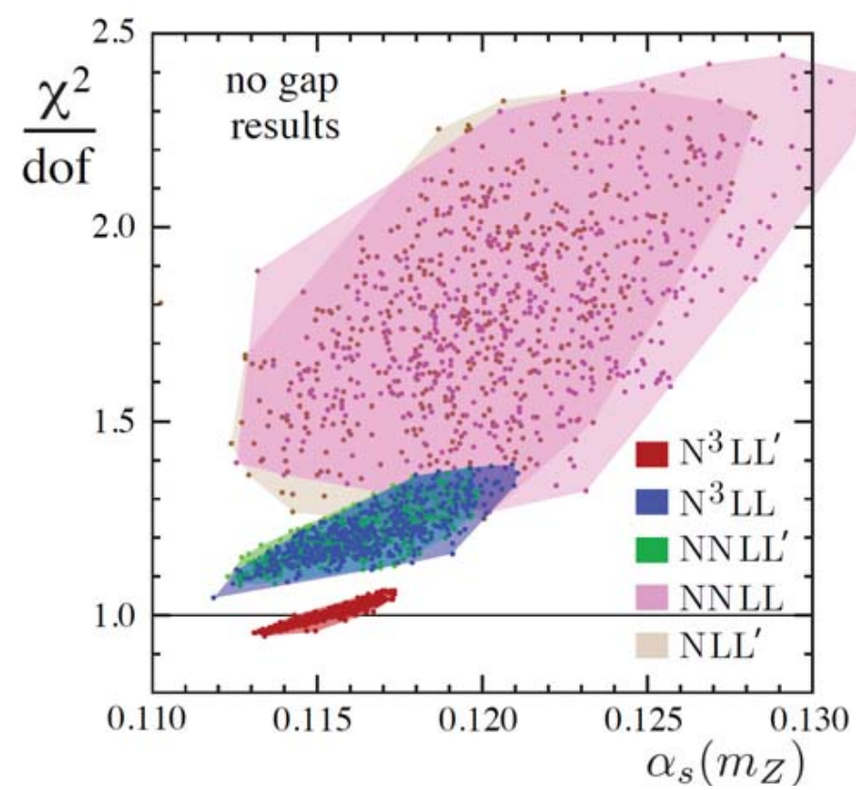
With renormalon



Renormalon-free results have smaller theory errors and better fits

Ω_1 determined to 16% accuracy





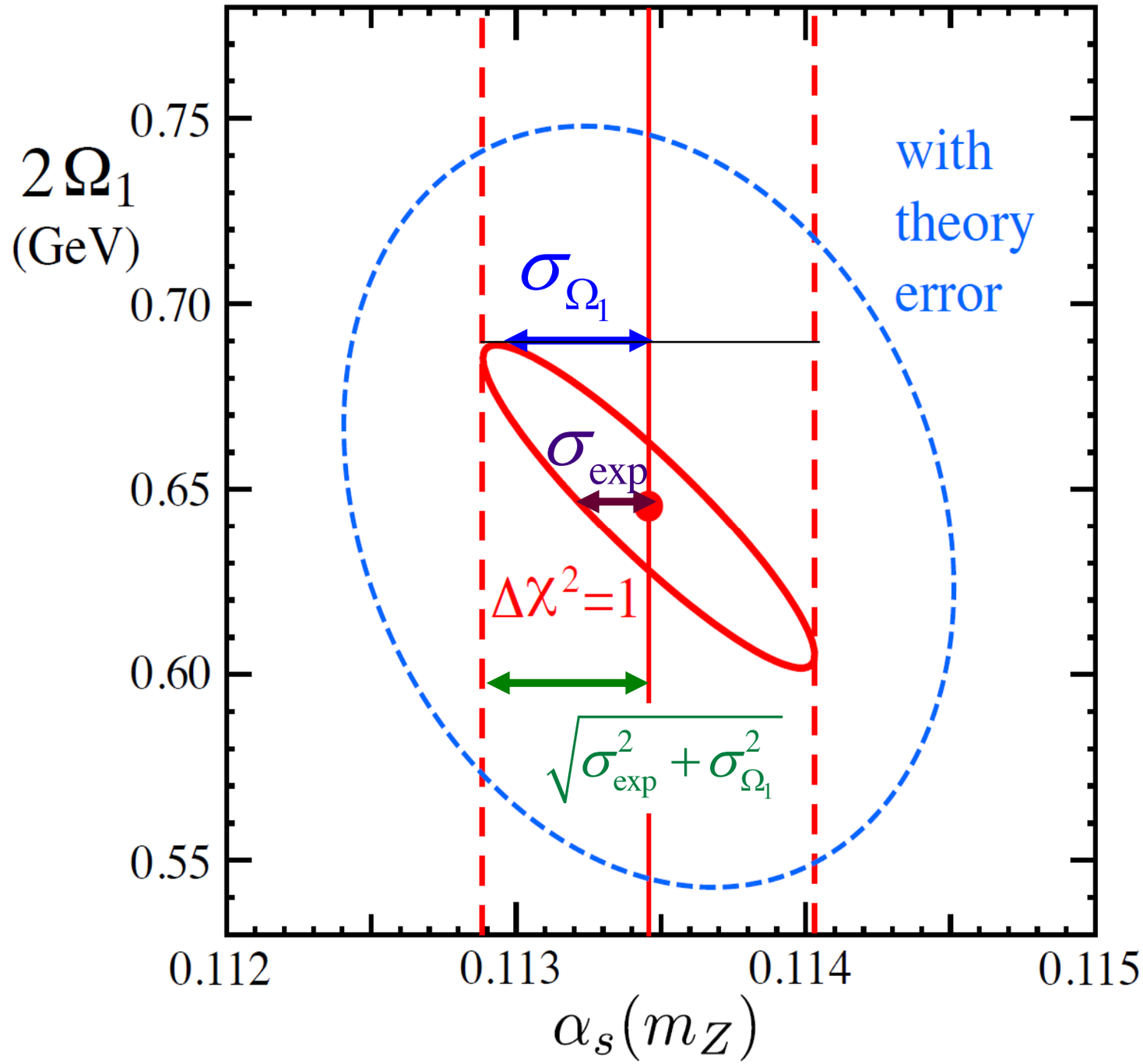
500 points random scan per order

Adding individual errors in quadrature gives similar (but smaller) error

Experimental error

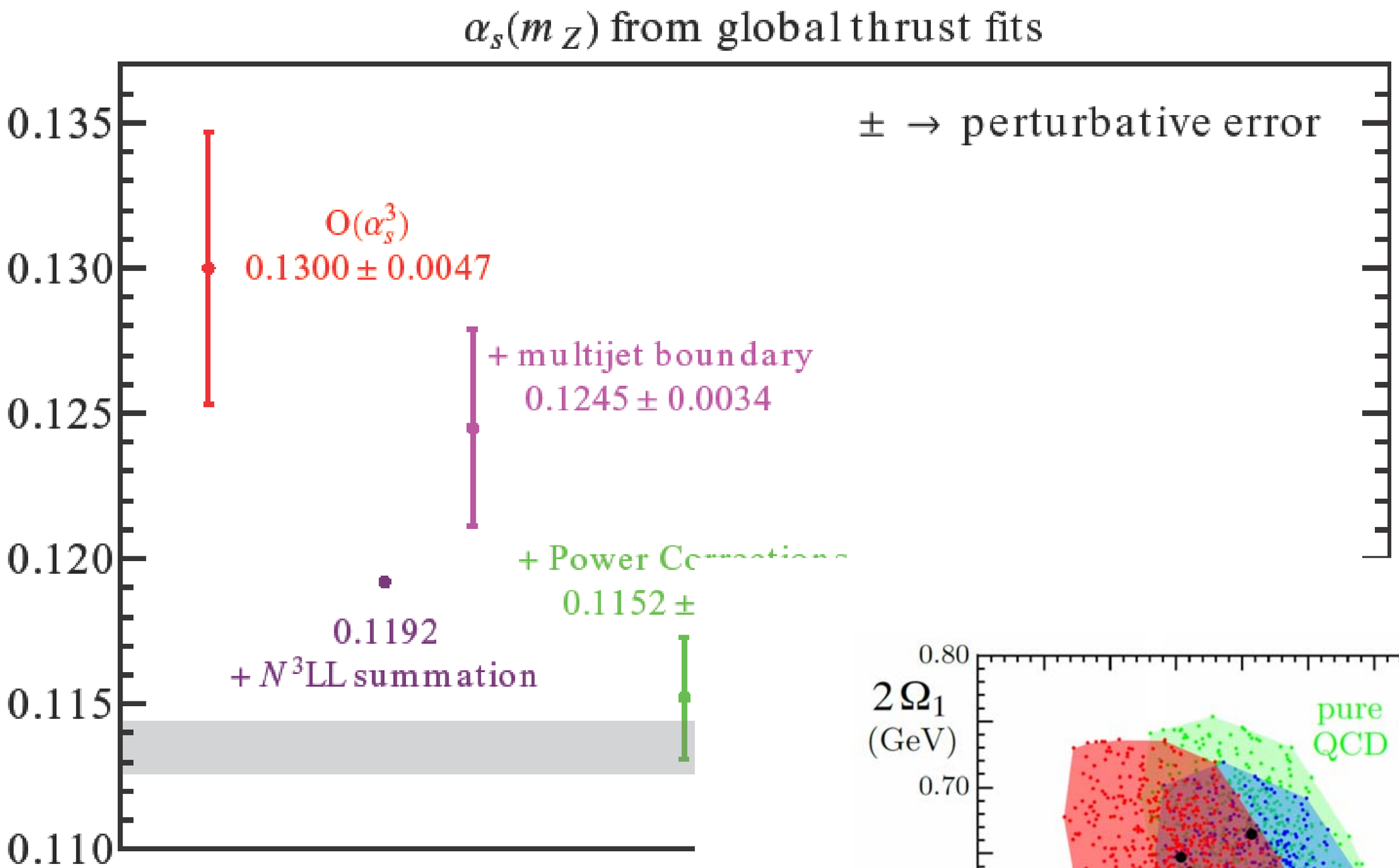
$$\frac{\chi^2}{\text{dof}} = \frac{440}{485} = 0.91$$

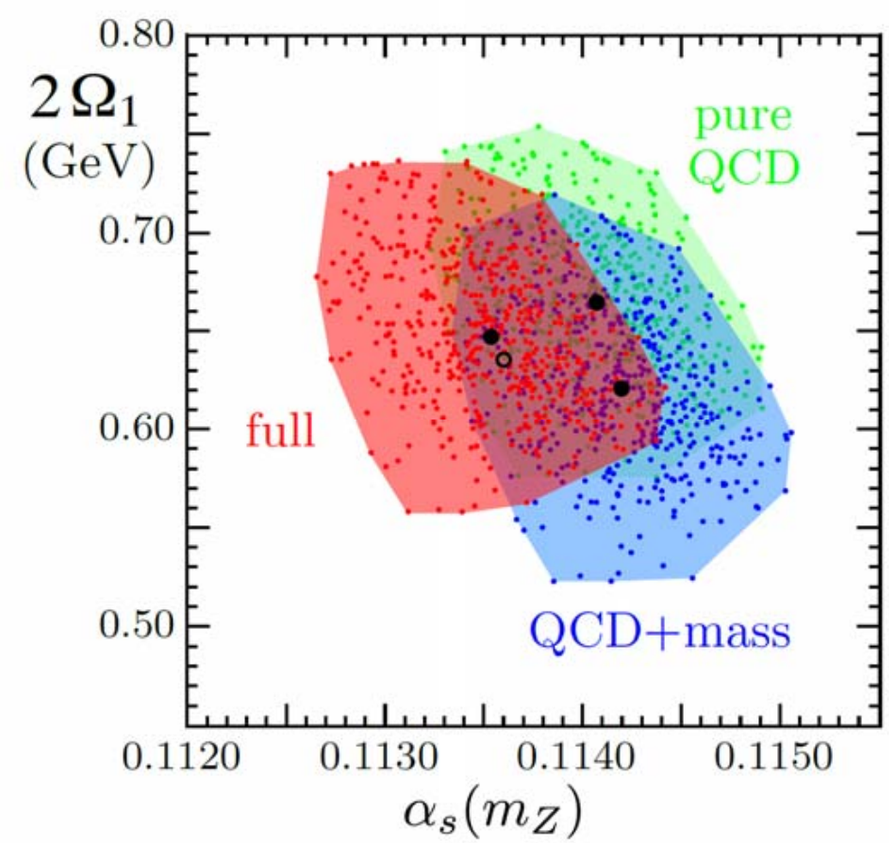
1σ (39% confidence level)



$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}}$$

mostly Ω_1, and includes Ω_2 — scan





- Resummation at N^3LL
- Multijet boundary condition
- Power correction, in a scheme free of the $O(\Lambda_{QCD})$ renormalon
- QED & bottom mass corrections