Fermionic Spectrum at Ultrasoft Region in a Hot QED/QCD Plasma

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Introduction

motivation: **quark spectrum** in the quark-gluon plasma (QGP)  
(collective excitation, quasi-particle picture)

• how many excitation modes?
• how are dispersion relation, decay width, strength?

very fundamental problems, but **not well investigated** even in the weak coupling region ( $g<<1$).

Introduction

Spectrum of boson-fermion system at high temperature
(Yukawa model, QED/QCD)

\( T \gg \text{any mass} \)

- **Bosonic**
  - Single particle excitation
  - Plasmon
  - Hydro mode

- **Fermionic**
  - Single particle excitation
  - Normal fermion
  - Plasmino

\( g^2T \)

\( gT \)

\( T \)

\( E \)

\( p/g^2T \)

Frequency of the bosonic sector: \( \omega = \sqrt{\omega_0^2 + k^2} \)

Dispersion relation in the fermionic sector

H. Weldon, PRD 26, 2789 (1982)
Introduction

spectrum of boson-fermion system at high temperature
(Yukawa model, QED/QCD)

T >> any mass

H. Weldon, PRD 26, 2789 (1982)
Ultrasoft fermionic mode

one-loop analysis \((m: \text{boson mass})\)

see also M. Kitazawa, T. Kunihiro and Y. Nemoto, PTP 117, 103 (2007).

a peak appears even in \(T >> m\) at the origin

(spectral function)

normal fermion

(anti) plasmino

spectral function in the fermion sector
Pinch singularity

in ultrasoft ($p \ll g^2 T$) region, the ordinary perturbation (Hard Thermal Loop (HTL) approximation) is not applicable.

1. $p \to 0$ limit can not be taken (pinch singularity)
   pinch singularity in computing transport coefficient:
   S. Jeon, PRD 52, 3591 (1995),

2. (one loop) $\ll$ (multi loop)

reorganizing perturbation expansion is necessary
Resummed Perturbation  
(V. V. Lebedev and A. V. Smilga, Annals Phys. 202, 229 (1990))

(1) resum thermal mass \( m_f, m_b = O(gT) \) and decay width \( \zeta_f, \zeta_b = O(g^2T) \) or \( O(g^4T) \) 

\[
g^2 \frac{k}{2p \cdot k + \delta m^2 + 2i \zeta k^0} \xrightarrow{p \to 0} O(g^0)
\]

\[
\delta m^2 = m_f^2 - m_b^2, \quad \zeta = \zeta_f + \zeta_b
\]

→ pinch singularity is regularized.

• Landau gauge
• residue is unity (same as bare one)
• no longitudinal mode


J. P. Blaizot and E. Iancu, PRL, 76, 3080 (1996)
Resummed Perturbation (V. V. Lebedev and A. V. Smilga, Annals Phys. 202, 229 (1990))

the singularity is regularized, but all of the ladder diagrams contribute at the same order.

\[
\frac{g^2}{2p \cdot k + \delta m^2 + 2i\zeta k^0} = O(g^0) \quad \left( \frac{g^2}{2p \cdot k + \delta m^2 + 2i\zeta k^0} \right)^2 = O(g^0) \quad (#\text{vertex})/(#\text{resummed propagator})=1
\]

\[\begin{align*}
\text{(QCD)} \\
\end{align*}\]

(2) sum up all the ladder diagrams.
we adopt this vertex:
Resummed Perturbation (V. V. Lebedev and A. V. Smilga, Annals Phys. 202, 229 (1990))

(1) and (2)

\[
\begin{align*}
\text{this diagram contains} & \quad \text{all of the leading order contribution. (}\mathcal{O}(p/g^2)\text{)}
\end{align*}
\]
Result


novel excitation is found in ultrasoft \((p, \omega << g^2 T)\) region.

<table>
<thead>
<tr>
<th>dispersion relation</th>
<th>Re(\omega = -p/3) (cf: anti-plasmino)</th>
</tr>
</thead>
<tbody>
<tr>
<td>decay width</td>
<td>Im(\omega = \zeta = O(g^2 T))</td>
</tr>
<tr>
<td>residue</td>
<td>(Z = \frac{g^2}{16\pi^2} \left( C_f + 8 \frac{\delta m^2}{g^2 T^2} \right)^2 = O(g^2)) (QED)</td>
</tr>
<tr>
<td></td>
<td>(= \left{ \frac{g^2}{164\pi^2} \right} \left( \frac{4N_f}{3} + \frac{13N}{6} + \frac{1}{2N} \right)^2) (QCD)</td>
</tr>
</tbody>
</table>

\((g=0.3)\)

(normal fermion (HTL result))

(new excitation is found!)

(anti) plasmino(HTL result)

dispersion relation in the fermionic sector
Check of gauge symmetry

Ward-Takahashi identity is derived from the gauge symmetry.

\[(k-p)_{\mu} \cdot \begin{pmatrix}
 k_{\mu} \\
 k-p \\
 p
\end{pmatrix} =
 \begin{pmatrix}
 k \\
 -1 \\
 k \\
 p \\
 -1 \\
 p
\end{pmatrix}
\]

\[S^{-1}(k) \simeq k \\
-S^{-1}(p) \simeq \Sigma(p)
\]

This identity relates vertex to self-energy.
Check of gauge symmetry

self-consistent eq. :

\[ \text{Diagram} = \text{Diagram} + \text{Diagram} \]
Check of gauge symmetry

self-consistent eq. :

\[(k-p)^\mu \cdot \begin{bmatrix} \end{bmatrix} = \begin{bmatrix} \end{bmatrix} - \begin{bmatrix} \end{bmatrix} - \begin{bmatrix} \end{bmatrix} \]
Check of gauge symmetry

self-consistent eq.  $\rightarrow$ W-T identity

Our vertex and self-energy satisfy this identity; We passed the consistency-check!
Summary

• we found the **novel fermionic mode** in ultrasoft ($\ll g^2 T$) region with a resummed perturbation in QED/QCD.

• we obtained the expression of the **dispersion relation**, **decay width** and **residue**.

• the obtained vertex and the fermion self-energy satisfy the **Ward-Takahashi identity**.
Future plan

• Systematic derivation of the resummed perturbation theory from the Kadanoff-Baym equation

• Effect to observables