Transport Properties of Strong-Interaction Matter

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Jochen Wambach

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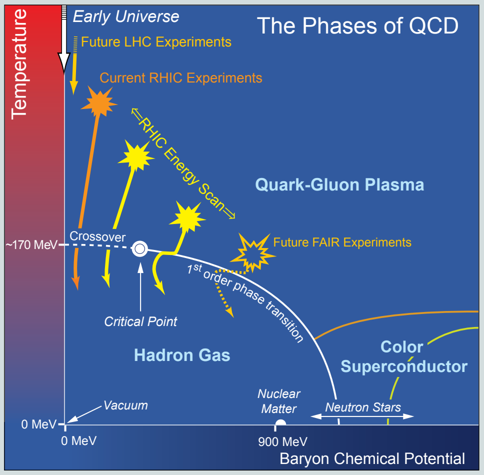
- ideal fluid @ RHIC and LHC?
- shear viscosity @ finite T
- pion dynamics @ low T
- effects of χS restoration
- use NJL model



Nambu-Jona-Lasinio model

bulk thermodynamics

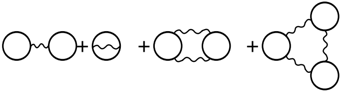
NJL Lagranian ($N_f = 2$)

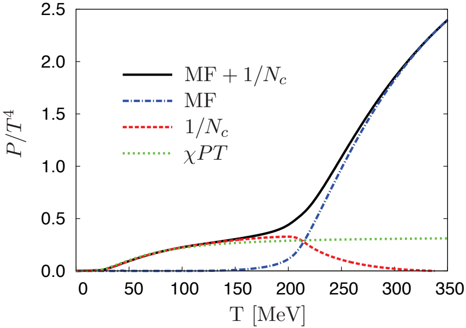
$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - m_0)\psi + g[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\vec{\tau}\psi)^2]$$

Nambu, Jona-Lasinio, Phys. Rev. 1961

thermodynamic potential

NLO in $1/N_c$-expansion

$\Omega =$  $+ \ldots$



Radzhabov et al., 2011, arXiv:1102.0664

Viscous hydrodynamics
(1^{st} order)

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asumptions:

system described locally by $u_\mu(x)$ local thermal equilibrium (EoS$\rightarrow p(\epsilon)$)

basic quantities:

energy-momentum tensor: $T^{\mu\nu}(x)$ 4-current: $J^\mu(x)$

conservation laws:

energy-momentum conservation: $\partial_\mu T^{\mu\nu}(x) = 0$ number conservation: $\partial_\mu J^\mu(x) = 0$

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - g^{\mu\nu}p + \pi^{\mu\nu}; \quad J^\mu = nu^\mu + \nu^\mu$$

$$\begin{aligned}
\pi^{\mu\nu} &= \eta\left(\partial^\mu u^\nu + \partial^\nu u^\mu - u^\mu u_\lambda \partial^\lambda u^\nu - u^\nu u_\lambda \partial^\lambda u^\mu\right) \\
&\quad + \left(\zeta - \frac{2}{3}\eta\right)\left(g^{\mu\nu} - u^\mu u^\nu\right)\partial_\lambda u^\lambda \\
\nu^\mu &= \kappa\left(\frac{nT}{\epsilon + p}\right)^2\left(\partial^\mu - u^\mu u^\nu \partial_\nu\right)\left(\frac{\mu}{T}\right)
\end{aligned}$$

Hydro Elliptic Flow

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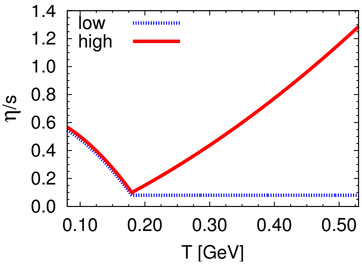
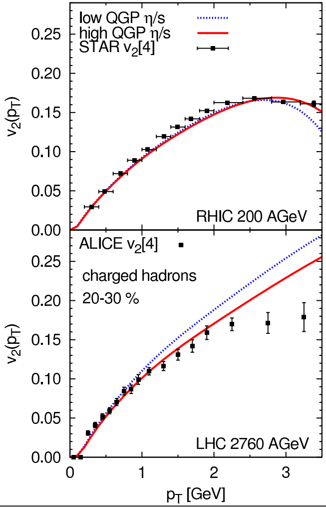


H. Niemi et al,. arXiv:1101.2442



Transport coefficients

linear response theory

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linear response theory describes how a system near equilibrium responds to a weak perturbation

$$\delta\langle O(x)\rangle_f = i\int_{t>0} d^4x' \underbrace{\langle[O(x), O(x')]\rangle}_{2p-correlator} \underbrace{f(x')}_{weak\ pert.}$$

viscosities:

$$\frac{1}{\omega}\lim_{\vec{q}\to 0}\int d^3r\int_0^\infty dt\ e^{i(\omega t-\vec{q}\cdot\vec{r})}\langle[T_{ik}(x), T_{lm}(0)]\rangle$$

$$= \eta(\omega)\left(\delta_{ik}\delta_{km} + \delta_{im}\delta_{kl} - \frac{2}{3}\delta_{ik}\delta_{lm}\right) + \zeta(\omega)\delta_{ik}\delta_{lm}$$

static limit:

$$\eta = \frac{1}{20}\lim_{\omega\to 0}\frac{1}{\omega}\int d^3r\int_0^\infty dt\ e^{i\omega t}\langle[T_{ij}(x), T_{ij}(0)]\rangle, \quad i \neq j$$

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int d^3r\int_0^\infty dt\ e^{i\omega t}\langle[T_{ii}(x), T_{jj}(0)]\rangle$$

$$\kappa = \frac{1}{6T}\lim_{\omega\to 0}\frac{1}{\omega}\int d^3r\int_0^\infty dt\ e^{i\omega t}\langle I_i(x), I_i(0)]\rangle ; \quad I_i = T_{0i} - \frac{\epsilon + p}{n}J_i$$

Quantum relativistic kinetic theory

equivalent to Green-Kubo in the dilute gas limit

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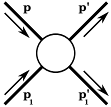
Boltzmann-Ueling-Uhlenbeck equation:

mean free path $\lambda = 1/n\sigma$ much larger than interaction range, c.f. $d = 1/m$ or $d = \sqrt{\sigma/\pi}$

$$\frac{\mathrm{d}}{\mathrm{d}t} f_a(\vec{x}, \vec{p}, t) = \sum_b C\,[f_a, f_b]$$



$$C\,[f_a, f_b] = \frac{1}{1+\delta_{ab}} \frac{g_b}{2E_b} \int \frac{\mathrm{d}^3 p'}{(2\pi)^3} \int \frac{\mathrm{d}^3 p_1}{(2\pi)^3} \int \frac{\mathrm{d}^3 p'_1}{(2\pi)^3} (2\pi)^4 \delta(p + p_1 - p' - p'_1) |\mathcal{M}_{ab}|^2 \times$$
$$\times \left[f'_a f'_{1b}(1+f_a)(1+f_{1b}) - f_a f_{1b}(1+f'_a)(1+f'_{1b}) \right]$$

- linearization of the BUU-equation (2^{nd}-order Chapman-Enskog expansion)

$$f_a = f_a^{(0)} + \epsilon f_a^{(1)} + \cdots\,; \qquad f_a^{(1)} = f_a^{(0)}(1 + f_a^{(0)})\phi_a$$

- shear viscosity

$$\eta = \sum_a \frac{g_a}{15} \frac{4\pi}{(2\pi)^3} \int_0^\infty dp \frac{p^4}{E_p} f_a^{(0)}(1 + f_a^{(0)}) \mathcal{B}_a(p)$$

- expansion of $\mathcal{B}_a(p)$ in orthogonal polynomials ('generalized Sonine functions')

Nambu-Jona-Lasinio model

quark and meson masses

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NJL Lagranian ($N_f = 2$)

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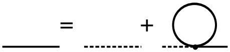
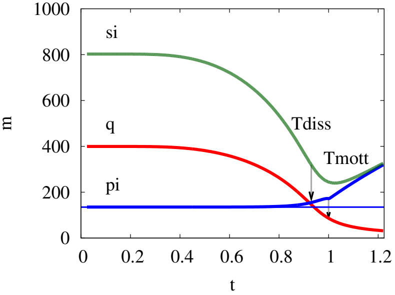
Nambu, Jona-Lasinio, Phys. Rev. 1961

Hartree quark propagator



RPA meson propagators

parameters: Buballa, Phys. Rept. 2005



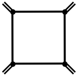
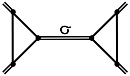
'dissociation' temperature $m_\sigma = 2m_\pi$

'Mott' temperature $m_\pi = 2m_q$

$$T_{diss} \approx 93\% T_{Mott}$$

Pion-Pion scattering

$\pi\pi$ scattering amplitude

$i\mathcal{M}_{\pi\pi}$ =  + 

Quack, et al., Phys. Lett. B 348 1 (1994)

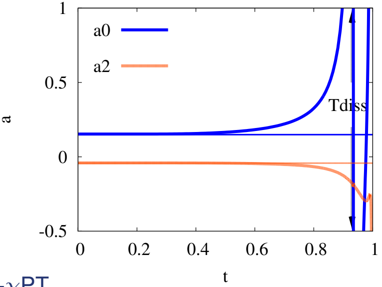
$\pi\pi$ scattering length

$$a^I = \frac{1}{32\pi m_\pi} \mathcal{M}^I_{\pi\pi}(s = 4m_\pi^2)$$

$\pi\pi$ cross section

$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}_{\pi\pi}|^2}{64\pi^2 E_{cm}^2}$$

$$|\mathcal{M}_{\pi\pi}|^2 = \frac{1}{9}\sum_I (2I+1)|\mathcal{M}^I_{\pi\pi}|^2$$



- LO-χPT

$$a_W^0 = \frac{7m_\pi}{32\pi f_\pi^2}; \qquad a_W^2 = -\frac{2m_\pi}{32\pi f_\pi^2}$$

Weinberg, Phys. Rev. Lett. 1966

- $a^{0,2}$ large at T_{diss}, T_{Mott}
  $\leftrightarrow$ ultra-cold atomic gases near 'Feshbach res.'

Cross section

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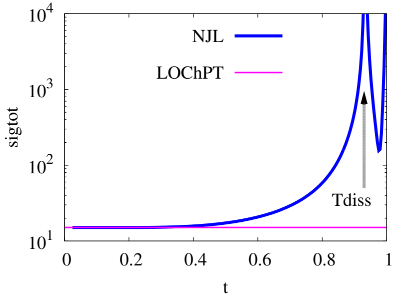
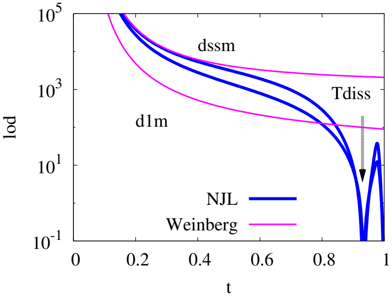
σ_{tot} at threshold



- in vacuum: NJL $\approx$ LO-χPT
- strong coupling at transition temperatures

validity of relativistic kinetic theory

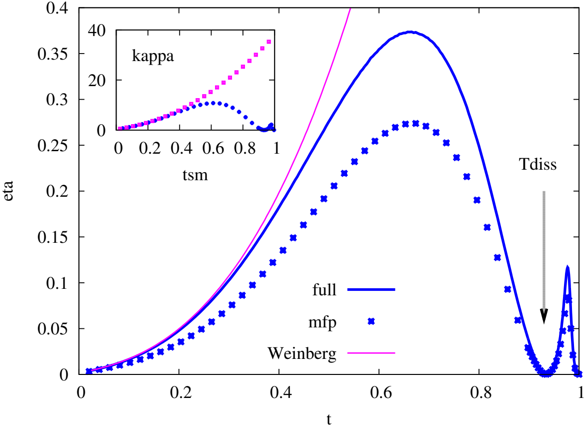


- Weinberg: valid in the whole T-range considered
- NJL: valid up to $\sim$ 88 % T_{Mott}

Validity check according to Itakura, et al., Phys. Rev. D 2008

Shear viscosity

and thermal conductivity



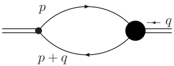
- η and κ similar behavior
- full RKT vs. mfp-estimate $\eta = \bar{p}/3\sigma$
- χPT & NJL agree for small T
- important modifications at higher T
- divergence of a^0 at T_{diss} visible
- strong coupling at T_{diss} and T_{Mott}

Shear viscosity

Green-Kubo formalism

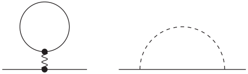
static shear viscosity using Green-Kubo formula

$$\eta = -\lim_{\omega\to 0}\lim_{\vec{q}\to 0}\frac{\mathrm{d}}{\mathrm{d}\omega}\mathrm{Im}\left(\quad \right)$$



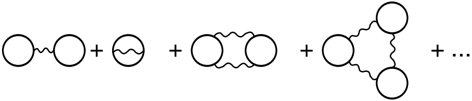
- quark spectral function
- quark vertex

quark self energy: (NLO)



entropy density: (NLO)

$$s = -\frac{\partial\Omega}{\partial T};\qquad \Omega =$$



Shear viscosity

$1/N_c$ - expansion

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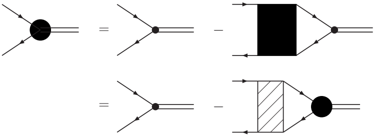
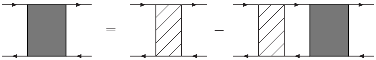
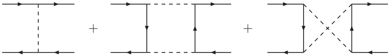
3-point vertex:



4-point vertex:



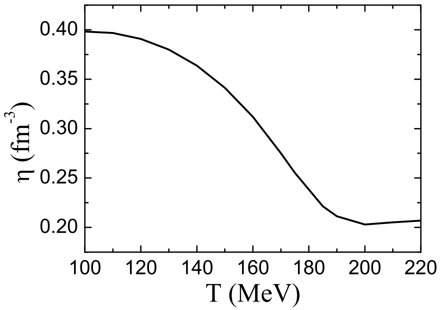
NLO Kernel:

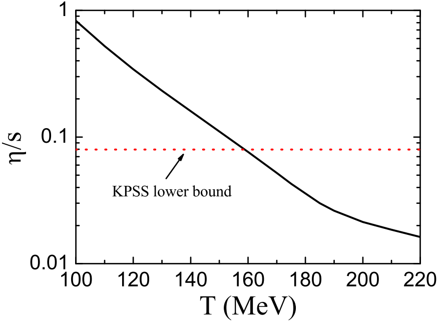


difficult to solve! for scalar O(N) theories Aarts and Resco, JHEP 2004

Shear viscosity

Kubo results





Wei-jie Fu, J. Wambach and Yue-liang Wu, in progress

Summary

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shear viscosity in the NJL model ($1/N_c$ - expansion)

- kinetic theory
  - in-medium $\pi\pi$ - scattering
  - scattering amplitude $\mathcal{M}$ from scattering length
  - agreement with χPT at low temperatures
  - σ large at T_{diss} and T_{Mott}
  - minima of η/s at transition temperatures
- Green-Kubo formalism
  - η strongly decreases near chiral restoration
  - η/s falls below ADS/CFT bound

future directions

- improve cross sections to include higher partial waves and more species
- PNJL calculation within Kubo formalism
- study finite chemical potential (CEP)