Top Quark Mass from Reconstruction

André H. Hoang

University of Vienna MPI Munich

PANIC 2011, Juli 25-29, 2011

Outline

Which Top Mass is Measured at the LHC ?

- Why the question is relevant.
- What top mass is measured (= $m_t^{\rm Pythia}$)
- What is the relation to any mass theorists know ?
  - Factorization Theorem in e+e-
  - First rough answer
  - Plans to go on …. toward LHC
- Outlook and Conclusions

Top Quark is Special !

- Heaviest known quark (related to SSB?)
- Important for quantum effects affecting many observables
- Very unstable, decays “before hadronization” $(\Gamma_t \approx 1.5\ \text{GeV})$

FERMILAB-TM-2504-E
TEVEWWG/top 2011/xx
CDF Note 10549
DØ Note 6222
July 2011

Combination of CDF and DØ results on the mass of the top quark using up to 5.8 fb^{-1} of data

The Tevatron Electroweak Working Group[1]
for the CDF and DØ Collaborations

Abstract

We summarize the top-quark mass measurements from the CDF and DØ experiments at Fermilab. We combine published Run I (1992–1996) measurements with the most precise published and preliminary Run II (2001-present) measurements using up to 5.8 fb^{-1} of data, adding new analyses (the $\not{E}_T$+Jets analysis) and updating old ones. Taking uncertainty correlations into account, and adding in quadrature the statistical and systematic uncertainties, the resulting preliminary Tevatron average mass of the top quark is $M_t = 173.2 \pm 0.9$ GeV$/c^2$.

$$M_t = 173.2 \pm 0.9\,\text{GeV}$$

~~$m_t = 172.4 \pm 1.2$ GeV~~

~~$m_t = 172.6 \pm 1.4$ GeV~~

~~$M_t = 170.9 \pm 1.8$ GeV$/c^2$~~

0.5% precision !

- How shall we theorists judge the error ?
- What is the theoretical error ?
- What mass is it ?

Concept of a Quark Mass

Quantum Field Theory:

Particles: Field-valued operators made from creation and annihilation operators

Lagrangian operators constructed using correspondence principle

Classic action: m is the rest mass

No other mass concept exists at the classic level.

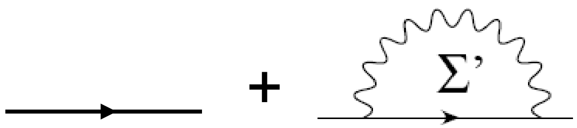
$$\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm classic} + \mathcal{L}_{\rm gauge-fix} + \mathcal{L}_{\rm ghost}$$

$$\mathcal{L}_{\rm classic} = -\frac{1}{4} F^A_{\alpha\beta} F^{\alpha\beta}_A + \sum_{\text{flavors } q} \bar{q}_\alpha (i\not{D} - m_q)_{\alpha\beta}\, q_b$$

Concept of a Quark Mass

Renormalization: UV-divergences in quantum corrections

Fields, couplings, masses in classic action are bare quantities that need to be renormalized to have (any) physical relevance



$$= \not{p} - m^0 + \Sigma(p, m^0)$$

$$m^0 \, \frac{\alpha_s}{\pi} \left[-\frac{1}{\epsilon} + \text{finite stuff} \right]$$

Mass Renormalization Schemes you know:

Pole mass: mass = classic rest mass

$$m^0 = m^{\text{pole}} + \delta m^{\text{pole}} \qquad \delta m^{\text{pole}} = \Sigma(m, m)$$

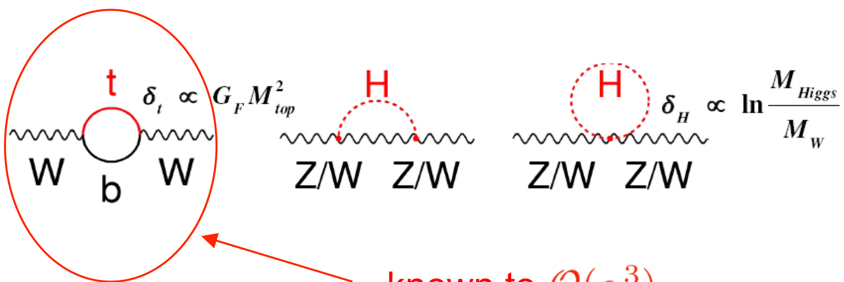
Unphysical due to confinement !

$\overline{\text{MS}}$ mass:

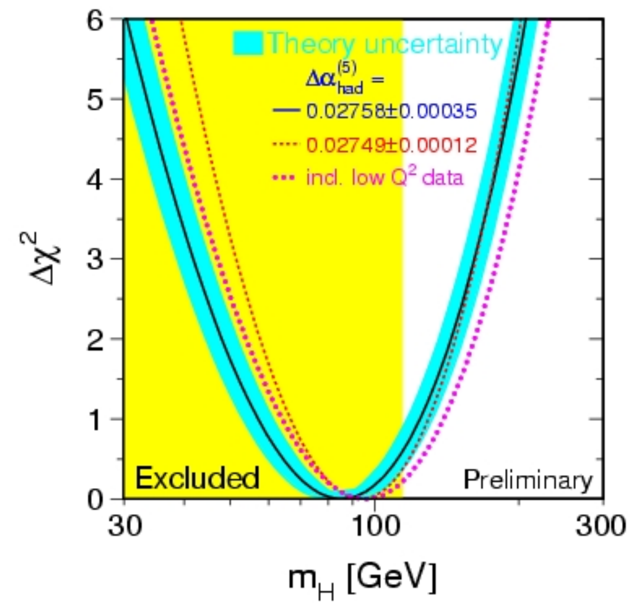
$$m^0 = \overline{m}(\mu) - \frac{\alpha_s}{\pi} \frac{1}{\epsilon}$$

Need for a precise Top mass

Fit to electroweak precision observables



known to $\mathcal{O}(\alpha_s^3)$



$$\sin\theta_W \times \left(1 + \delta(m_t, m_H, \ldots)\right)$$

$$= 1 - \frac{M_W^2}{M_Z^2}$$

$$m_t = 170.9 \pm 1.8 \text{ GeV}$$

2 GeV error: 15% change in m_H

$$m_H = 76^{+33}_{-24} \text{ GeV}$$

$$m_H < 182 \text{ GeV} \;\; (95\%\text{CL})$$

Best convergence using the $\overline{\text{MS}}$ top scheme:

$$\overline{m}_t(\overline{m}_t)$$

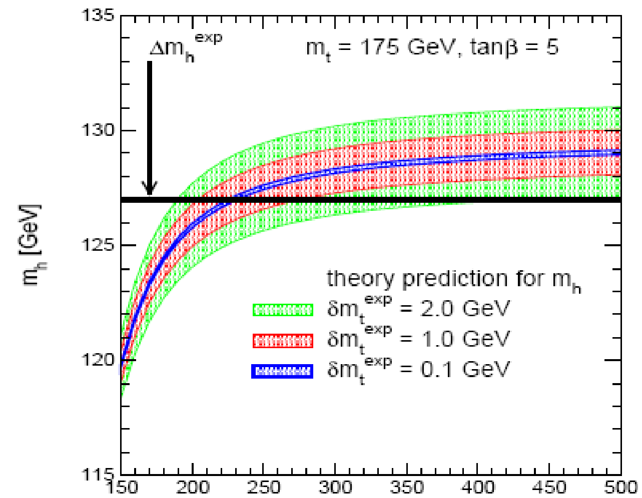
Need for a precise Top mass

Blick in die Zukunft:

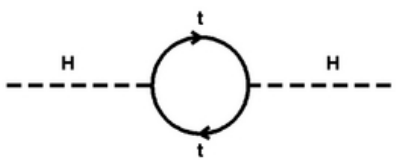
Minimales Supersymmetrisches Standard Model

5 Higgs Bosonen:

m_h (skalar, neutral)
m_H (skalar, neutral)
m_A (speudoskalar, neutral)
$m_H^{\pm}$ (geladen)



Heinemeyer, Kraml, Porod, Weiglein



$$m_h^2 \simeq M_Z^2 + \frac{G_F\, m_t^4}{\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

Corrections known to $\mathcal{O}(\alpha_s^3)$

Best convergence using the $\overline{\text{MS}}$ top scheme:

$$\overline{m}_t(\sqrt{M_{\text{SUSY}}\overline{m}_t})$$

Haber, Hempfling, Hoang

Top Quark Pole Mass

Top decay width

$$\Gamma_0 = \frac{G_F m_t^3 |V_{tb}|^2}{8\sqrt{2}\pi}$$

$$\Gamma_t(t \to bW) = \Gamma_0^{pole}[1 - 0.10\epsilon - 0.02\epsilon^2]$$

$m_t^{\rm pole}$

$$\Gamma(t \to bW) = \bar{\Gamma}_0[1 - 0.04\epsilon - 0.003\epsilon^2]$$

$\overline{m}_t(\overline{m}_t)$

Rho parameter

$$x_t \equiv 3\frac{G_F m_t^2}{8\sqrt{2}\pi^2}$$

$$\Delta\rho = x_t^{pole}[1 - 0.098\epsilon - 0.017\epsilon^2]$$

$$\Delta\rho = \bar{x}_t[1 - 0.007\epsilon - 0.007\epsilon^2]$$

Top $\overline{\text{MS}}$ mass preferred for electroweak precision fits and most top mass related predictions.

Pole mass leads to artificially large perturbative corrections.

Main Methods at Tevatron

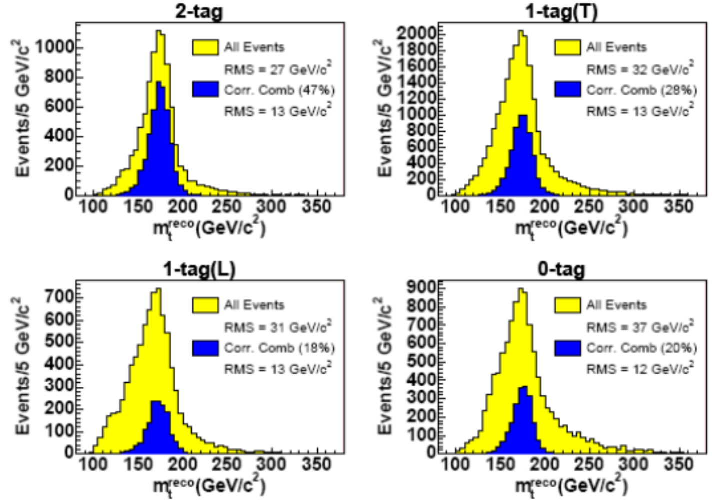
Template Method

- Principle: perform kinematic fit and reconstruct top mass event by event. E.g. in lepton+jets channel:

$$\chi^2 = \sum_{i=\ell,4jets} \frac{(p_T^{i,fit} - p_T^{i,meas})^2}{\sigma_i^2} + \sum_{j=x,y} \frac{(p_j^{UE,fit} - p_j^{UE,meas})^2}{\sigma_j^2} + \frac{(M_{\ell\nu} - M_W)^2}{\Gamma_W^2} + \frac{(M_{jj} - M_W)^2}{\Gamma_W^2} + \frac{(M_{b\ell\nu} - m_t^{reco})^2}{\Gamma_t^2} + \frac{(M_{bjj} - m_t^{reco})^2}{\Gamma_t^2}$$

Usually pick solution with lowest χ^2.

Lepton+jets (≥1 b-tag); Signal-only templates



Dynamics Method

- Principle: compute event-by-event probability as a function of m_t making use of all reconstructed objects in the events (integrate over unknowns). Maximize sensitivity by:

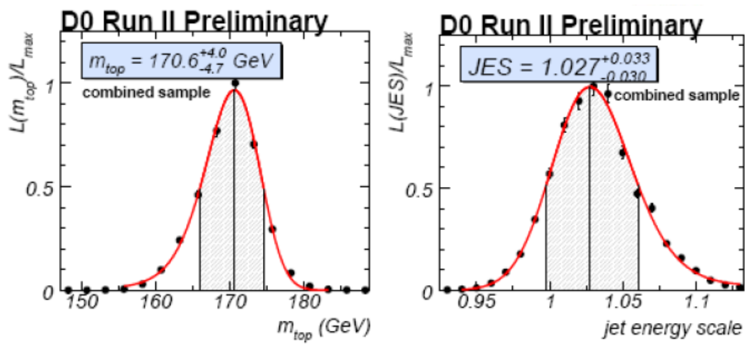
$$P(x;m_t) = \frac{1}{\sigma}\int d^n\sigma(y;m_t)\,dq_1 dq_2\, f(q_1) f(q_2)\, W(x|y)$$

parton distribution functions

differential cross section (LO matrix element)

transfer function: mapping from parton-level variables (y) to reconstructed-level variables (x)

Lepton+jets (370 pb^{-1})



Main Methods at Tevatron

Template Method

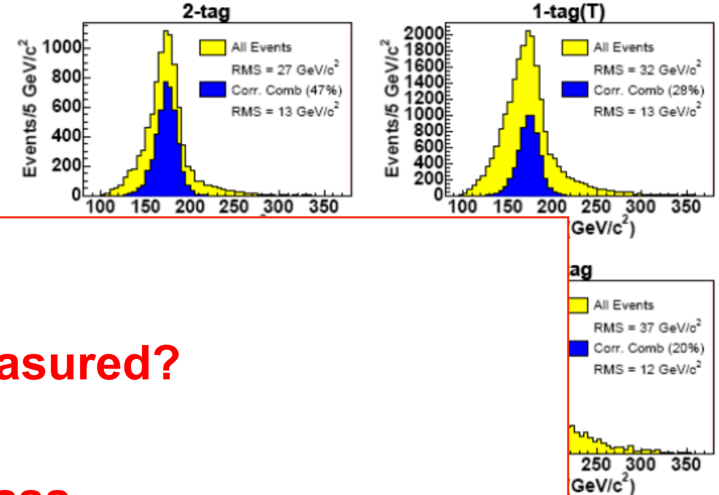
- Principle: perform kinematic fit and reconstruct top mass event by event. E.g. in lepton+jets channel:

$$\chi^2 = \sum_{i=\ell,4jets} \frac{(p_T^{i,fit} - p_T^{i,meas})^2}{\sigma_i^2} + \sum \frac{(p_j^{UE,fit} - p_j^{UE,meas})^2}{\sigma_j^2}$$

$$+ \frac{(M_{\ell\nu} - M_{\dots}}{\Gamma_W^2}$$

Usually p

Lepton+jets (≥1 b-tag); Signal-only templates



What mass is measured?

The Pythia Mass

Dynam

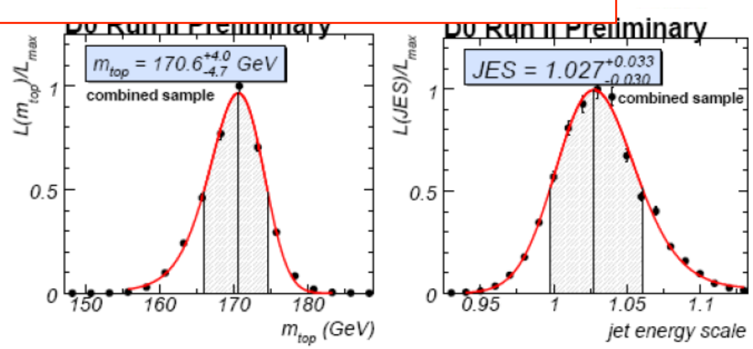
- Principle
a functio
objects in the events (integrate over unknowns).
Maximize sensitivity by:

$$P(x;m_t) = \frac{1}{\sigma}\int d^n\sigma(y;m_t)\,dq_1 dq_2\, f(q_1) f(q_2)\, W(x\,|\,y)$$

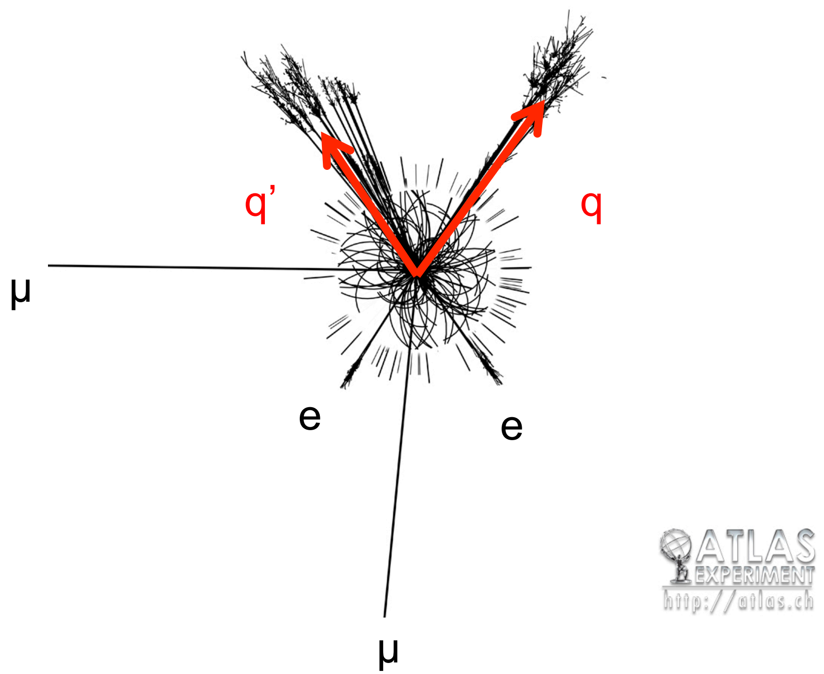
parton distribution functions

differential cross section (LO matrix element)

transfer function: mapping from parton-level variables (y) to reconstructed-level variables (x)



Description of Jets



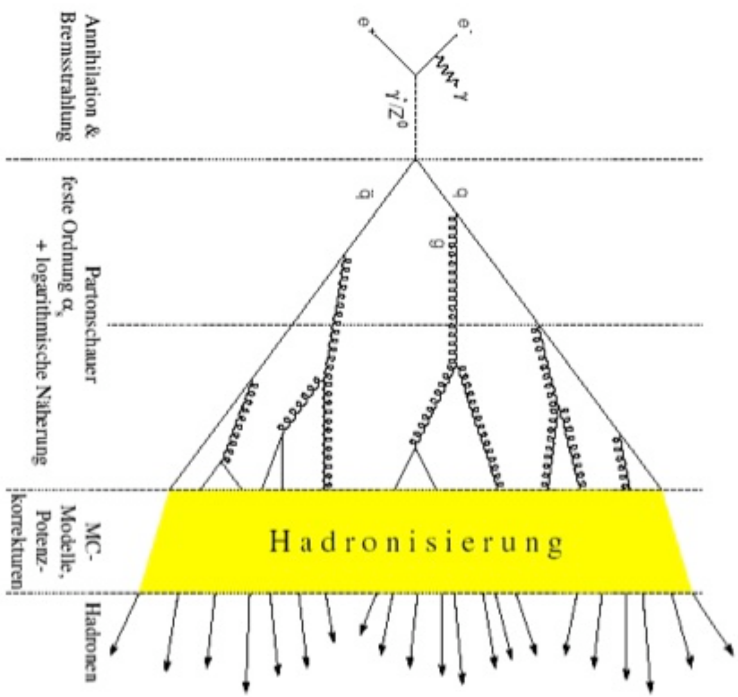
Description of Jets

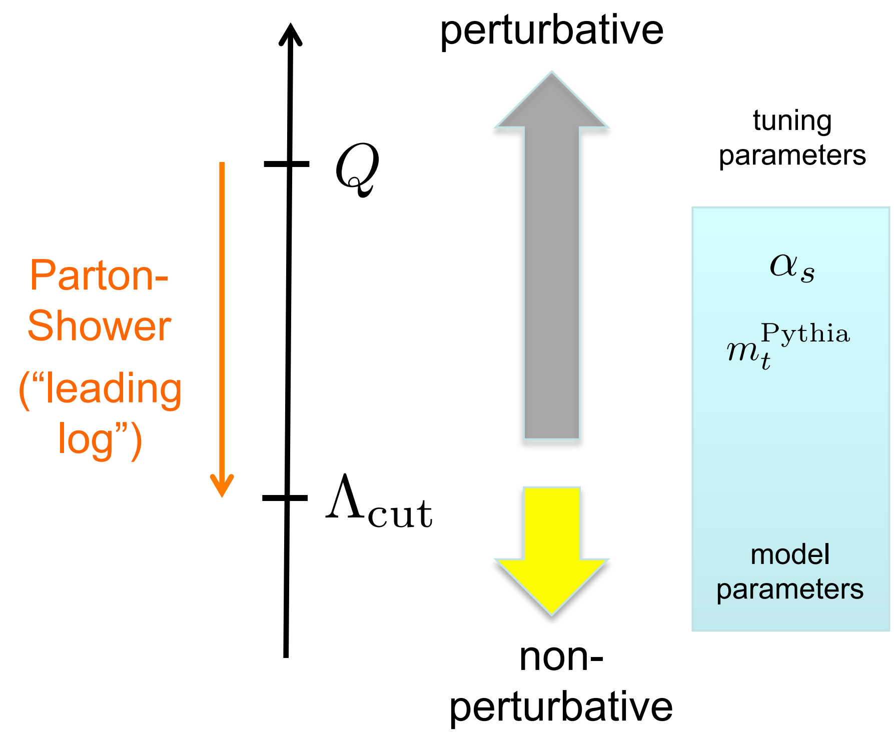
$$m_t^{\text{Pythia}} \neq m_t^{\text{pole}}$$

Monte Carlo generators:

Universal instrument to describe hadronic final states.

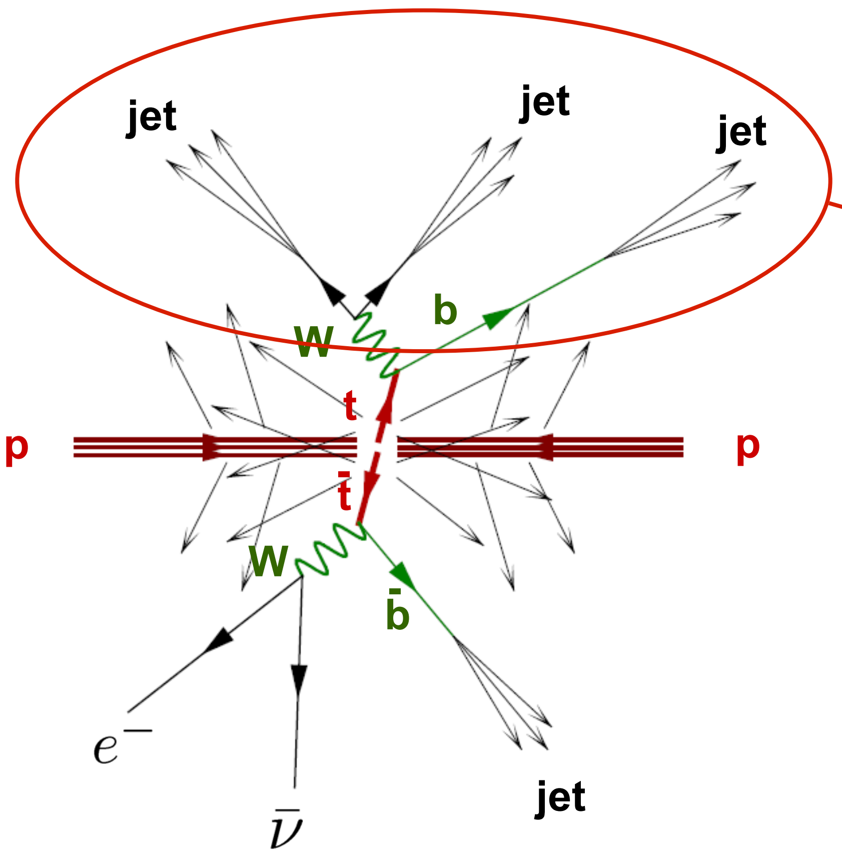
- Hadronization model and α_s are "tuned" to experimental data.





Description of Jets

LHC:



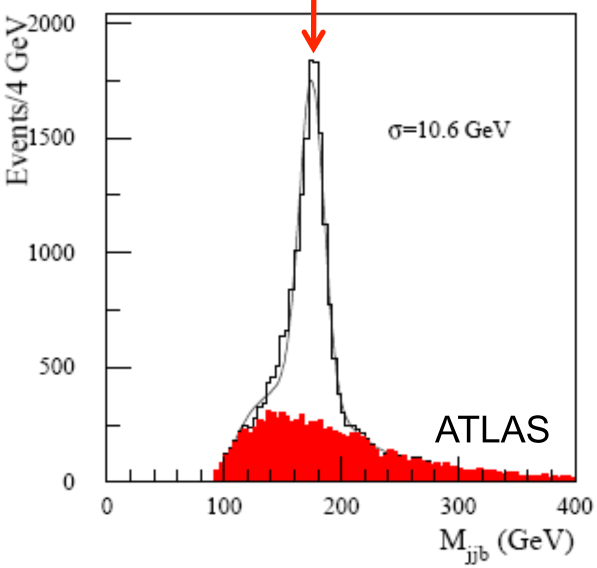
Principle of mass measurements:

Identification of the top decay products

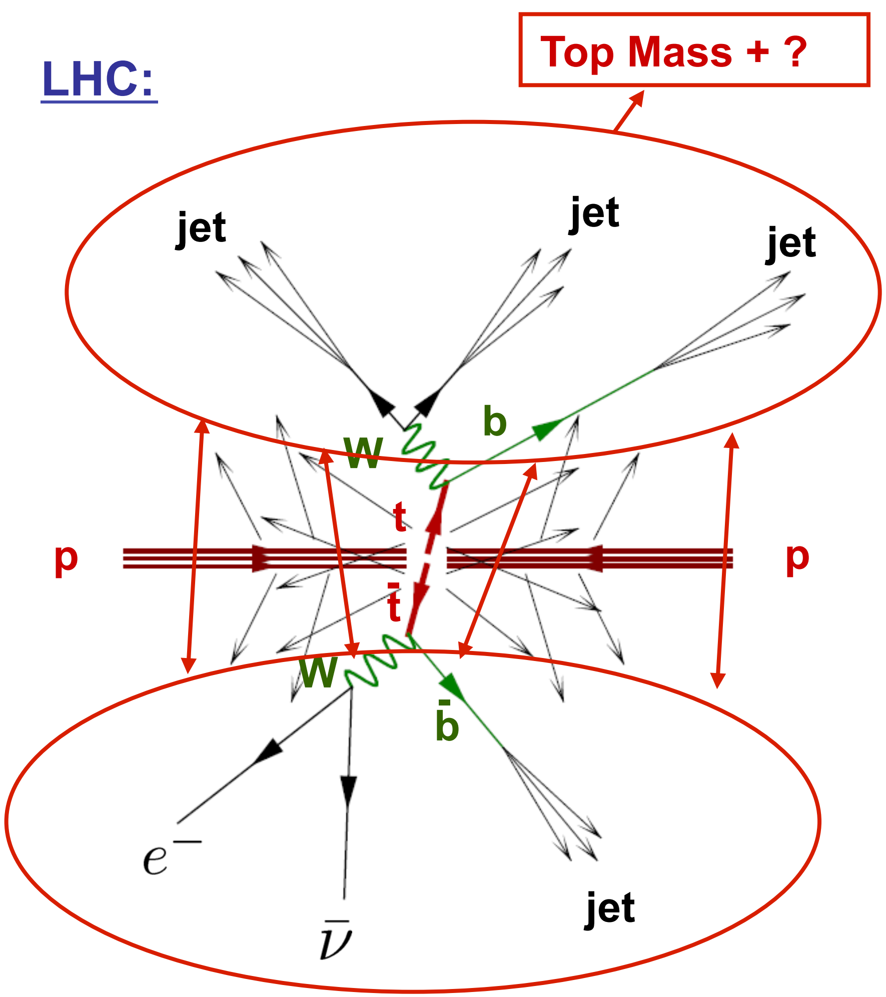
$$“\ m_{\text{top}}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2\ ”$$

Invariant mass distribution

$m_t + \Delta m_t^{\text{non-pert.}}$



Description of Jets



Principle of mass measurements:

Identification of the top decay products

$$“\ m_{\text{top}}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2\ ”$$

Problem is non-trivial !

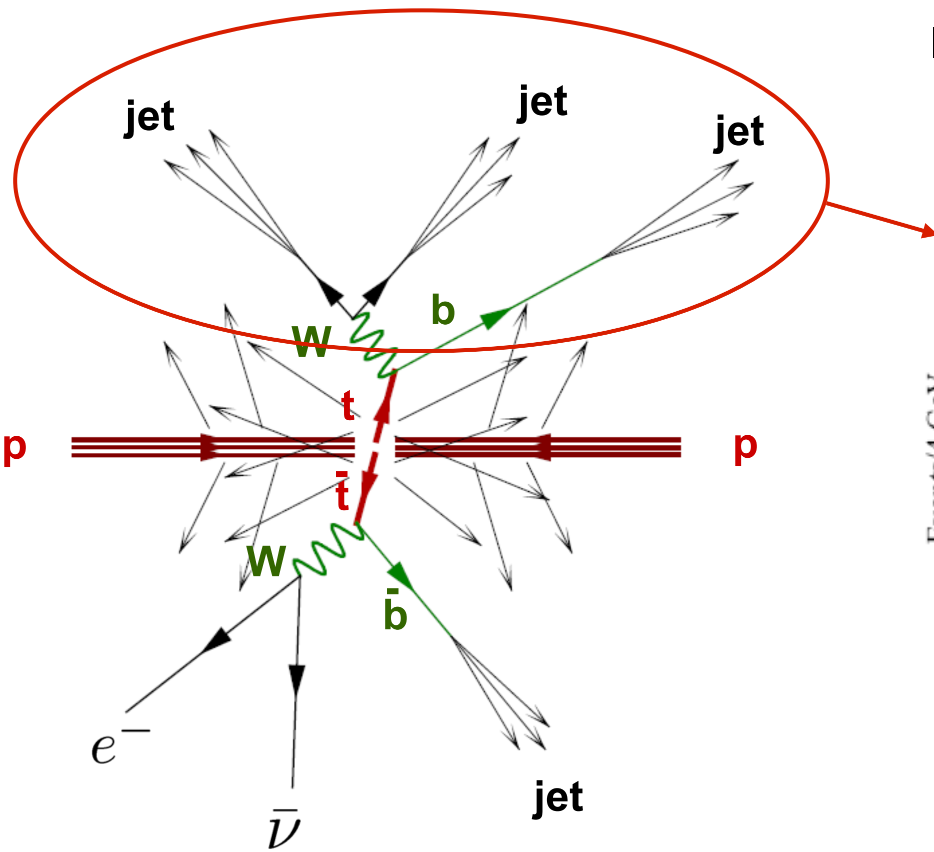
- Measured object does not exist a priori, but only through the experimental prescription for the measurement. **Quantum effects !!**

The idea of a - by itself - well defined object having a well defined mass is incorrect !!

Details and uncertainties of the parton shower and the hadronization models in den MC's influence the measured top quark mass.

Description of Jets

LHC:



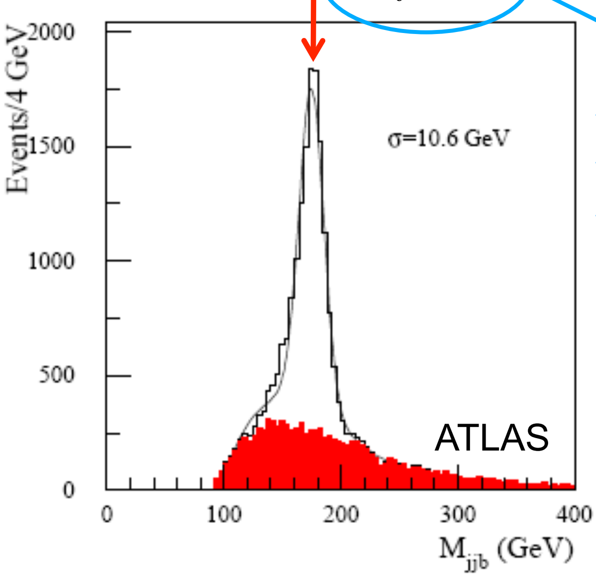
Principle of mass measurements:

Identification of the top decay products

$$\text{“}\; m_{\text{top}}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2 \;\text{”}$$

Invariant mass distribution

$m_t + \Delta m_t^{\text{non-pert}}$

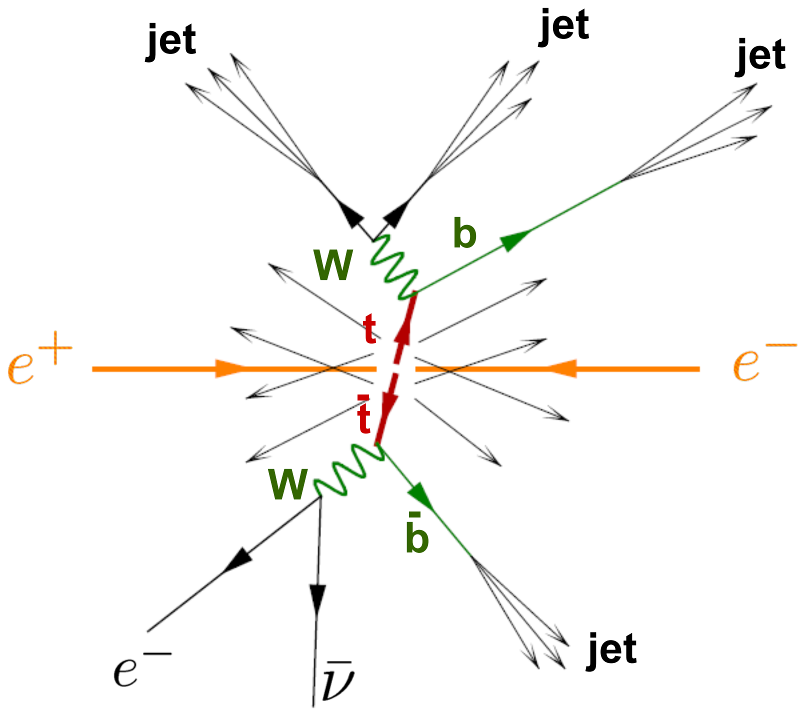


depends on:

* mass definition
* jet definition
* Monte Carlo

QCD Factorization

Top Invariant Mass Distribution:



Simpler case:

$$e^+e^- \to t\bar{t}$$

$$Q \gg m_t \quad (p_T \gg m_t)$$

→ We need: QCD factorization in the final state

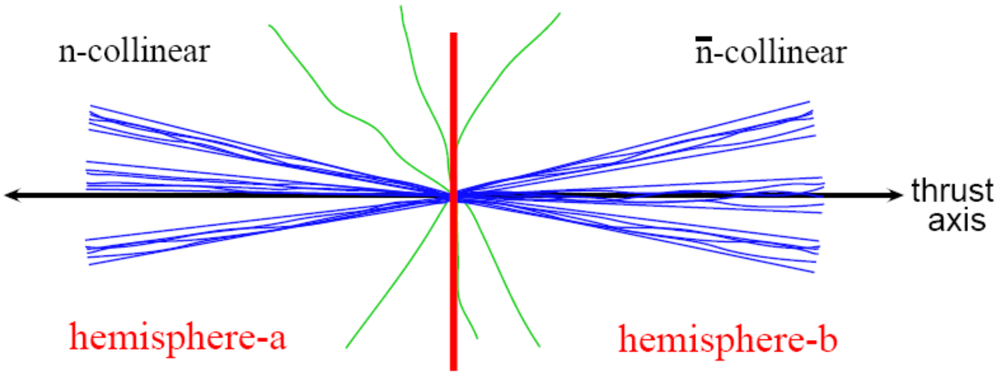
QCD Factorization

Top Invariant Mass Distribution:

Definition of the observable

Fleming, Mantry, Stewart, AHH

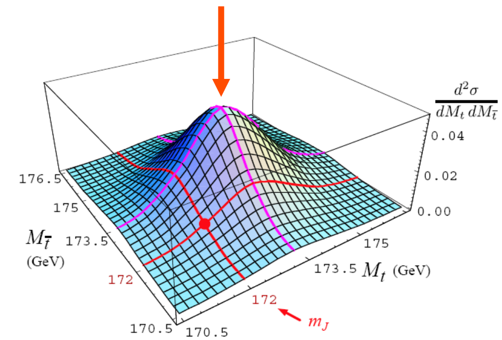
Phys.Rev.D77:074010,2008
Phys.Rev.D77:114003,2008
Phys.Lett.B660:483-493,2008



$$M_t^2 = \left(\sum_{i\,\epsilon\,a} p_i^\mu\right)^2 \qquad M_{\bar t}^2 = \left(\sum_{i\,\epsilon\,b} p_i^\mu\right)^2$$

resonance region:

$$M_{t,\bar t} - m_t \sim \Gamma$$



$$\frac{d^2\sigma}{dM_t\,dM_{\bar t}}$$ Double differential hemisphere mass distribution

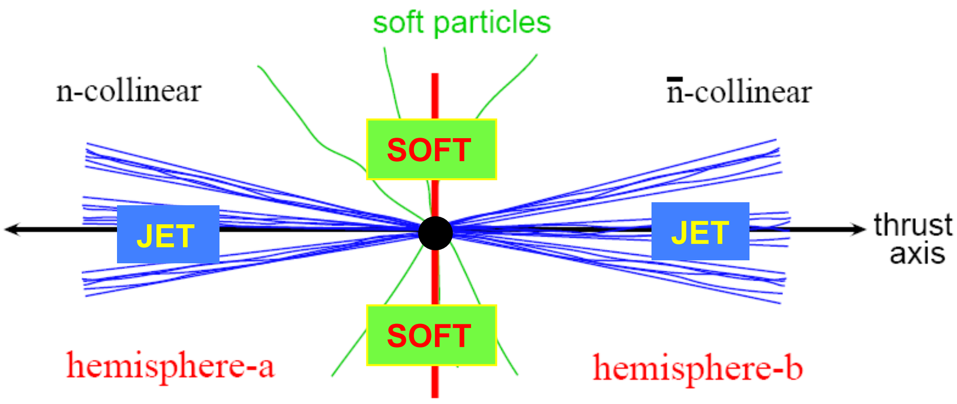
QCD Factorization

Fleming, Mantry, Stewart, Hoang

Phys.Rev.D77:074010,2008
Phys.Rev.D77:114003,2008
Phys.Lett.B660:483-493,2008

$$Q \gg m_t \gg \Gamma_t > \Lambda_{\rm QCD}$$

- Soft-Collinear-Effective Theory
- Heavy Quark Effective Theory
- Unstable Particle Effective Theory



$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^- \, B_+\left(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

$$\hat s = \frac{M_t^2 - m_J^2}{m_J}$$

JET JET SOFT

QCD Factorization

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Jet functions:

$$B_+(\hat s, \Gamma_t, \mu) = {\rm Im}\left[\frac{-i}{12\pi m_J}\int d^4x\, e^{ir.x}\, \langle 0|\, T\,\{\bar h_{v_+}(0) W_n(0)\, W_n^\dagger(x) h_{v_+}(x)\,\}\, |0\rangle\right]$$

- perturbative
- dependent on mass, width, color charge

$$B_\pm^{\rm Born}(\hat s, \Gamma_t) = \frac{1}{\pi m_t}\frac{\Gamma_t}{\hat s^2 + \Gamma_t^2} \qquad \hat s = \frac{M^2 - m_t^2}{m_t}$$

Soft function:

$$S_{\rm hemi}(\ell^+, \ell^-, \mu) = \frac{1}{N_c}\sum_{X_s} \delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger(0)|0\rangle$$

- non-perturbative
- analogous to the pdf's
- dependent on color charge, kinematics

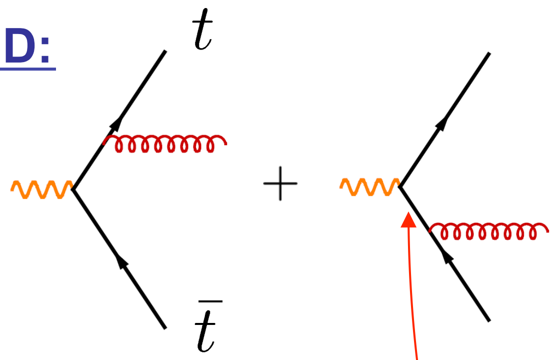
Independent of the mass !

⟶ event shapes

$k_+ = k_0 - k_3$
$k_- = k_0 + k_3$

QCD Factorization

full QCD:

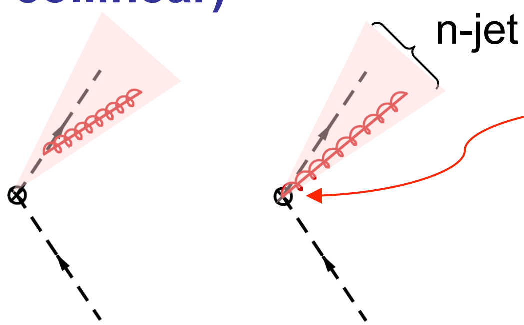


3 phase space regions: $\lambda \sim m_t/Q$

- n-collinear: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, 1, \lambda)$
- n̄-collinear: $(k_+, k_-, k_\perp) \sim Q(1, \lambda^2, \lambda)$
- soft: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2)$

$$\frac{1}{(p_{\bar{t}}+k)^2 - m_t^2} \qquad (p_{\bar{t}}^2 \approx m_t^2\,,\ \ \bar{n}^2 = 0)$$

Gluon collinear to the top:
(n-collinear)



$$\frac{1}{Q\ \ \bar{n}.k}$$

$$W_n^\dagger(\infty, x) = \mathrm{P}\ \exp\left(ig\int_0^\infty ds\bar{n}\cdot A_+(ns+x)\right)$$

$h_{v_+}(x)$ → **gauge dependent**

$W_n^\dagger(\infty, x)\, h_{v_+}(x)$ → **gauge independent**

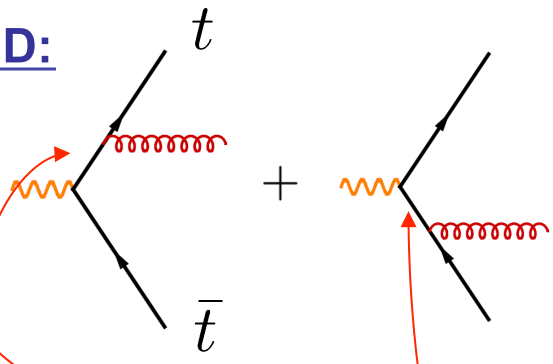
$$B_+(\hat{s}, \Gamma_t, \mu)\ =\ \mathrm{Im}\left[\frac{-i}{12\pi m_J}\int d^4x\, e^{ir.x}\,\langle 0|\, T\,\{\bar{h}_{v_+}(0) W_n(0)\, W_n^\dagger(x) h_{v_+}(x)\,\}\,|0\rangle\right]$$

$k_+ = k_0 - k_3$

$k_- = k_0 + k_3$

QCD Factorization

full QCD:



3 phase space regions: $\lambda \sim m_t/Q$

- n-collinear: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, 1, \lambda)$
- n̄-collinear: $(k_+, k_-, k_\perp) \sim Q(1, \lambda^2, \lambda)$
- soft: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2)$

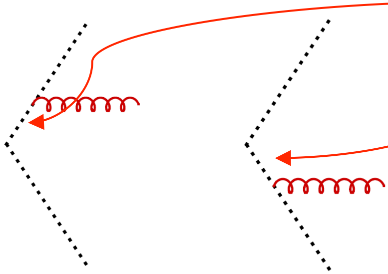
$$\frac{1}{(p_{t,\bar{t}}+k)^2 - m_t^2}$$

$$(p_{t,\bar{t}}^2 \approx m_t^2\,,\ \ n^2 = 0\,,\ \ \bar{n}^2 = 0)$$

Gluon soft:

$$\frac{1}{Q\ \ n.k}$$

$$\frac{1}{Q\ \ \bar{n}.k}$$



$$Y_n(x) = \overline{\mathrm{P}}\ \exp\Big(-ig \int_0^\infty ds\, n\cdot A_s(ns+x)\Big)$$

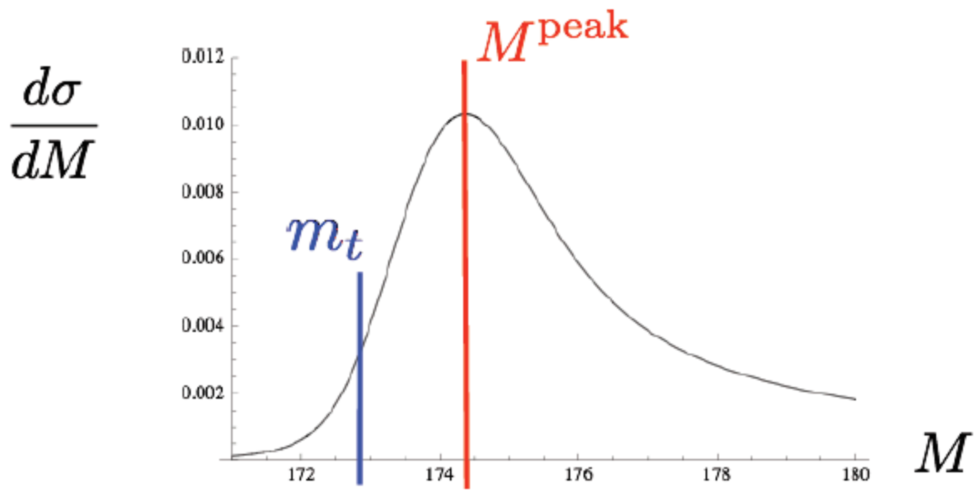
$$\overline{Y_{\bar{n}}}(x) = \overline{\mathrm{P}}\ \exp\Big(-ig \int_0^\infty ds\, \bar{n}\cdot \overline{A}_s(\bar{n}s+x)\Big)$$

$$S_{\mathrm{hemi}}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \langle 0|\, (\overline{Y}_{\bar{n}})^{cd}\, (Y_n)^{ce}(0)\, \delta(\ell^- - (\hat{P}_a^+)^\dagger)\delta(\ell^- - \hat{P}_b^-)\, (Y_n^\dagger)^{ef} (\overline{Y}_{\bar{n}}^\dagger)^{df}(0)\, |0\rangle$$

QCD Factorization

$$k_+ = k_0 - k_3$$
$$k_- = k_0 + k_3$$

Fleming, Mantry, Stewart, AHH
Phys.Rev.D77:074010,2008



$$M^{\rm peak} = m_t + \Gamma_t(\alpha_s + \alpha_s^2 + \ldots) + \frac{Q}{m_t}\,\Omega_1 + \mathcal{O}\left(\frac{m_t \Lambda_{\rm QCD}}{Q}\right)$$

first moment of the soft function: $\Omega_1 = \int d\ell\, \ell\, S(\ell, \mu)$

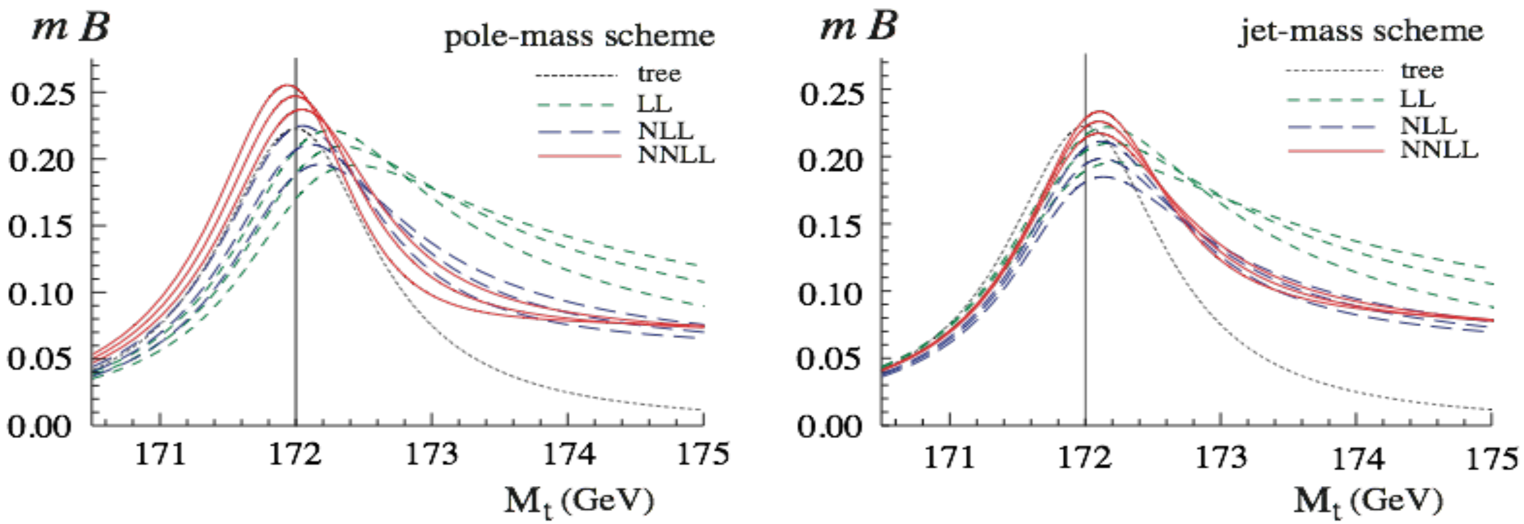
QCD Factorization

Higher Orders & Top Mass Scheme:

Fleming, Mantry, Stewart, AHH
Phys.Rev.D77:074010,2008

$$\mathcal{B}_\pm(\hat{s},0,\mu,\delta m) = -\frac{1}{\pi m}\,\frac{1}{\hat{s}+i0}\left\{1+\frac{\alpha_s C_F}{4\pi}\left[4\ln^2\left(\frac{\mu}{-\hat{s}-i0}\right)+4\ln\left(\frac{\mu}{-\hat{s}-i0}\right)+4+\frac{5\pi^2}{6}\right]\right\}$$

Jain, Scimemi, Stewart
PRD77, 094008(2008)



$$m_{\rm pole} = m_J(\mu) + e^{\gamma_E} R\frac{\alpha_s(\mu)\,C_F}{\pi}\left[\ln\frac{\mu}{R}+\frac{1}{2}\right]+\mathcal{O}(\alpha_s^2)$$

$$R \sim \Gamma_t$$

Theory Issues for $pp \to t\bar{t} + X$

- ★ definition of jet observables → Hadron event shapes
- ★ initial state radiation
- ★ final state radiation
- • underlying events → Soft function ?
- ★ color reconnection & soft gluon interactions
- ★ beam remnant
- ★ parton distributions
- ★ summing large logs $Q \gg m_t \gg \Gamma_t$
- ★ relation to Lagrangian short distance mass

$$T_{\perp,g} \equiv \max_{\vec{n}_T} \frac{\sum_i |\vec{q}_{\perp i} \cdot \vec{n}_T|}{\sum_i q_{\perp i}}$$

Banfi, Salam, Zanderighi

★ **Can be addressed in the framework of a LC.**

★ **Requires extensions of LC concepts and other known concepts**

MC Top Mass

→ Use analogies between MC set up and factorization theorem

Final State Shower

- Start: at transverse momenta of primary partons, evolution to smaller scales.
- Shower cutoff $R_{sc} \sim 1\ \mathrm{GeV}$
- Hadronization models fixed from reference processes

Factorization Theorem

- Renormalization group evolution from transverse momenta of primary partons to scales in matrix elements.
- Subtraction in jet function that defines the mass scheme
- Soft function model extracted from another process with the same soft function

Additional Complications:

Initial state shower, underlying events, combinatorial background, etc

} Let's assume that these aspects are treated correctly in the MC

MC Top Mass

Conclusion (quick answer):

constant of order unity

$$m_t^{\rm MC}(R_{sc}) \,=\, m_t^{\rm pole} \,-\, R_{sc}\, c \left[\frac{\alpha_s}{\pi}\right]$$

Determination of the MSbar mass:

$$m_t^{\rm TeV} \,=\, m_t^{\rm MC}(R_{sc}) \,=\, 172.6 \,\pm\, 0.8(stat) \,\pm\, 1.1(syst)$$

3-loop R-evolution equation

AHH,Jain, Scimemi, Stewart
PRL 101,151602(2008)

$$\overline{m}_t(\overline{m}_t) \,=\, 163.0 \,\pm\, 1.3\,^{+0.6}_{-0.3}\ {\rm GeV} \qquad (c = 3^{+6}_{-2})$$

More systematic study needed for final answer!

The exercise just carried out does not account for possible conceptual uncertainties!

Outlook & Conclusion

Conclusion:

→ Current top mass measurements from the Tevatron refer to the top mass parameter in Pythia m_t^{Pythia} .

$m_t^J(2\ \mathrm{GeV})$

→ For a high energy Linear Collider we have a factorization theorem to do MC independent short-distance Lagrangian top mass measurements (jet mass)

→ The analogy between MC generators and factorization theorem indicates that the m_t^{Pythia} is a short-distance mass like the jet mass (and not the pole mass).

m_t^{Pythia}	$m_t^J(2\ \mathrm{GeV})$		
	1-loop	2-loop	3-loop
160.00			
165.00			
170.00			

Plans:

→ "Measure" the m_t^{Pythia} in terms of the Jet mass $m_t^J(2\ \mathrm{GeV})$ using thrust and other event shapes

→ Derivation of eventshape-like factorization theorems for Tevatron/LHC

→ "Measure" m_t^{Pythia} for LHC-Pythia