Top Quark Mass from Reconstruction

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Outline

Which Top Mass is Measured at the LHC?

• Why the question is relevant.
• What top mass is measured (= $m_t^\text{Pythia}$)
• What is the relation to any mass theorists know?
  - Factorization Theorem in e+e-
  - First rough answer
  - Plans to go on .... toward LHC

• Outlook and Conclusions
Top Quark is Special!

- Heaviest known quark (related to SSB?)
- Important for quantum effects affecting many observables
- Very unstable, decays “before hadronization” ($\Gamma_t \approx 1.5$ GeV)

Combination of CDF and DØ results on the mass of the top quark using up to 5.8 $\text{fb}^{-1}$ of data

The Tevatron Electroweak Working Group$^1$
for the CDF and DØ Collaborations

$$M_t = 173.2 \pm 0.9 \text{ GeV}$$

$$m_t = 172.4 \pm 1.2 \text{ GeV}$$

$$m_t = 172.6 \pm 1.4 \text{ GeV}$$

$$M_t = 170.9 \pm 1.8 \text{ GeV}/c^2$$

0.5% precision!

- How shall we theorists judge the error?
- What is the theoretical error?
- What mass is it?
Concept of a Quark Mass

Quantum Field Theory: Particles: Field-valued operators made from creation and annihilation operators

Lagrangian operators constructed using correspondence principle

Classic action: \( m \) is the rest mass

No other mass concept exists at the classic level.

\[
\mathcal{L}_{QCD} = \mathcal{L}_{\text{classic}} + \mathcal{L}_{\text{gauge-fix}} + \mathcal{L}_{\text{ghost}}
\]

\[
\mathcal{L}_{\text{classic}} = -\frac{1}{4} F_{\alpha\beta}^A F_A^{\alpha\beta} + \sum_{\text{flavors } q} \bar{q}_\alpha (i\not{D} - m_q)_{\alpha\beta} q_b
\]
Concept of a Quark Mass

Renormalization: UV-divergences in quantum corrections

Fields, couplings, masses in classic action are bare quantities that need to be renormalized to have (any) physical relevance

\[ \phi - m^0 + \Sigma(p, m^0) \]

Mass Renormalization Schemes you know:

Pole mass: mass = classic rest mass

\[ m^0 = m^{\text{pole}} + \delta m^{\text{pole}} \]

\[ \delta m^{\text{pole}} = \Sigma(m, m) \]

MS mass: 

\[ m^0 = \bar{m}(\mu) - \frac{\alpha_s}{\pi} \frac{1}{\epsilon} \]

Unphysical due to confinement!
Need for a precise Top mass

Fit to electroweak precision observables

\[
\Delta \chi^2 = \ln \frac{M_{\text{Higgs}}}{M_W}
\]

\[
\sin \theta_W \times \left(1 + \delta(m_t, m_H, \ldots)\right) = 1 - \frac{M_W^2}{M_Z^2}
\]

\[
m_H = 76^{+33}_{-24} \text{ GeV}
\]

\[
m_H < 182 \text{ GeV (95\% CL)}
\]

\[
m_t = 170.9 \pm 1.8 \text{ GeV}
\]

2 GeV error: 15\% change in \(m_H\)

Best convergence using the \(\overline{\text{MS}}\) top scheme:

\[
\overline{m}_t(\overline{m}_t)
\]
Need for a precise Top mass

Blick in die Zukunft:

Minimales Supersymmetrisches Standard Model

5 Higgs Bosonen:

$m_h$ (skalar, neutral)
$m_H$ (skalar, neutral)
$m_A$ (speudoskalar, neutral)
$m_H^\pm$ (geladen)

\[
m_h^2 \simeq M_Z^2 + \frac{G_F m_t^4}{\pi^2 \sin^2 \beta} \ln \left( \frac{m_{t_1} m_{t_2}}{m_t^2} \right)
\]

Corrections known to $O(\alpha_s^3)$

Best convergence using the MS top scheme:

$\overline{m}_t \left( \sqrt{M_{SUSY}} \overline{m}_t \right)$

Haber, Hempfling, Hoang
Top Quark Pole Mass

Top decay width

\[ \Gamma_t(t \rightarrow bW) = \Gamma_0^{pole} [1 - 0.10\varepsilon - 0.02\varepsilon^2] \]

\[ \Gamma(t \rightarrow bW) = \bar{\Gamma}_0 [1 - 0.04\varepsilon - 0.003\varepsilon^2] \]

Rho parameter

\[ \Delta \rho = x_t^{pole} [1 - 0.098\varepsilon - 0.017\varepsilon^2] \]

\[ \Delta \rho = \bar{x}_t [1 - 0.007\varepsilon - 0.007\varepsilon^2] \]

Top MS mass preferred for electroweak precision fits and most top mass related predictions.

Pole mass leads to artificially large perturbative corrections.
Main Methods at Tevatron

**Template Method**

- **Principle**: perform kinematic fit and reconstruct top mass event by event. E.g. in lepton+jets channel:

\[
\chi^2 = \sum_{i=L,E,jets} \frac{(p_{iT,i,fit} - p_{iT,i,meas})^2}{\sigma_i^2} + \sum_{j=x,y} \frac{(p_{j,i,UE,fit} - p_{j,i,UE,meas})^2}{\sigma_j^2} + \frac{(M_{W} - M_{W})^2}{\Gamma_W^2} + \frac{(M_{j,j} - M_{W})^2}{\Gamma_W^2} + \frac{(M_{bt,b} - m_{t}^{reco})^2}{\Gamma_t^2} + \frac{(M_{bj,j} - m_{j}^{reco})^2}{\Gamma_j^2}
\]

Usually pick solution with lowest \(\chi^2\).

**Dynamics Method**

- **Principle**: compute event-by-event probability as a function of \(m_t\) making use of all reconstructed objects in the events (integrate over unknowns). Maximize sensitivity by:

\[
P(x;m_t) = \frac{1}{\sigma} \int d^8 \sigma(y;m_t) dq_1 dq_2 f(q_1) f(q_2) W(x | y)
\]

**Lepton+jets (≥1 b-tag); Signal-only templates**

**Lepton+jets (370 pb⁻¹)**
Main Methods at Tevatron

Template Method

- **Principle**: perform kinematic fit and reconstruct top mass event by event. E.g. in lepton+jets channel:

\[
\chi^2 = \sum_{i \in \ell, 4j, ets} \frac{(p_T^{i, \text{fit}} - p_T^{i, \text{meas}})^2}{\sigma_i^2} + \sum \left( \frac{(p_T^{UE, \text{fit}} - p_T^{UE, \text{meas}})^2}{\sigma_{UE}^2} \right) + \frac{(M_W - M_{WW})^2}{\Gamma_W^2}
\]

Usually preferred method.

Dynamics

- **Principle**: fit a function to the mass of objects in the events (integrate over unknowns). Maximize sensitivity by:

\[
P(x;m_t) = \frac{1}{\sigma} \int d^4 \sigma(y;m_t) dq dq_z f(q) f(q_z) W(x | y)
\]

parton distribution functions

differential cross section (LO matrix element)

transfer function: mapping from parton-level variables \(y\) to reconstructed-level variables \(x\)
Description of Jets
Description of Jets

Monte Carlo generators:

Universal instrument to describe hadronic final states.

- Hadronization model and $\alpha_s$ are “tuned” to experimental data.

Parton-Shower ("leading log")

$Q$  $\Lambda_{\text{cut}}$

$perturbative$

$tuning$

parameters

$\alpha_s$

$m_t^{\text{Pythia}} \neq m_t^{\text{pole}}$

$\alpha_s$

model

parameters

non-
perturbative
Description of Jets

LHC:

Principle of mass measurements:

Identification of the top decay products

\[ m_{\text{top}}^2 = p_t^2 = (\sum_i p_i^\mu)^2 \]

Invariant mass distribution

![Invariant mass distribution graph](image)
**Description of Jets**

**LHC:**

- Measured object does not exist a priori, but only through the experimental prescription for the measurement. **Quantum effects!!**

**Problem is non-trivial!**

- Measured object does not exist a priori, but only through the experimental prescription for the measurement. **Quantum effects!!**

The idea of a - by itself - well defined object having a well defined mass is incorrect!!

Details and uncertainties of the parton shower and the hadronization models in den MC's influence the measured top quark mass.

**Principle of mass measurements:**

Identification of the top decay products

\[
\begin{align*}
    m_{\text{top}}^2 &= p_t^2 = \left(\sum_i p_i^\mu\right)^2
\end{align*}
\]

- **LHC:**
  - jet
  - W
  - b
  - p
  - t
  - e^-
  - \bar{\nu}
  - \bar{\nu}
  - jet

Top Mass + ?
Description of Jets

LHC:

Principle of mass measurements:

Identification of the top decay products

\[ m_{\text{top}}^2 = p_t^2 = \left( \sum_i p_i^\mu \right)^2 \]

Invariant mass distribution

depends on:
* mass definition
* jet definition
* Monte Carlo

ATLAS
QCD Factorization

Top Invariant Mass Distribution:

We need: QCD factorization in the final state

Simpler case:

\[ e^+ e^- \rightarrow t\bar{t} \]

\[ Q \gg m_t \quad (p_T \gg m_t) \]
**QCD Factorization**

**Top Invariant Mass Distribution:**

Definition of the observable

\[ M_t^2 = \left( \sum_{i \in a} p_i^\mu \right)^2 \]

\[ M_{t,\bar{t}}^2 = \left( \sum_{i \in b} p_i^\mu \right)^2 \]

Double differential hemisphere mass distribution

\[ \frac{d^2 \sigma}{dM_t dM_{\bar{t}}} \]

Fleming, Mantry, Stewart, AHH


resonance region:

\[ M_{t,\bar{t}} - m_t \sim \Gamma \]
QCD Factorization

\[ Q \gg m_t \gg \Gamma_t > \Lambda_{QCD} \]

- Soft-Collinear-Effective Theory
- Heavy Quark Effective Theory
- Unstable Particle Effective Theory

\[
\left( \frac{d^2 \sigma}{dM_t^2 \ dM_t^2} \right)_{\text{hemi}} = \sigma_0 H_Q(Q, \mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu \right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^- \ B_+ \left( \hat{s}_t - \frac{Q \ell^+}{m}, \Gamma, \mu \right) B_- \left( \hat{s}_t - \frac{Q \ell^-}{m}, \Gamma, \mu \right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]

\[ \hat{S} = \frac{M_t^2 - m_J^2}{m_J} \]
Jet functions:

\[ B_+ (\hat{s}, \Gamma_t, \mu) = \text{Im} \left[ \frac{-i}{12\pi m_J} \int d^4 x e^{i r.x} \langle 0 | T \{ \bar{h}_{v+} (0) W_n (0) W_n^\dagger (x) h_{v+} (x) \} | 0 \rangle \right] \]

- perturbative
- dependent on mass, width, color charge

\[ B_{\pm}^{\text{Born}} (\hat{s}, \Gamma_t) = \frac{1}{\pi m_t} \frac{\Gamma_t}{\hat{s}^2 + \Gamma_t^2} \quad \hat{s} = \frac{M^2 - m_t^2}{m_t} \]

Soft function:

\[ S_{\text{semi}} (\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta (\ell^+ - k_s^{+a}) \delta (\ell^- - k_s^{-b}) \langle 0 | \bar{Y}_n Y_n (0) | X_s \rangle \langle X_s | Y_n \bar{Y}_n (0) | 0 \rangle \]

- non-perturbative
- analogous to the pdf’s
- dependent on color charge, kinematics
  - Independent of the mass !
  - \( \rightarrow \) event shapes
full QCD:

\[ \frac{1}{(p_t+k)^2 - m_t^2} \]

3 phase space regions:

- n-collinear: \( (k_+, k_-, k_\perp) \sim Q(\lambda^2, 1, \lambda) \)
- \( \bar{n}\)-collinear: \( (k_+, k_-, k_\perp) \sim Q(1, \lambda^2, \lambda) \)
- soft: \( (k_+, k_-, k_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2) \)

\[ p_t^2 \approx m_t^2 , \bar{n}^2 = 0 \]

\[ W_n^\dagger(\infty, x) = P \exp \left( ig \int_0^\infty ds \bar{n} \cdot A_+ (ns + x) \right) \]

\[ h_{v+}(x) \rightarrow \text{gauge dependent} \]

\[ W_n^\dagger(\infty, x) h_{v+}(x) \rightarrow \text{gauge independent} \]
QCD Factorization

full QCD:

\[ \begin{align*}
\frac{1}{(p_{t,\bar{t}}+k)^2-m_t^2} & \quad \text{3 phase space regions:} \quad \lambda \sim m_t/Q \\
& \quad \bullet \text{n-collinear: } (k_+, k_-, k_{\perp}) \sim Q(\lambda^2, 1, \lambda) \\
& \quad \bullet \text{\bar{n}-collinear: } (k_+, k_-, k_{\perp}) \sim Q(1, \lambda^2, \lambda) \\
& \quad \bullet \text{soft: } (k_+, k_-, k_{\perp}) \sim Q(\lambda^2, \lambda^2, \lambda^2) \\
& \quad (p_{t,\bar{t}}^2 \approx m_t^2, \, n^2 = 0, \, \bar{n}^2 = 0)
\end{align*} \]

Gluon soft:

\[ \begin{align*}
\frac{1}{Q \, n.k} & \quad \frac{1}{Q \, \bar{n}.k} \\
\Rightarrow & \quad Y_n(x) = \bar{P} \exp \left( -ig \int_0^\infty ds \, n \cdot A_s(ns+x) \right) \\
\Rightarrow & \quad \bar{Y}_n(x) = \bar{P} \exp \left( -ig \int_0^\infty ds \, \bar{n} \cdot \bar{A}_s(\bar{n}s+x) \right)
\end{align*} \]

\[ S_{\text{hemi}}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \langle 0 | (\bar{Y}_n)^{ce}(Y_n)^{cd}(0) \, \delta(\ell^- - (\hat{P}_a^+)^\dagger) \, \delta(\ell^- - \hat{P}_b^-) \, (Y_n^\dagger)^{ef}(\bar{Y}_n)^{\dagger df}(0) | 0 \rangle \]
\[ \frac{d\sigma}{dM} \]

\[ M_{\text{peak}} = m_t + \Gamma_t (\alpha_s + \alpha_s^2 + \ldots) + \frac{Q}{m_t} \Omega_1 + \mathcal{O}\left(\frac{m_t \Lambda_{\text{QCD}}}{Q}\right) \]

first moment of the soft function:

\[ \Omega_1 = \int d\ell \, \ell \, S(\ell, \mu) \]
QCD Factorization

Higher Orders & Top Mass Scheme:

\[ B_{\pm}(\hat{s}, 0, \mu, \delta m) = -\frac{1}{\pi m} \frac{1}{\hat{s} + i0} \left\{ 1 + \frac{\alpha_s C_F}{4\pi} \left[ 4 \ln^2 \left( \frac{\mu}{\hat{s} + i0} \right) + 4 \ln \left( \frac{\mu}{\hat{s} + i0} \right) + 4 + \frac{5\pi^2}{6} \right] \right\} \]

\[ m_{\text{pole}} = m_J(\mu) + e^{\gamma_E} R \frac{\alpha_s(\mu) C_F}{\pi} \left[ \ln \frac{\mu}{R} + \frac{1}{2} \right] + \mathcal{O}(\alpha_s^2) \]

\[ R \sim \Gamma_t \]
Theory Issues for $pp \rightarrow t\bar{t} + X$

- definition of jet observables → Hadron event shapes
- initial state radiation
- final state radiation
- underlying events → Soft function?
- color reconnection & soft gluon interactions
- beam remnant
- parton distributions
- summing large logs $Q \gg m_t \gg \Gamma_t$
- relation to Lagrangian short distance mass

$T_{\perp,g} \equiv \max_{\vec{n}_T} \sum_i \frac{|\vec{q}_{\perp i} \cdot \vec{n}_T|}{\sum_i q_{\perp i}}$

Banfi, Salam, Zanderighi

Can be addressed in the framework of a LC.

Requires extensions of LC concepts and other known concepts
Use analogies between MC set up and factorization theorem

**Final State Shower**

- **Start**: at transverse momenta of primary partons, evolution to smaller scales.
- Shower cutoff $R_{sc} \sim 1$ GeV
- Hadronization models fixed from reference processes

**Factorization Theorem**

- Renormalization group evolution from transverse momenta of primary partons to scales in matrix elements.
- Subtraction in jet function that defines the mass scheme
- Soft function model extracted from another process with the same soft function

**Additional Complications:**

Initial state shower, underlying events, combinatorial background, etc

Let’s assume that these aspects are treated correctly in the MC
MC Top Mass

Conclusion (quick answer):

\[ m_t^{MC}(R_{sc}) = m_t^{pole} - R_{sc} c \left( \frac{\alpha_s}{\pi} \right) \]

Determination of the MSbar mass:

\[ m_t^{\text{TeV}} = m_t^{MC}(R_{sc}) = 172.6 \pm 0.8(\text{stat}) \pm 1.1(\text{syst}) \]

\[ \overline{m}_t(\overline{m}_t) = 163.0 \pm 1.3^{+0.6}_{-0.3} \text{ GeV} \quad (c = 3^{+6}_{-2}) \]

More systematic study needed for final answer!
The exercise just carried out does not account for possible conceptual uncertainties!
Conclusion:

→ Current top mass measurements from the Tevatron refer to the top mass parameter in Pythia $m_t^{\text{Pythia}}$.

→ For a high energy Linear Collider we have a factorization theorem to do MC independent short-distance Lagrangian top mass measurements (jet mass).

→ The analogy between MC generators and factorization theorem indicates that the $m_t^{\text{Pythia}}$ is a short-distance mass like the jet mass (and not the pole mass).

<table>
<thead>
<tr>
<th>$m_t^{\text{Pythia}}$</th>
<th>$m_t^J$ (2 GeV)</th>
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<tbody>
<tr>
<td>160.00</td>
<td></td>
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<tr>
<td>165.00</td>
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<tr>
<td>170.00</td>
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Plans: → “Measure” the $m_t^{\text{Pythia}}$ in terms of the Jet mass $m_t^J$ (2 GeV) using thrust and other event shapes

→ Derivation of eventshape-like factorization theorems for Tevatron/LHC

→ “Measure” $m_t^{\text{Pythia}}$ for LHC-Pythia