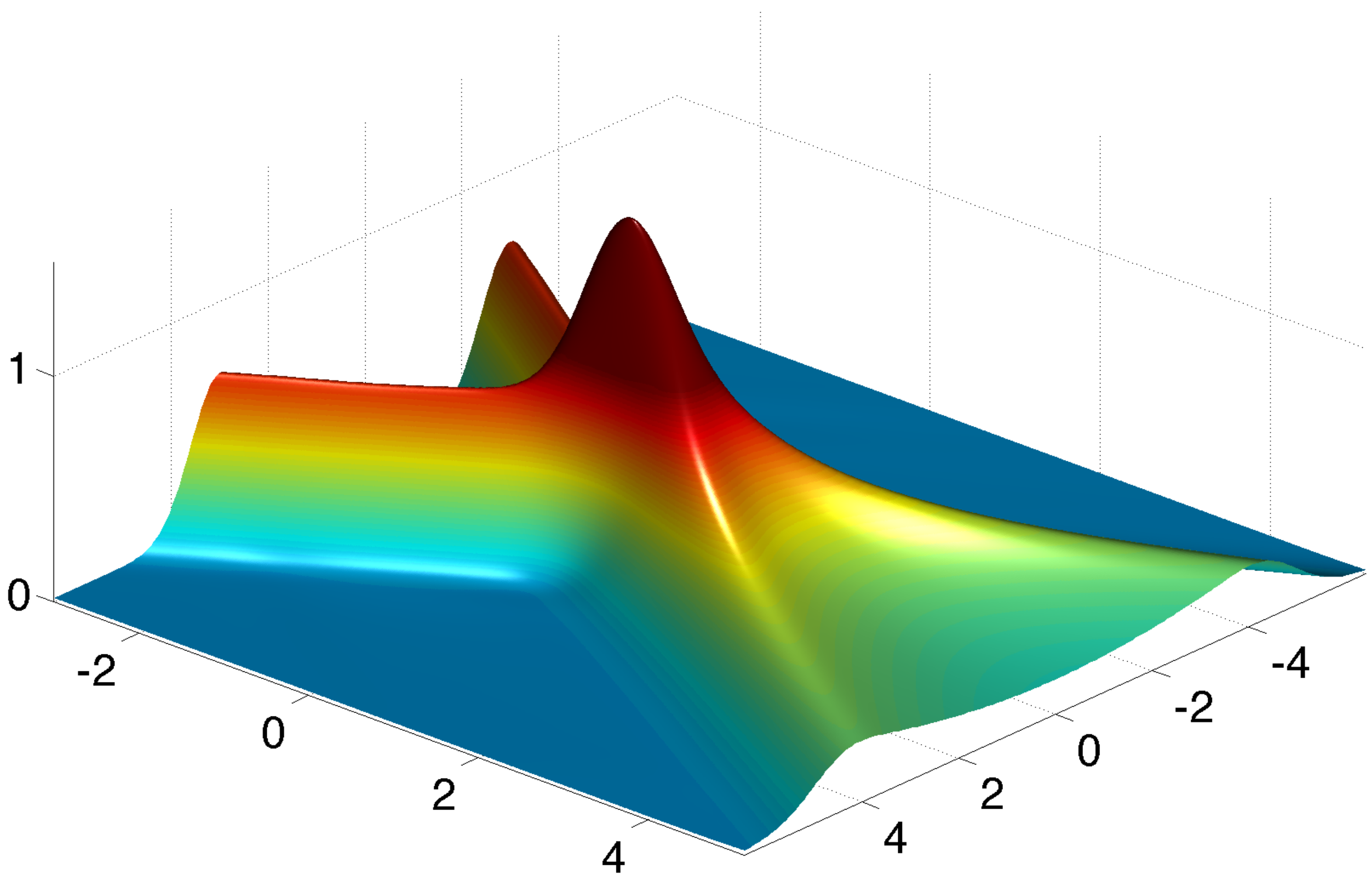
Gravitational collapse and far-from-equilibrium dynamics in holographic conformal field theories

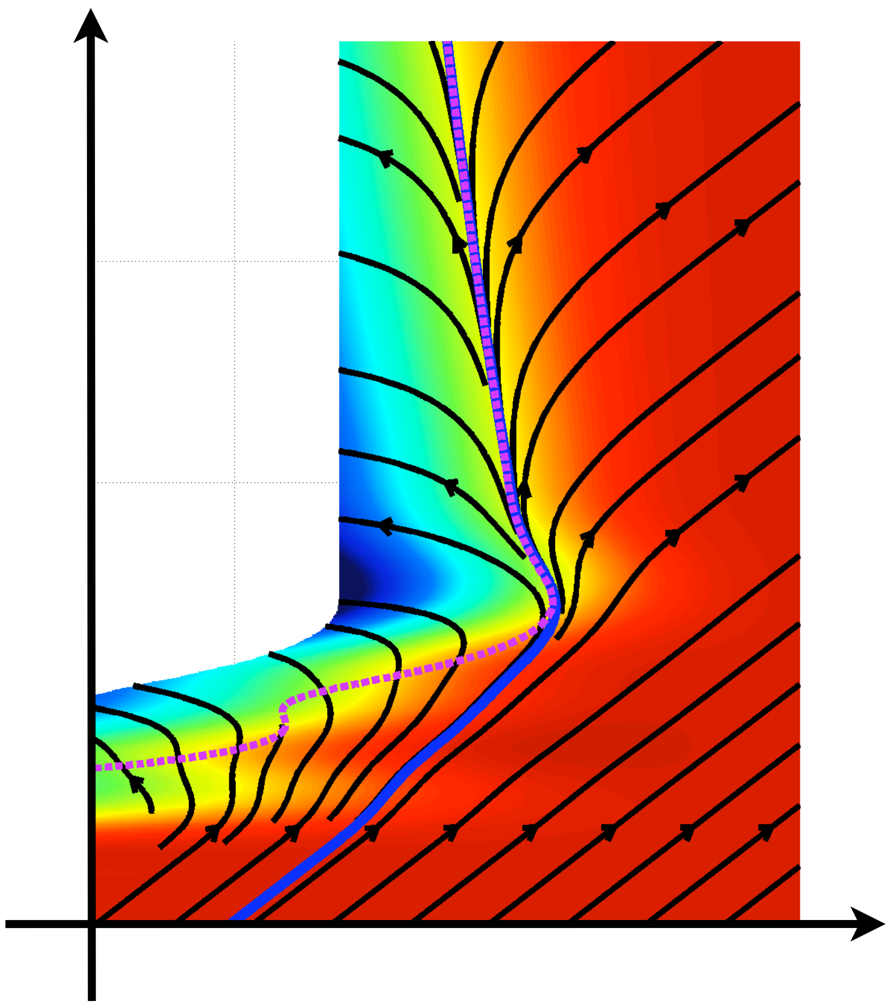
Paul Chesler

Massachusetts Institute of Technology

Work done with Larry Yaffe, Derek Teaney & Simon Caron-Huot.

[1011:3562, 1102.1073]

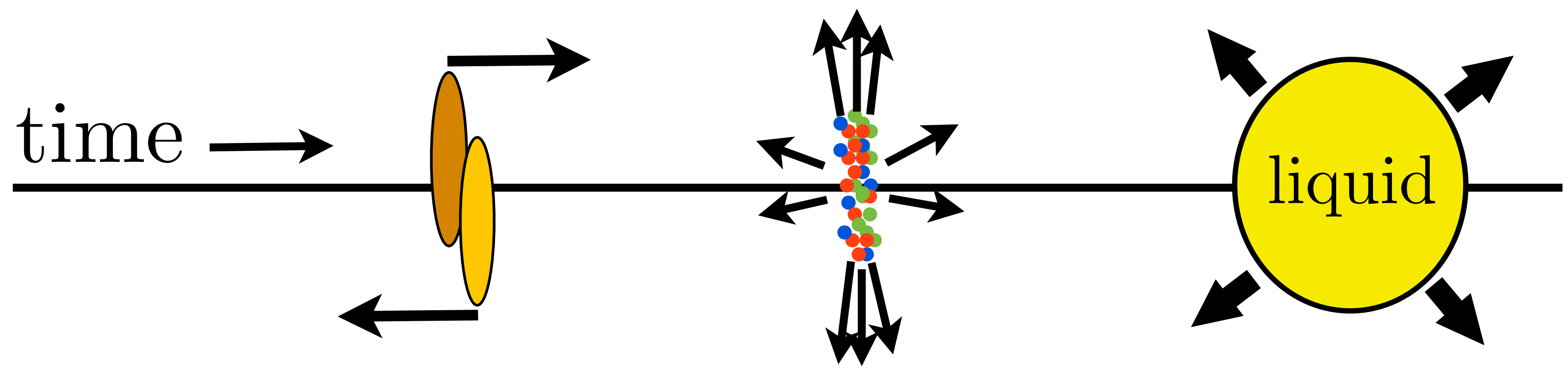




Two seemingly different processes

Far from equilibrium dynamics in QFTs

- Initial state: nuclei.
- "Final" state: QGP.



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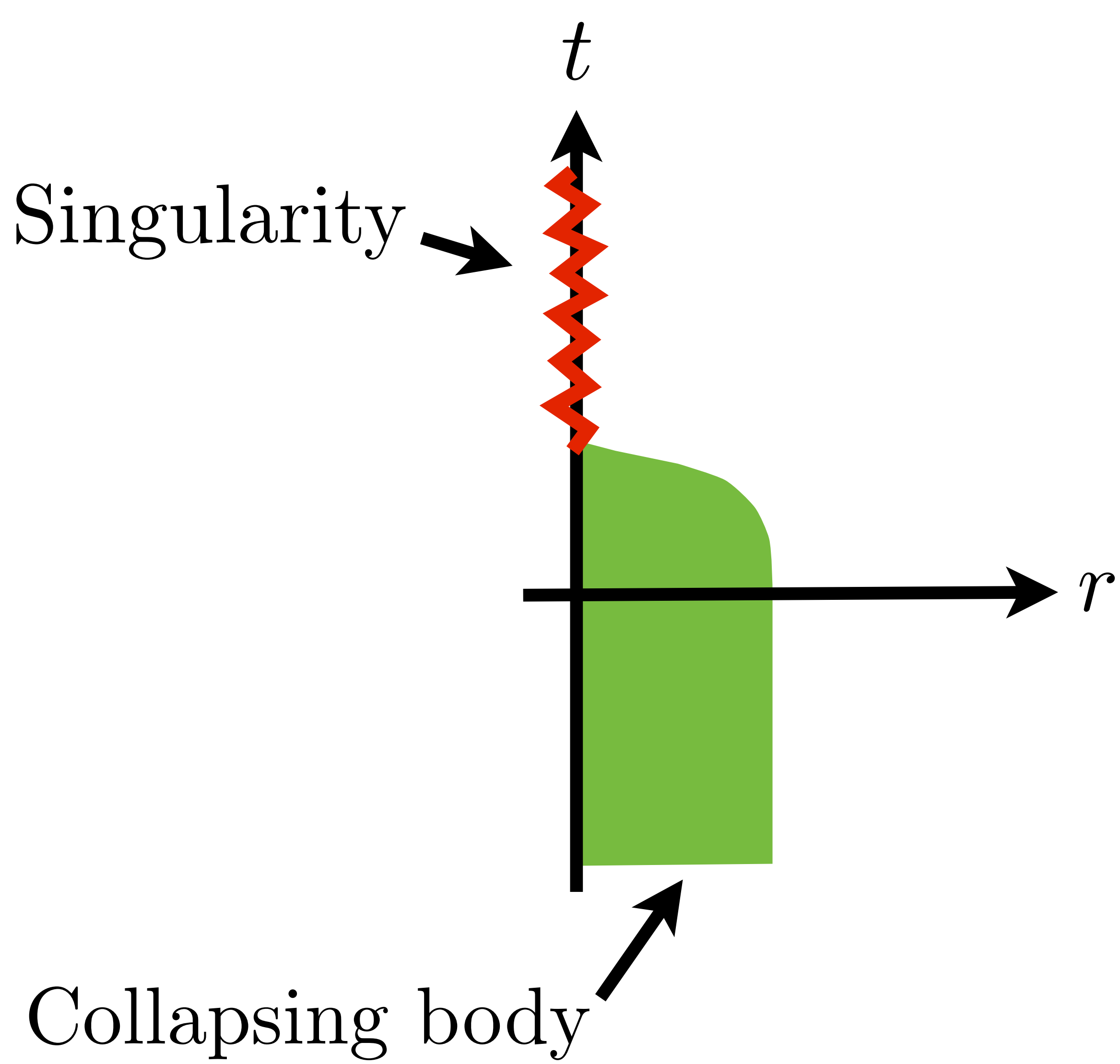

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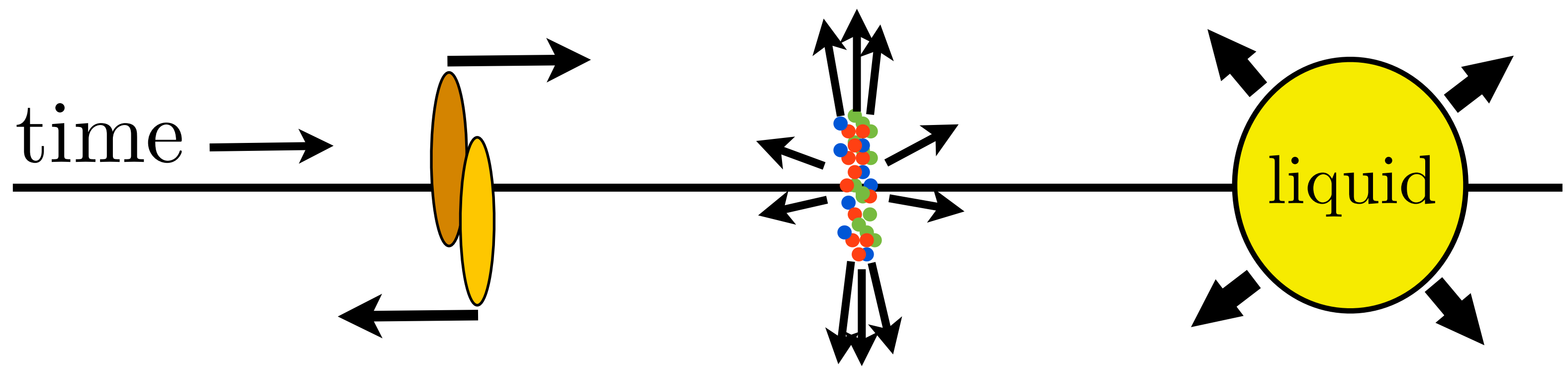

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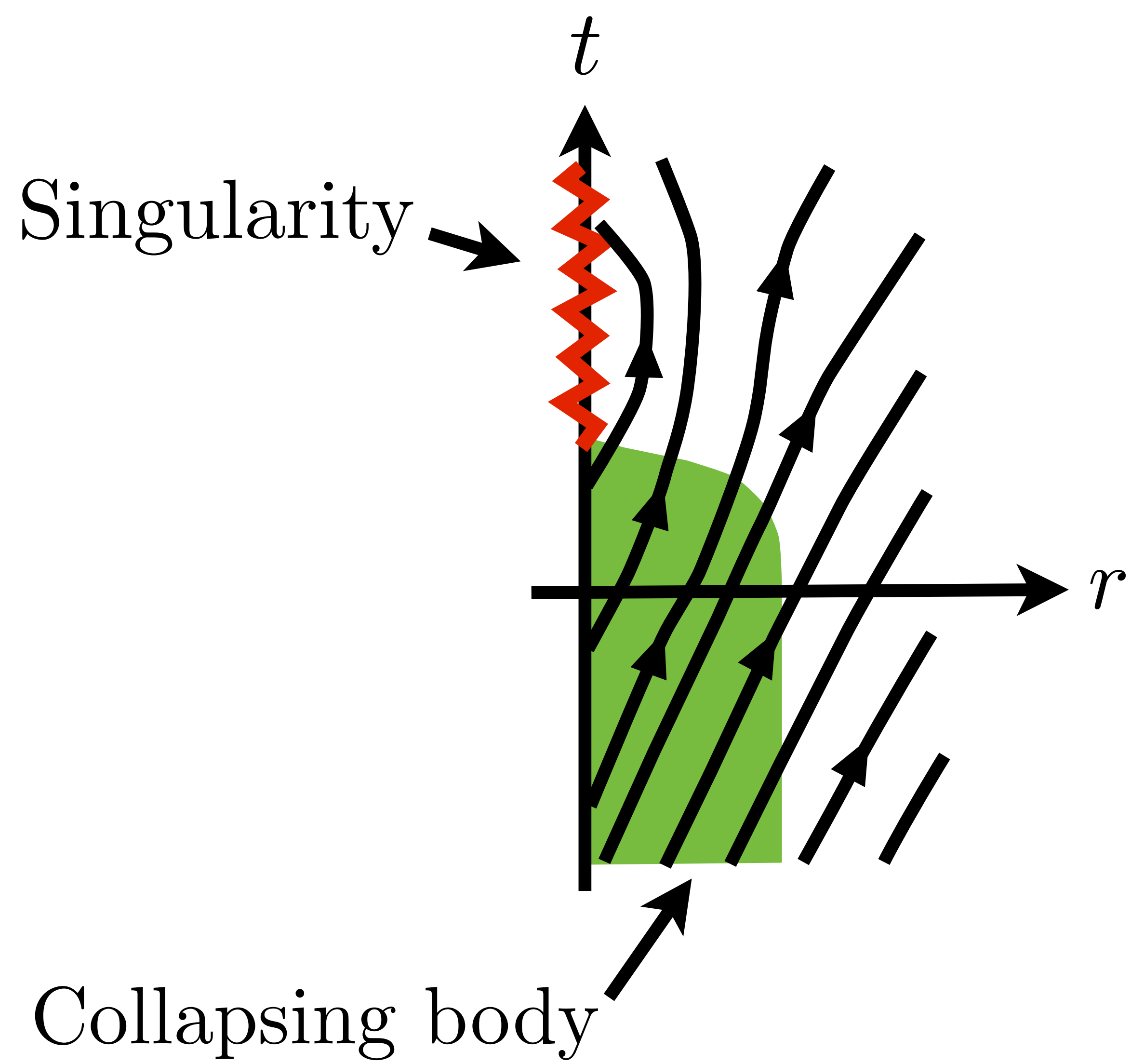

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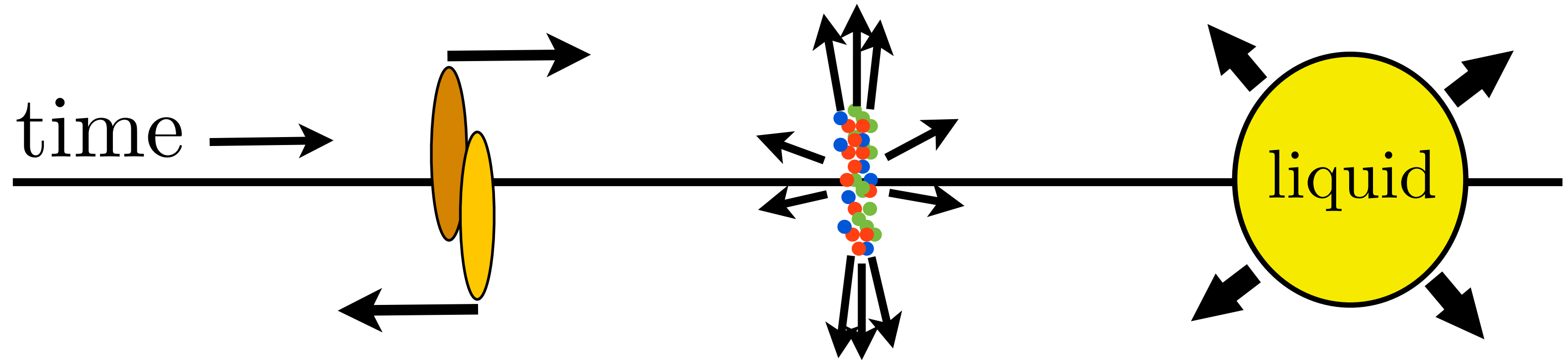

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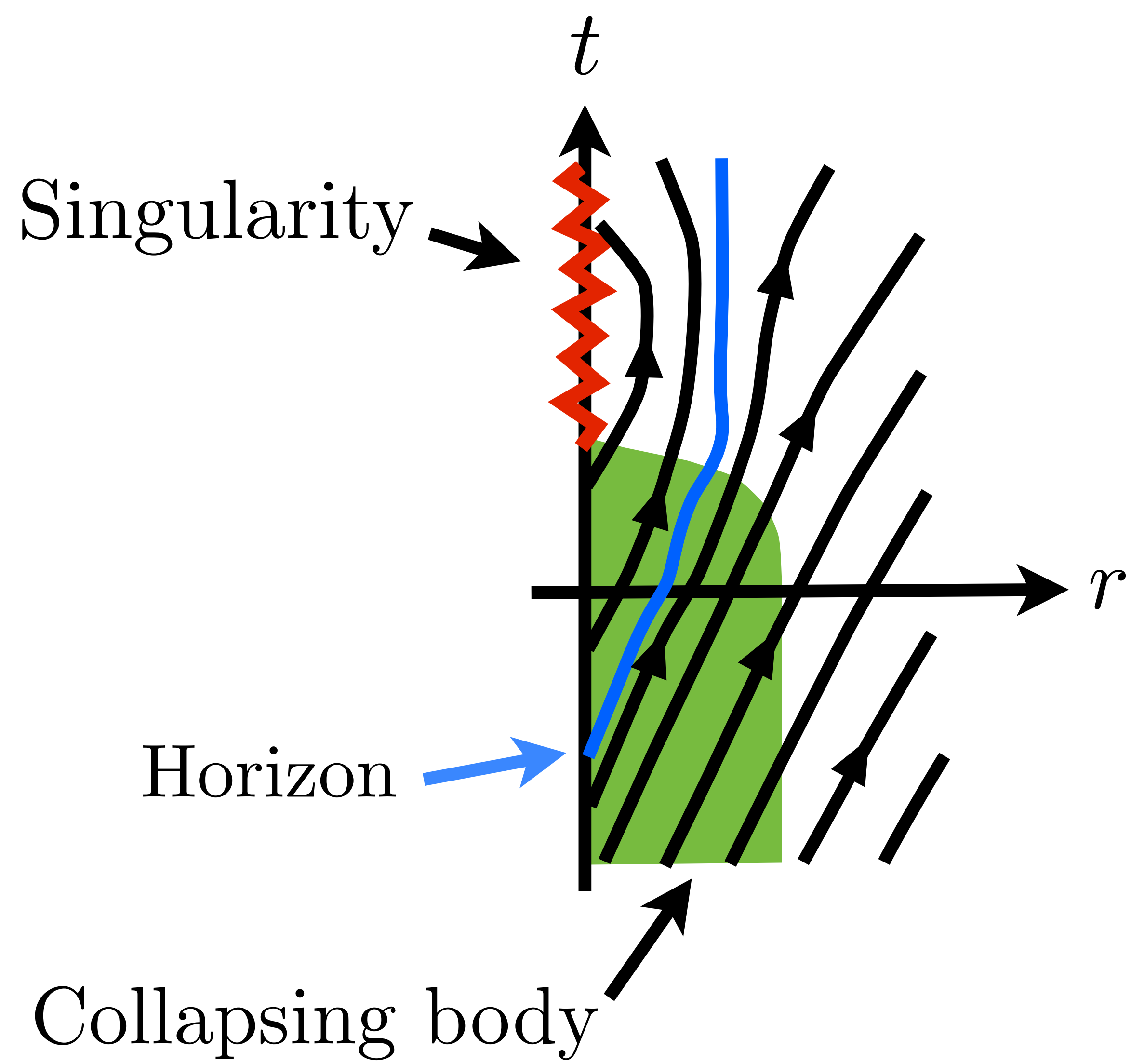

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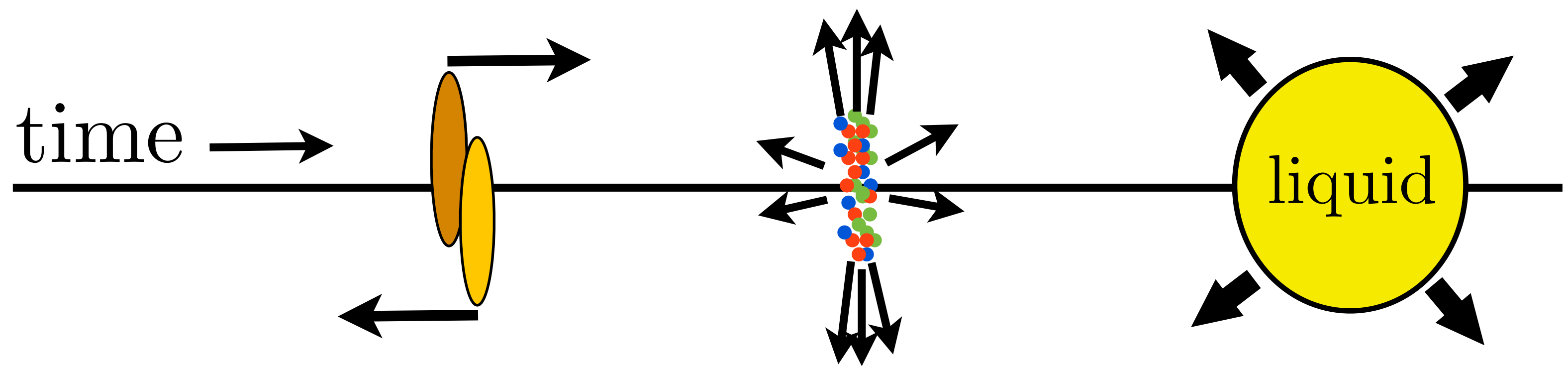

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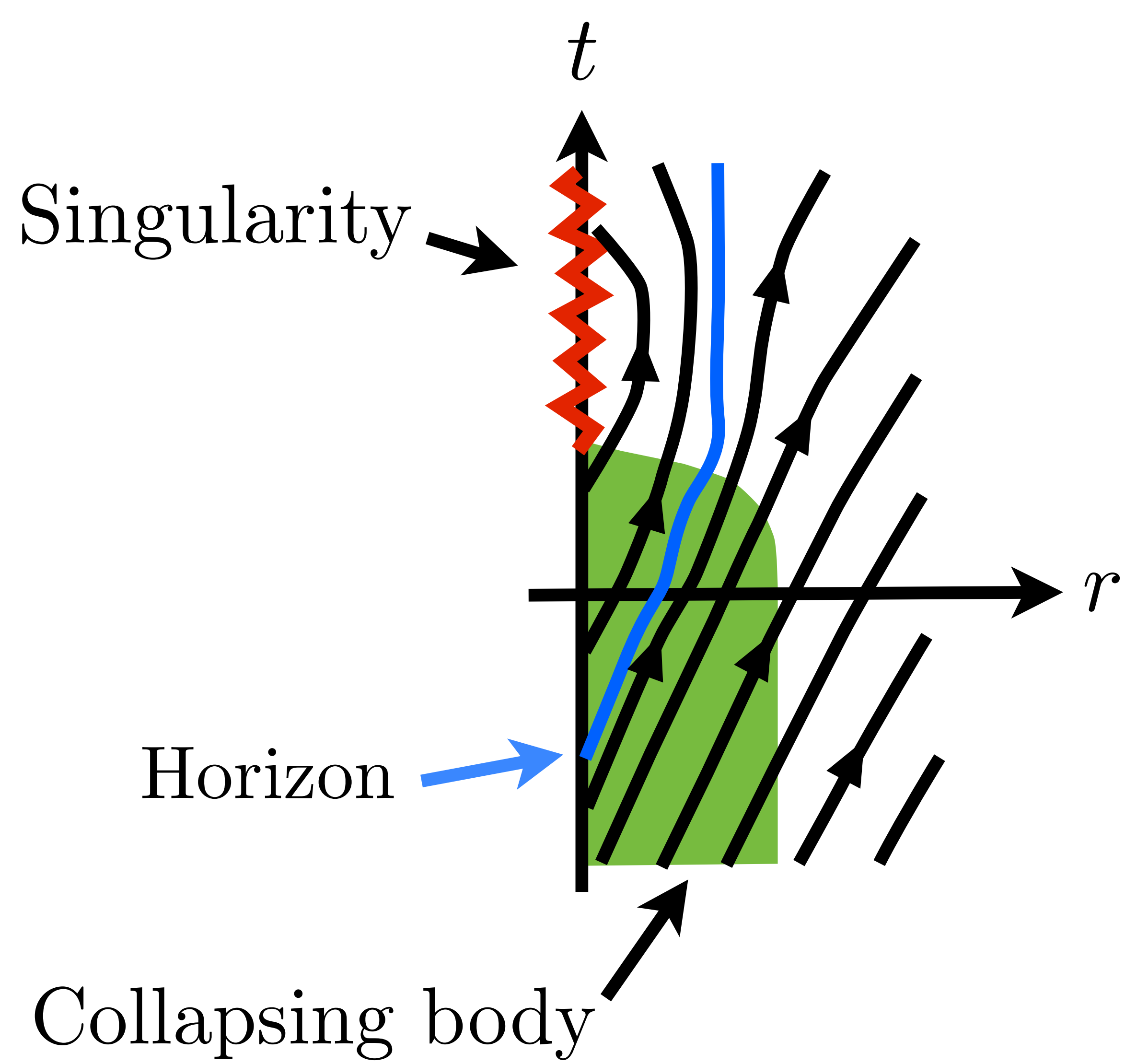


Similarities

- Far-from-equilibrium dynamics.
- Entropy production.

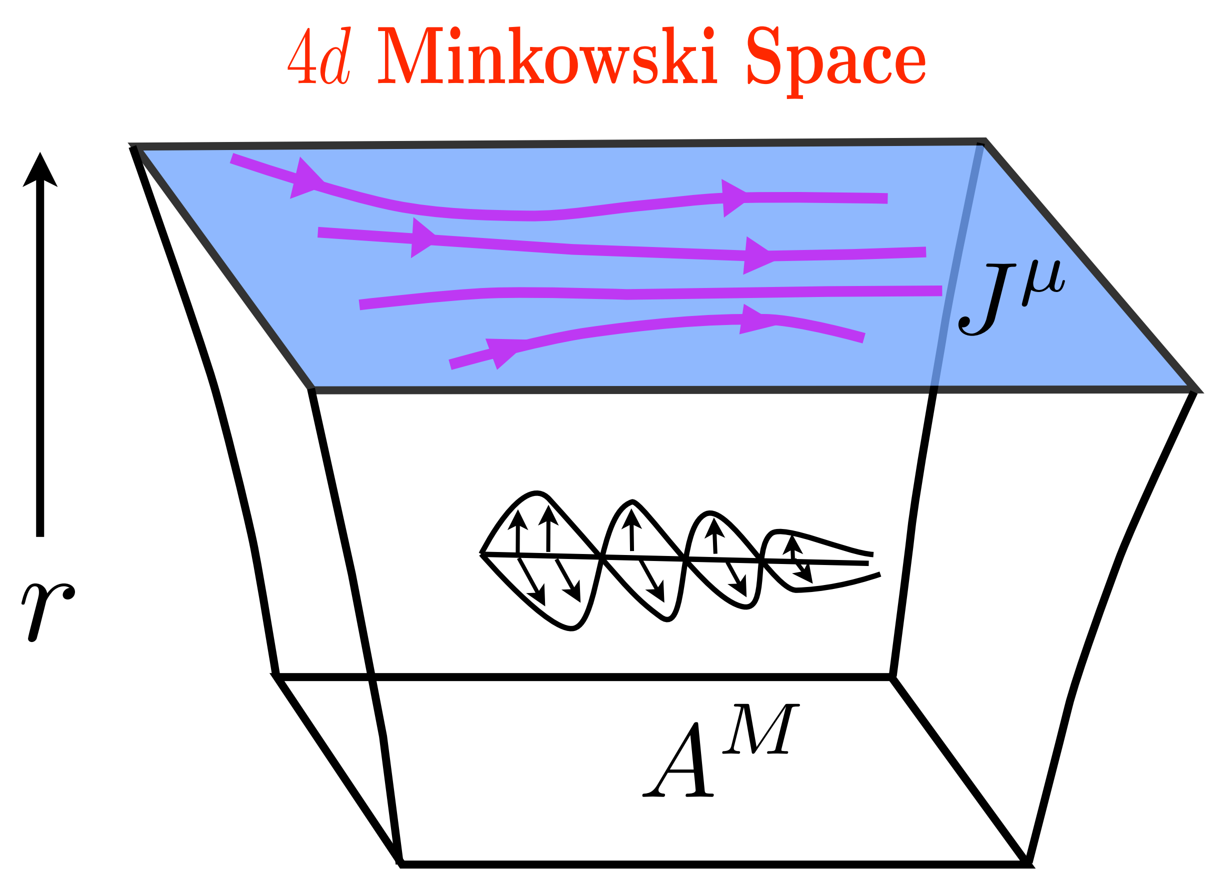
String theory and holographic CFTs

String theory on asymptotically AdS_5 $\Longleftrightarrow$ $4d$ CFT

$$ds^2 = r^2\left[-dt^2 + d\mathbf{x}^2\right] + \frac{dr^2}{r^2}$$

Dictionary

$5d$ **object**		**CFT observables**
A_M	$\Longleftrightarrow$	J^μ
G_{MN}	$\Longleftrightarrow$	$T_{\mu\nu}$
black holes	$\Longleftrightarrow$	QGP
$\vdots$		$\vdots$



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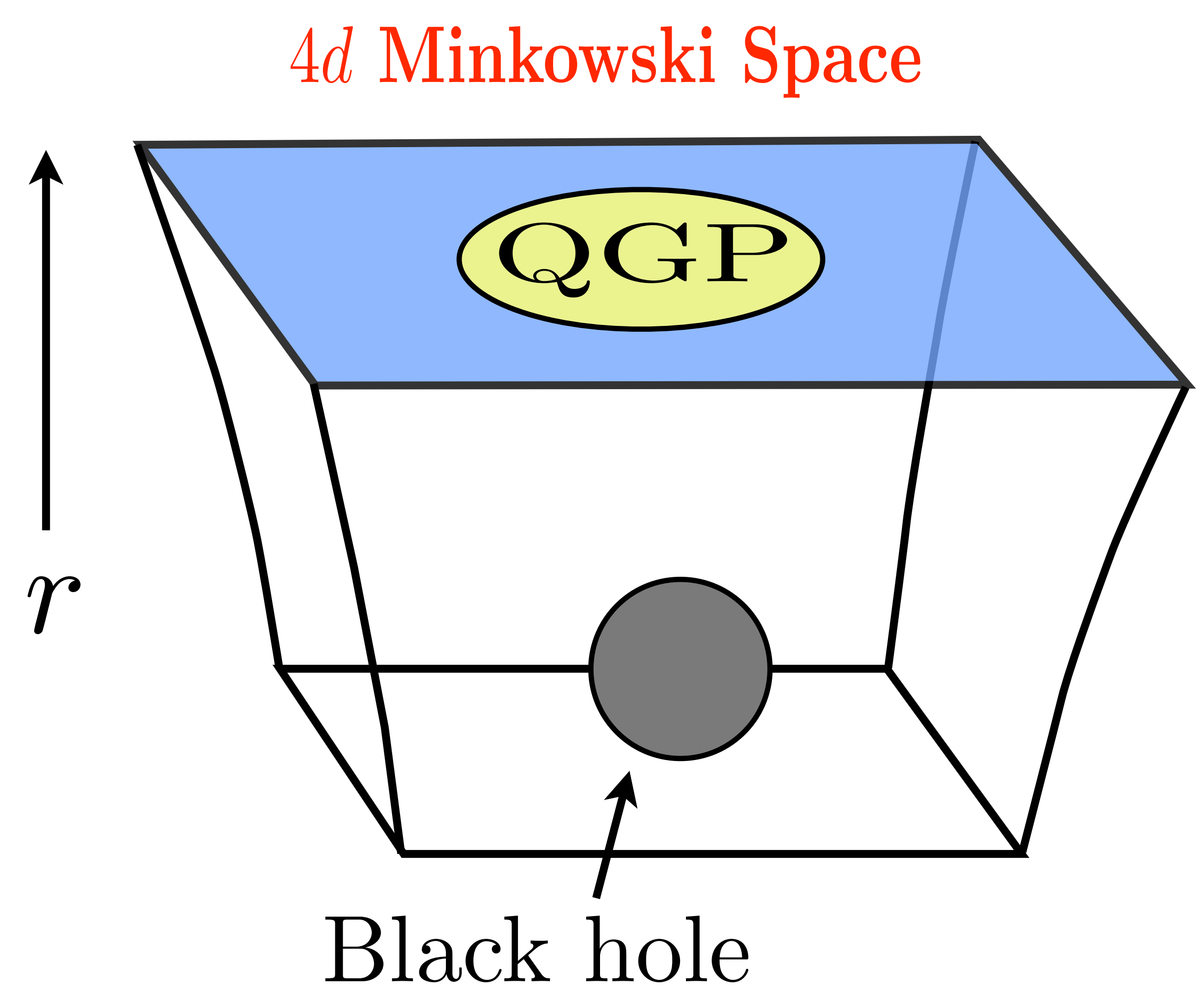
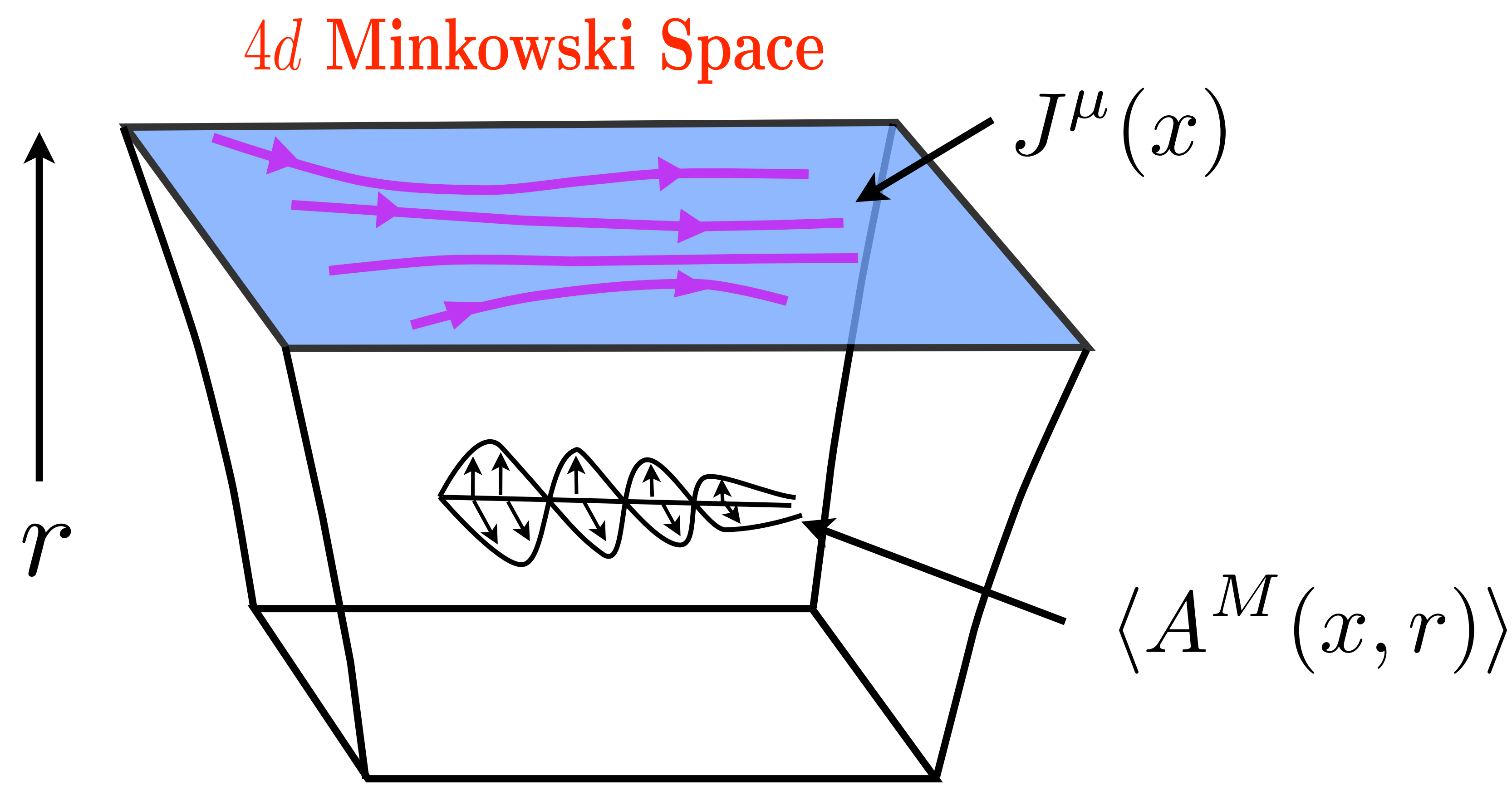


Connecting $5d$ physics to $4d$ physics



- Near-boundary behavior of $\langle\phi(x,r)\rangle \Rightarrow \langle\mathcal{O}(x)\rangle$.
- Near-boundary behavior of $\langle\phi(x,r)\phi(x',r')\rangle \Rightarrow \langle\mathcal{O}(x)\mathcal{O}(x')\rangle$.

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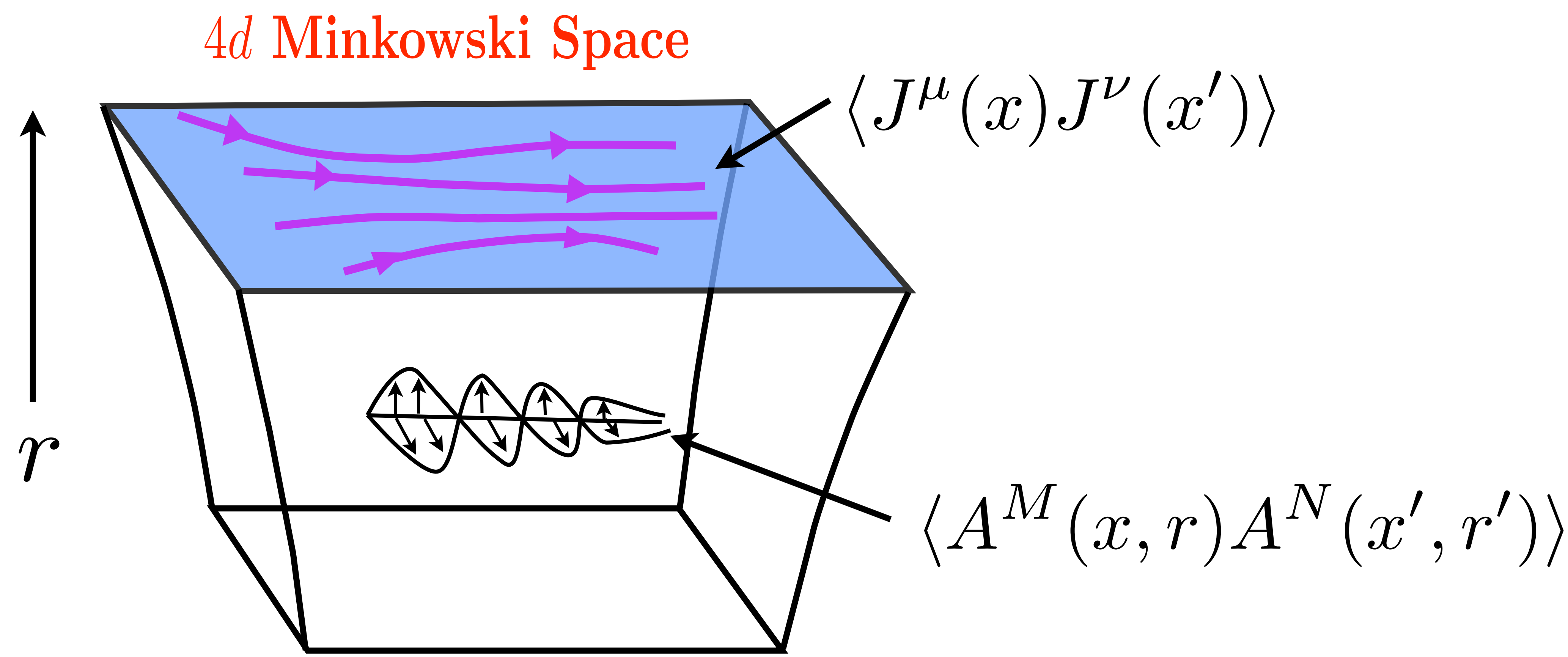

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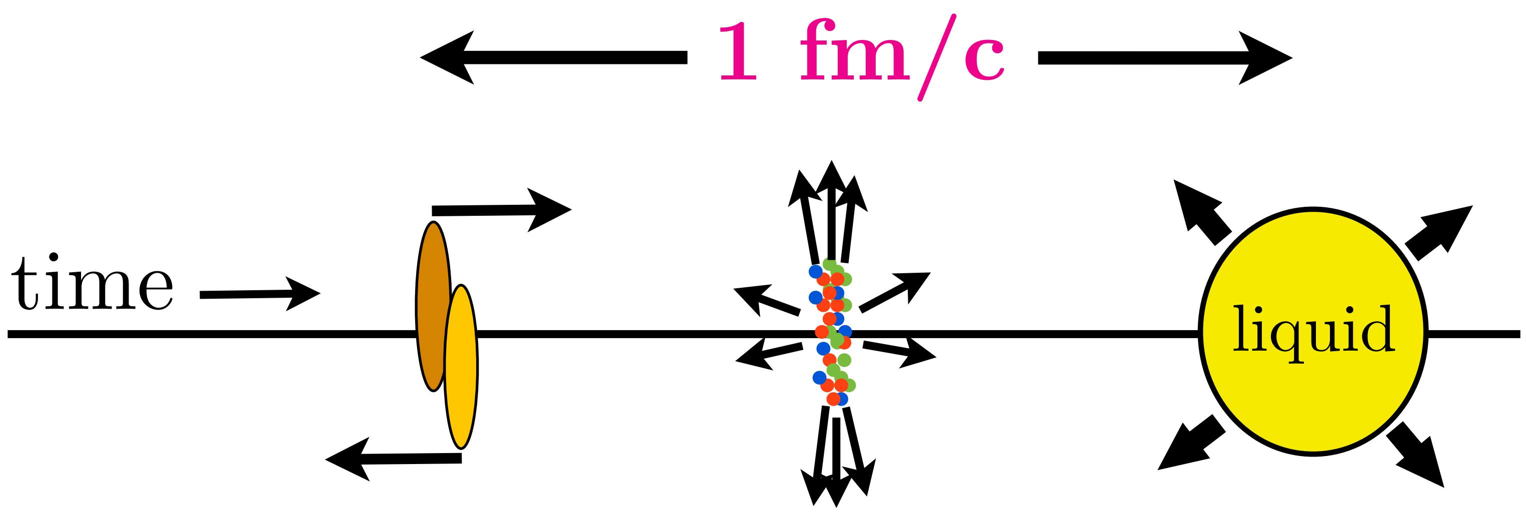
What can holography teach us about far-from-equilibrium dynamics?

Thermalization times in strongly coupled CFTs

- Asymptotic freedom $\Rightarrow \tau_{\rm CFT} < \tau_{QCD}$.



Signatures of local equilibrium:

1. **Hydrodynamic Constitutive Relations**

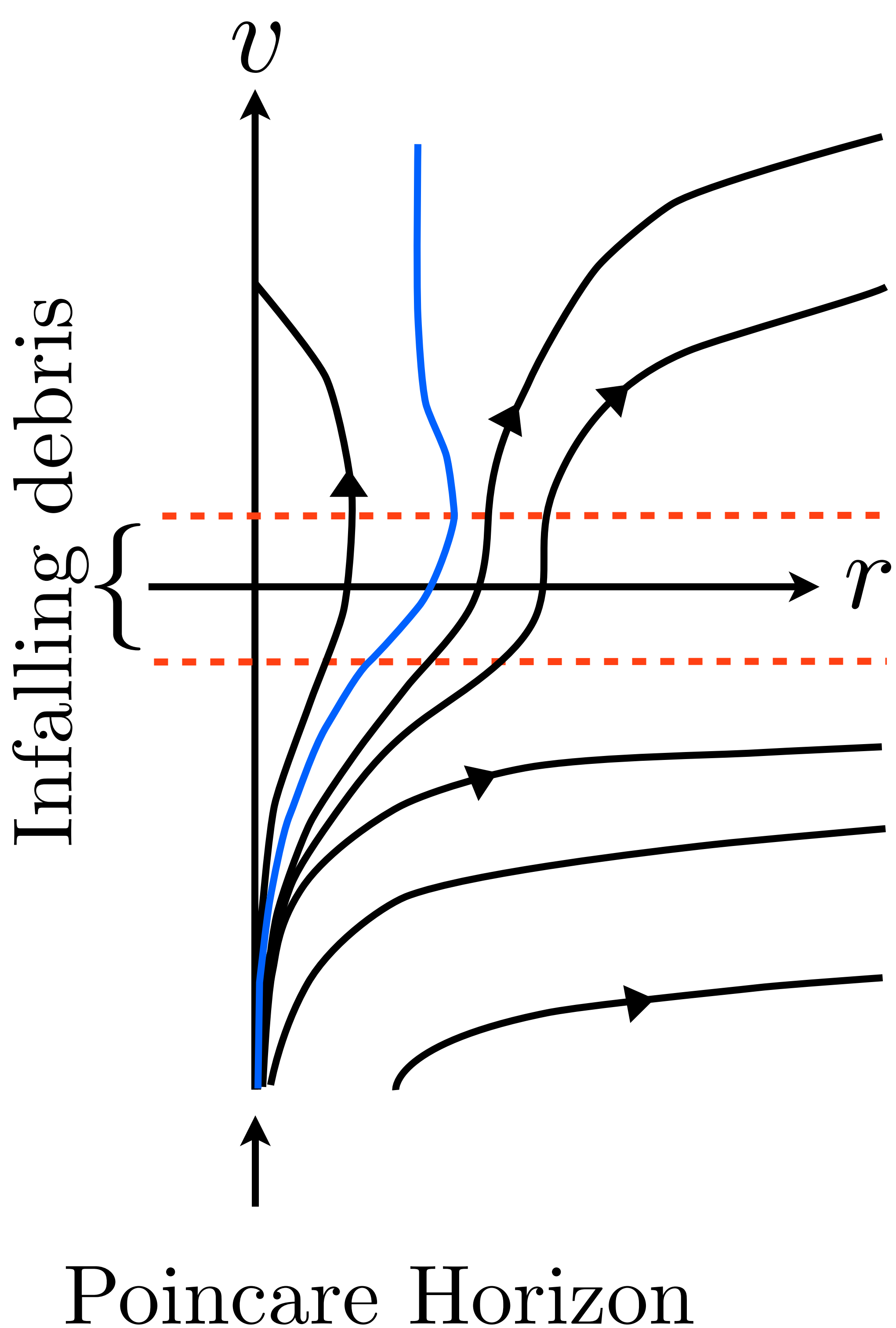
$$T^{\mu\nu} = (\epsilon+p)g^{\mu\nu} + pu^{\mu}u^{\nu} - \eta\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta+\partial_\beta u_\alpha-\tfrac{2}{3}g_{\alpha\beta}\partial\cdot u\right) + \ldots.$$

2. **Local Fluctuation-Dissipation Theorem:**

$$G_{\rm sym}(\omega,\bar{t}|\mathbf{q},\bar{\mathbf{x}}) = -(1+2n){\rm Im}\; G_{\rm ret}(\omega,\bar{t}|\mathbf{q},\bar{\mathbf{x}}).$$

Black brane formation in asymptotically AdS_5

Asymptotic AdS_5 metric: $ds^2 = r^2[-dv^2 + d\mathbf{x}^2] + 2dvdr.$



- Horizon exists *before* debris falls!
- Late time evolution = hydro. [0712.2456]
- Relaxation of BH governed by causality.

$$\tau_{\text{relax}} \sim r_h \sim 1/T.$$

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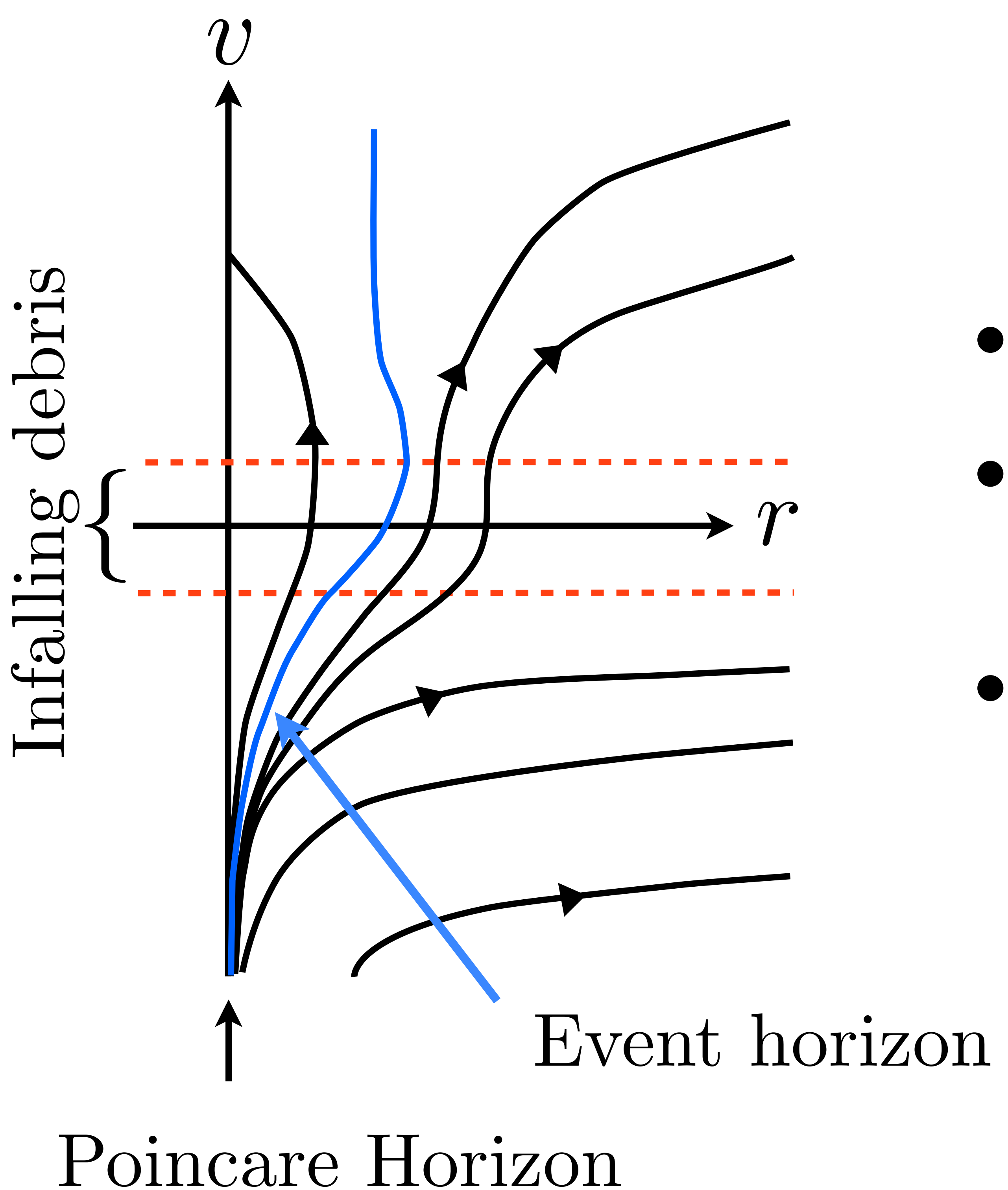

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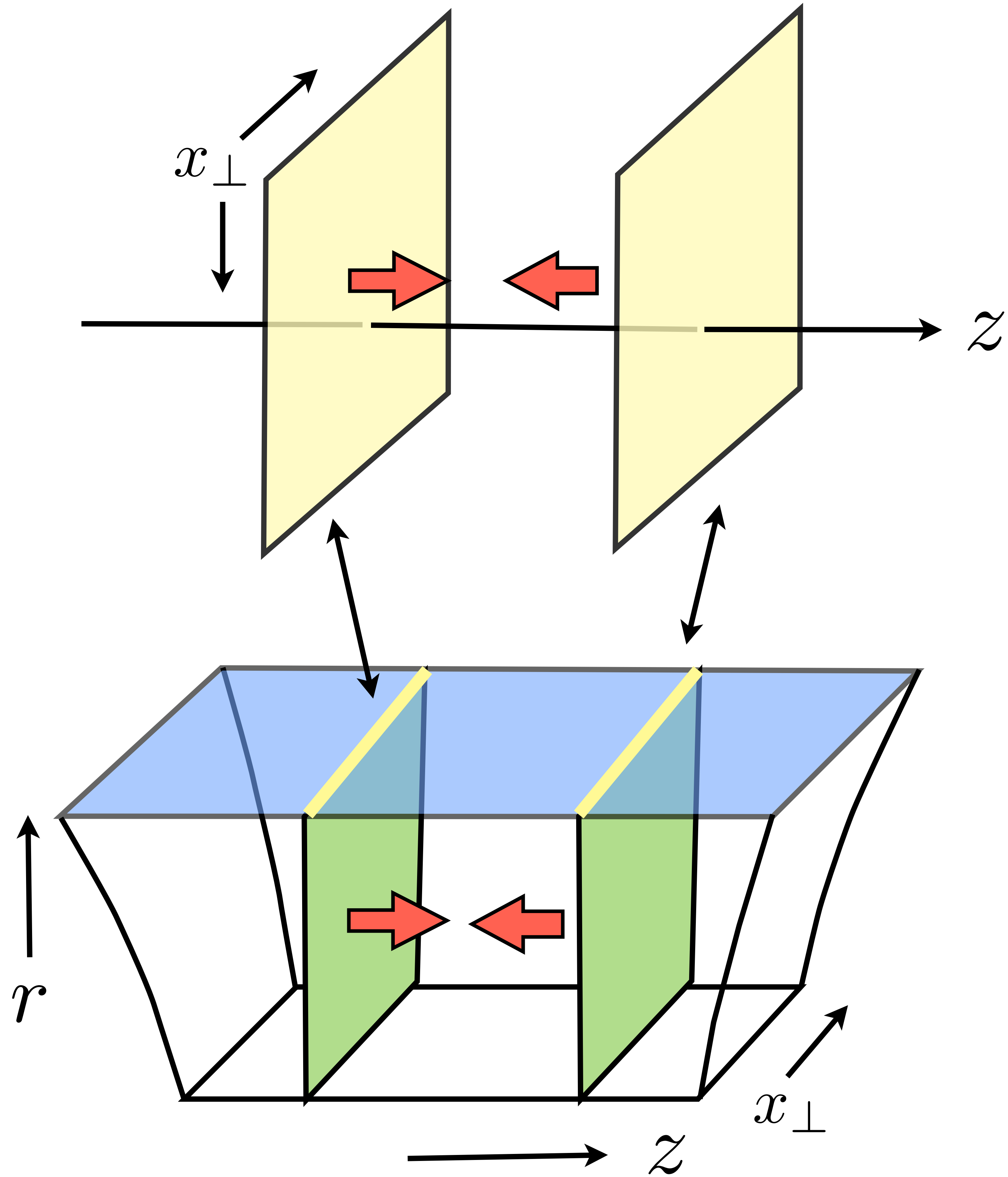
$2 + 1d$ problems — Colliding sheets of matter

CFT description:

- Colliding sheets.
- $\Rightarrow 1 + 1d$ problem.

Gravity description:

- Colliding grav. waves.
- $\Rightarrow 2 + 1d$ problem.



Initial data

Pre-collision metric:

$$ds^2 = r^2 \left[-dx_+ dx_- + dx_\perp^2 + \frac{1}{r^4}\varphi(x_+)dx_+^2 + \frac{1}{r^4}\varphi(x_-)dx_-^2 \right] + \frac{dr^2}{r^2}.$$

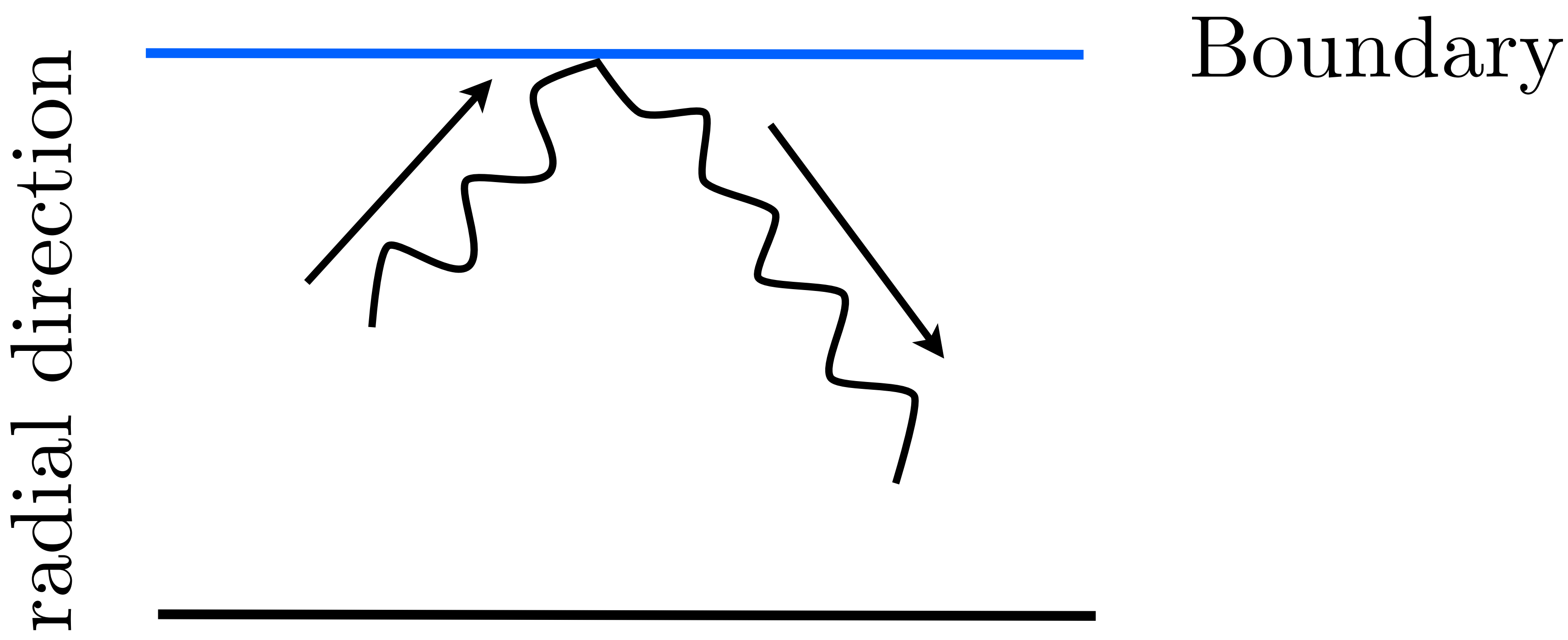
where $x_\pm = t \pm z$.

Properties:

- Exact solution to sourceless Einstein when $\varphi(x_\pm)$ don't overlap.
- Field theory stress:

$$\mathcal{E} = \varphi(x_+) + \varphi(x_-), \quad \mathcal{P}_\perp = 0,$$
$$\mathcal{S} = \varphi(x_+) - \varphi(x_-), \quad \mathcal{P}_{||} = \varphi(x_+) + \varphi(x_-).$$

Choosing a coordinate system



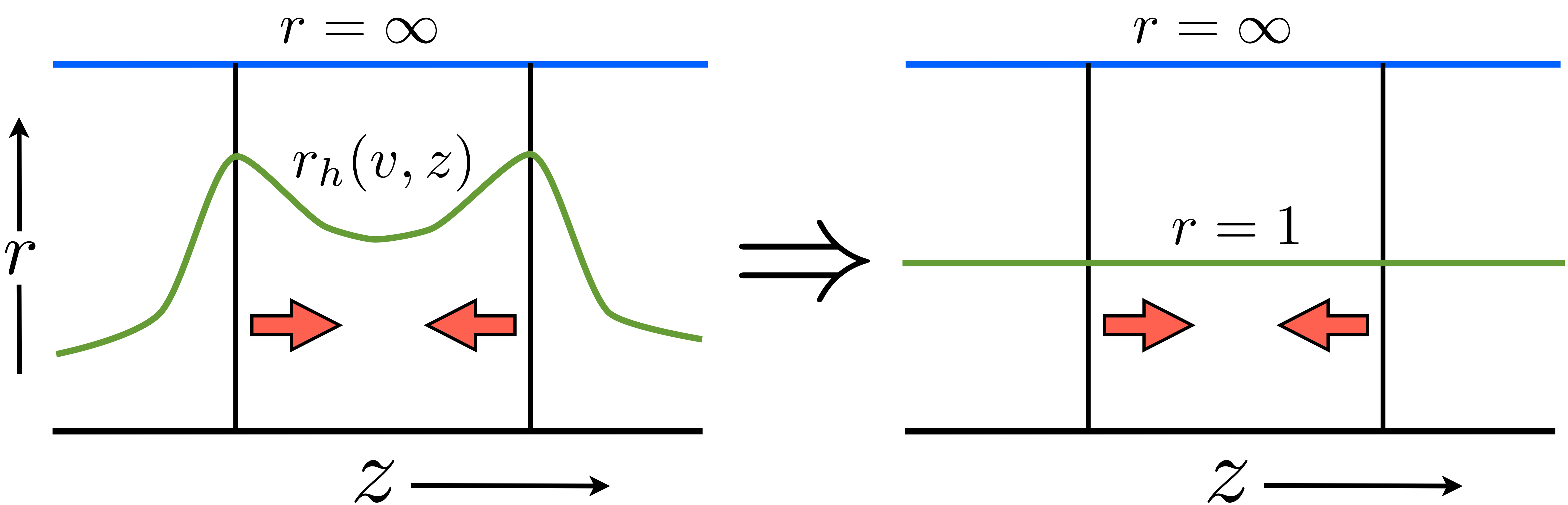
Desirable features:

- Natural implementation of BC at boundary.
- Idealized for infalling radiation.
- Regularity at horizon.

Metric ansatz: $ds^2 = -Adv^2 + \Sigma^2 \left[e^B dx_\perp^2 + e^{-2B} dz^2\right] + 2drdv + 2Fdzdv.$

Residual gauge freedom: $r \to r + \xi(v, z)$

Removing the residual gauge freedom and horizon excision.



Residual gauge freedom: $r \to r + \xi(v, z)$.

Gauge transformation: $\xi(v, z) = -1 + r_h(v, z)$.

Parameters

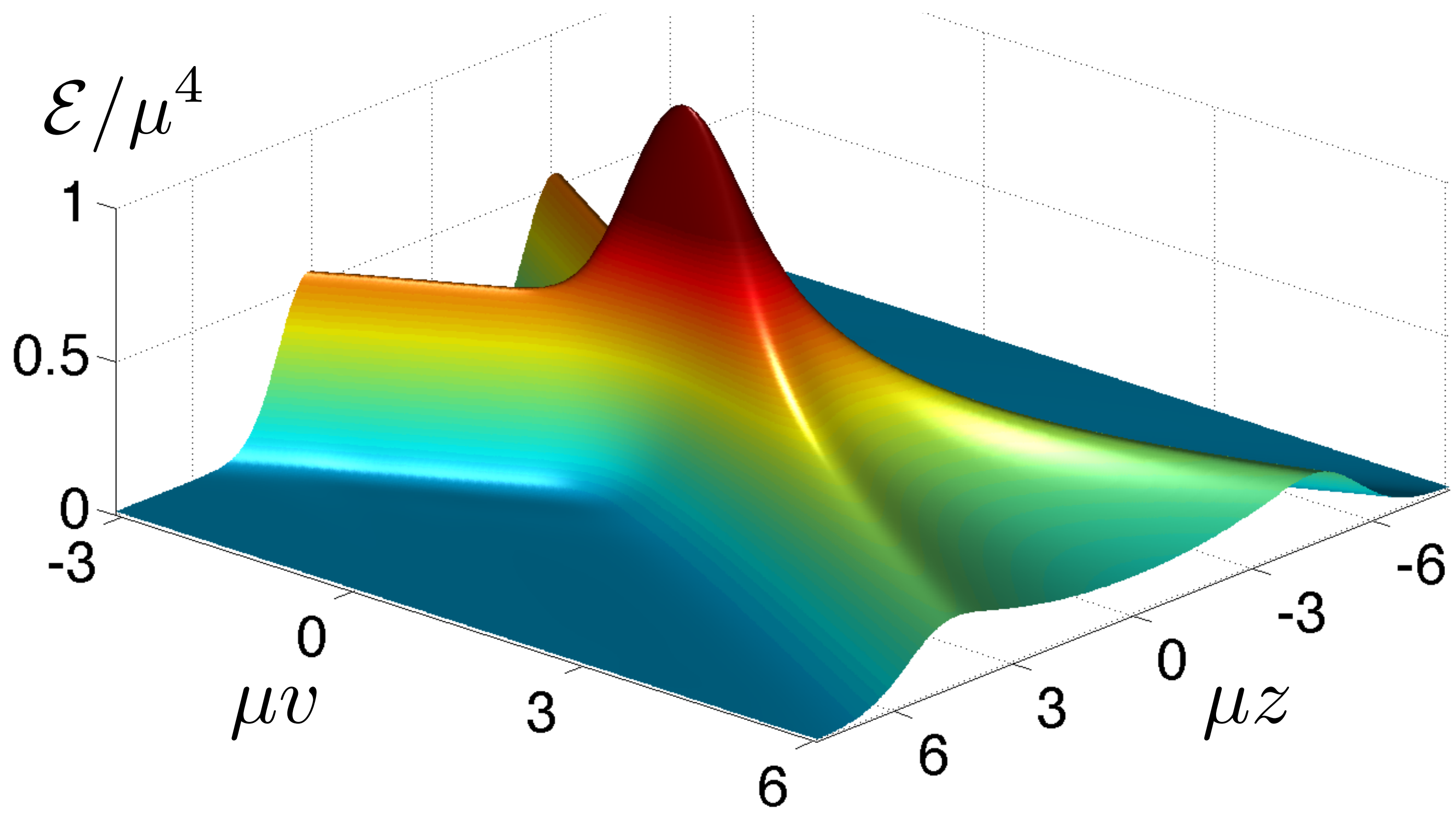
- "Shock" profile: $\varphi(z) = \frac{\mu^3}{\sqrt{2\pi}\sigma} e^{-z^2/(2\sigma^2)}$.

- Energy per unit area $= \frac{3\mu^3 N_{\rm c}^2}{8\pi^2}$.

- Width $\sigma = 0.75/\mu$

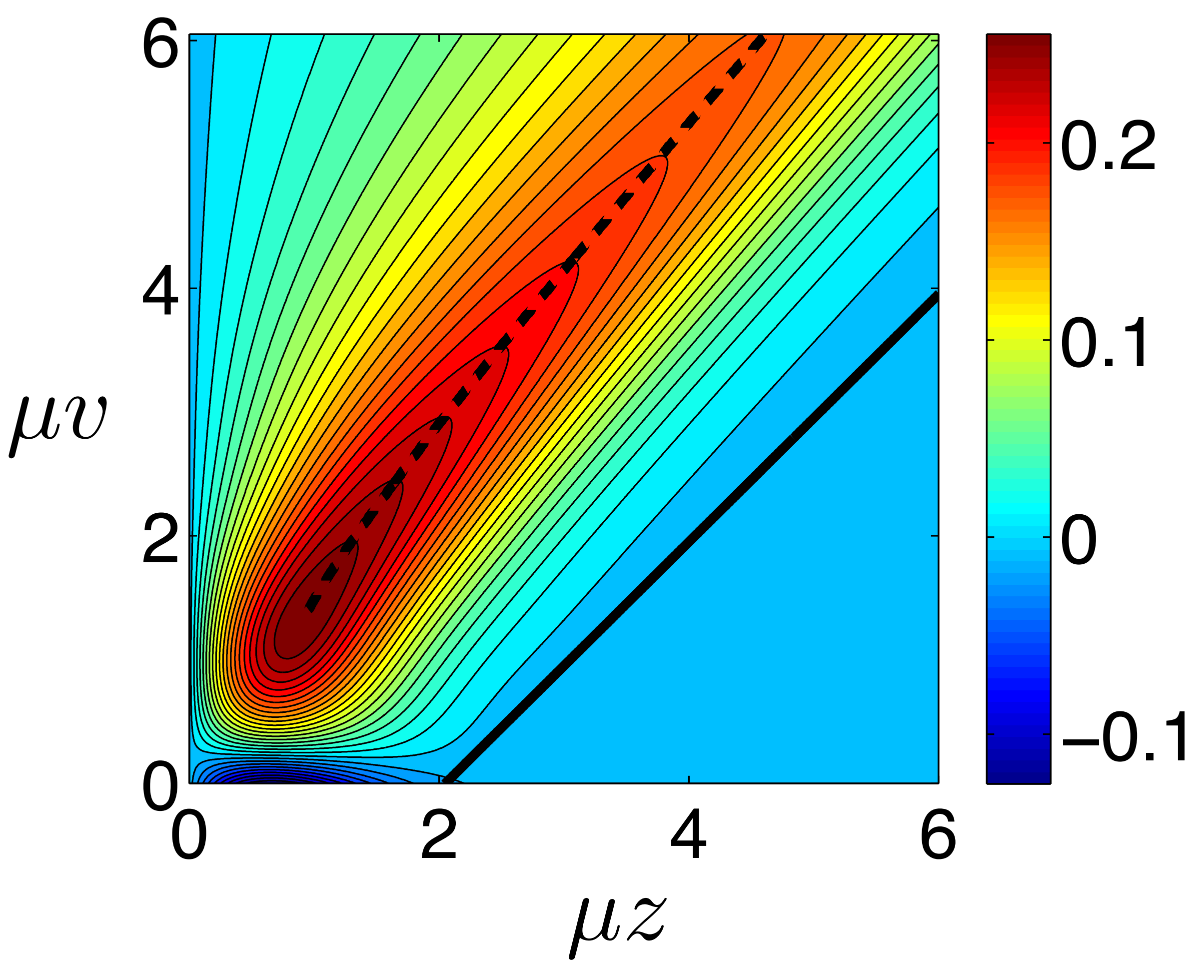
Results illustrated

[Chesler & Yaffe: 1011.3562]



Energy density

Energy flux



Hydrodynamic constituent relations

Hydrodynamic ingredients:

- Proper energy density ϵ.
- Fluid velocity 3-velocity V.
- Transport coefficients $\frac{\eta}{s} = \frac{1}{4\pi}$,
- EOS $\epsilon = 3p$.

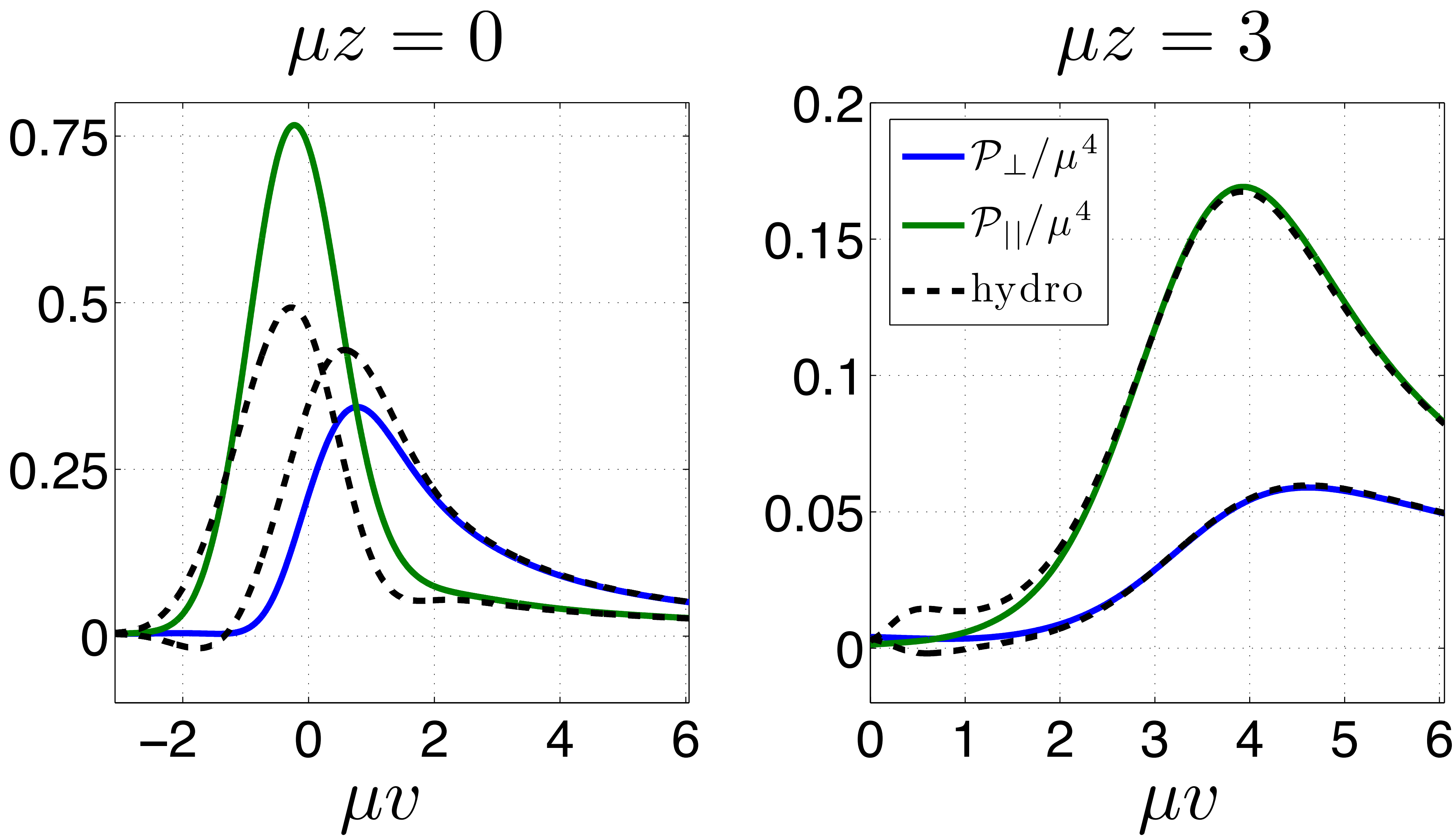
Constituent relations:

$$\mathcal{P}_{\perp}^{\text{hydro}} = \tfrac{1}{3}\epsilon + \frac{2\eta\gamma^3}{3}\left[\partial_z V + V\partial_v V\right] + \ldots,$$
$$\mathcal{P}_{||}^{\text{hydro}} = \tfrac{1}{3}\gamma^2(1+3V^2)\epsilon - \frac{4\eta\gamma^5}{3}\left[\partial_z V + V\partial_v V\right] + \ldots,$$

where

$$V = 2\mathcal{S}/\left[\mathcal{P}_{||} + \mathcal{E} + \sqrt{(\mathcal{P}_{||}+\mathcal{E})^2 - 4\mathcal{S}^2}\right],$$
$$\epsilon = \tfrac{1}{2}\left[\mathcal{E} - \mathcal{P}_{||} + \sqrt{(\mathcal{E}+\mathcal{P}_{||})^2 - 4\mathcal{S}^2}\right].$$

Comparing to 1st order hydrodynamics



- Hydro works within 15% for $v > 2.4/\mu$.
  - Estimate for RHIC: $\tau_{\text{hydro}} \sim 0.35$ fm/c.
- $\mathcal{P}_\perp \gtrsim 2\mathcal{P}_{||}$ at $z = 0 \Rightarrow$ viscous effects are important.

Previous recipes for computing correlation functions

Son & Starinets: hep-th/0205051:

+ Recipe for computing equilibrium correlation functions.

- Only works in equilibrium.

Skenderis & van Rees: 0805.0150:

+ Recipe for computing correlation functions in any state.

- Requires analytic continuation to imaginary time.

Balaburamanian et al: 1012.4753:

+ Recipe for computing correlation functions in any state.

- Valid only for large conformal dimension operators.

A new look at an old recipe

[Caron-Huot, Chesler & Teaney: 1102.1073]

Example: dilaton:

$$g_{\rm sym}(1|2) = \tfrac{1}{2}\langle\{\phi(1),\phi(2)\}\rangle.$$

Equations of motion:

$$-D_1^2 g_{\rm sym}(1|2) = -D_2^2 g_{\rm sym}(1|2) = 0.$$

Formal solution:

$$g_{\rm sym}(1|2) = \int_{v_1'=v_1'=-\infty} G_{\rm ret}(1|1')G_{\rm ret}(2|2')\overleftrightarrow{\partial}_{v_1'}\overleftrightarrow{\partial}_{v_2'} g_{\rm sym}(1'|2')\Big|_{v_1'=v_2'=-\infty}$$

where $-D_1^2 G_{\rm ret}(1|1') = \delta(1-1')$.

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All info about density matrix in initial conditions.

Hawking radiation and horizon correlators

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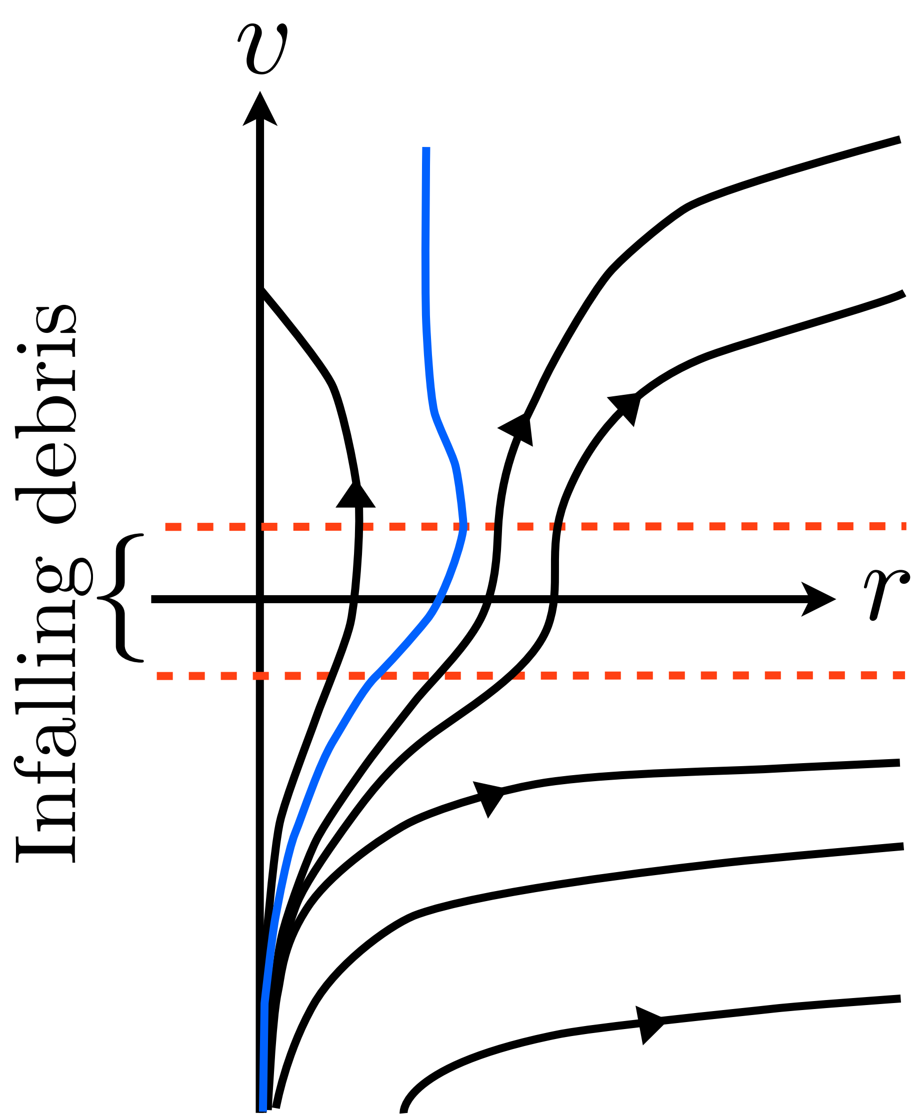

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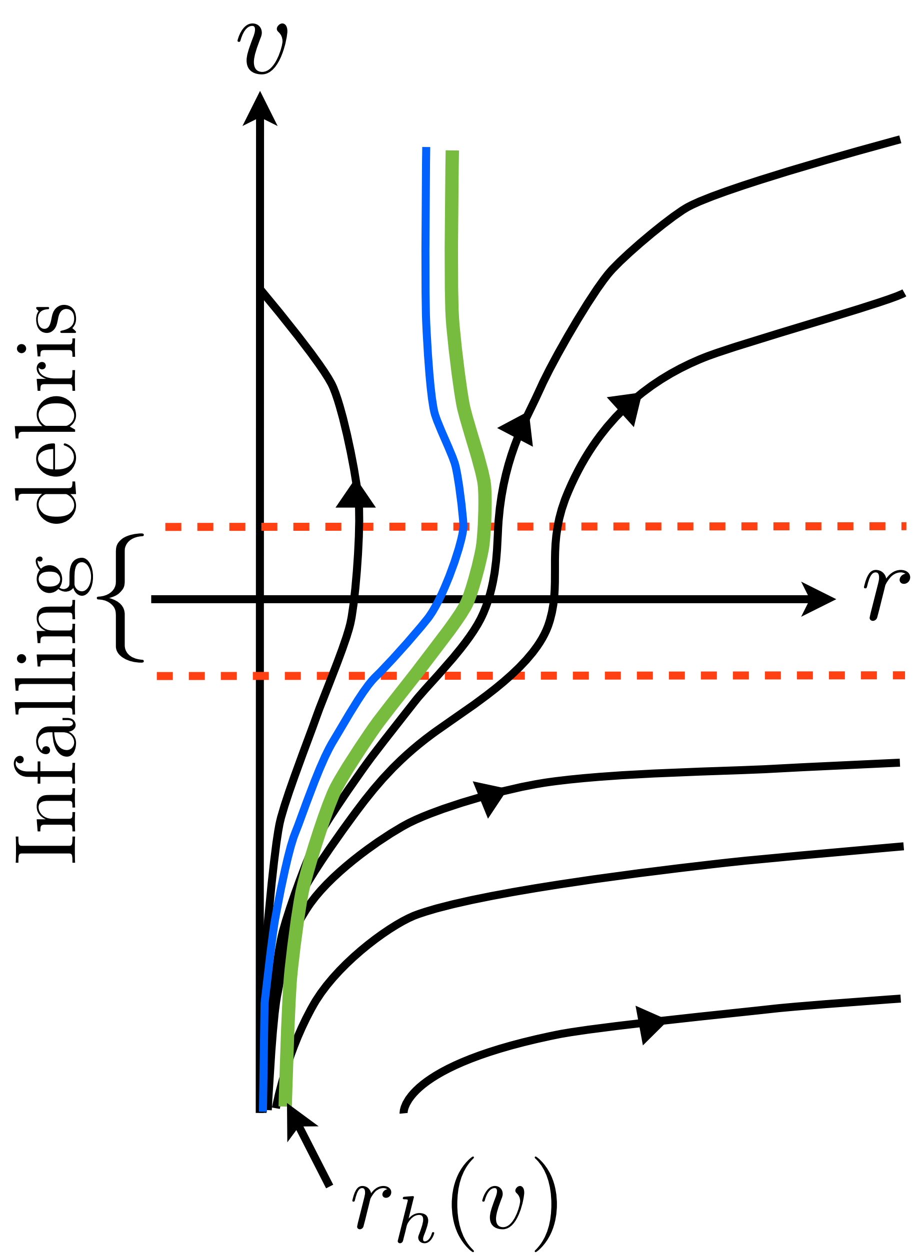

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&= \int_{r_1'=r_h(v_1'),r_2'=r_h(v_2')} G_{\mathrm{ret}}(1|1')G_{\mathrm{ret}}(2|2')G^h_{\mathrm{sym}}(1'|2').
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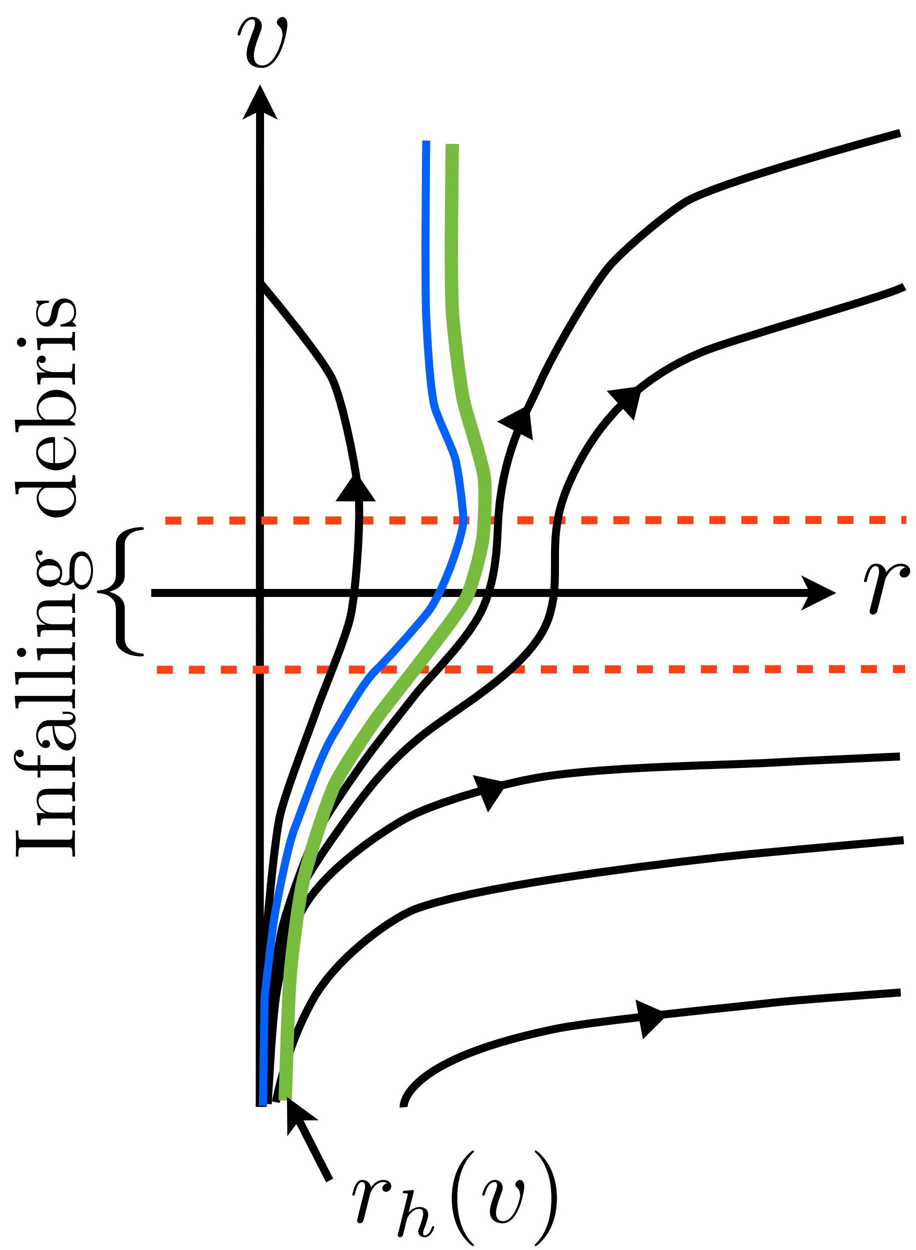

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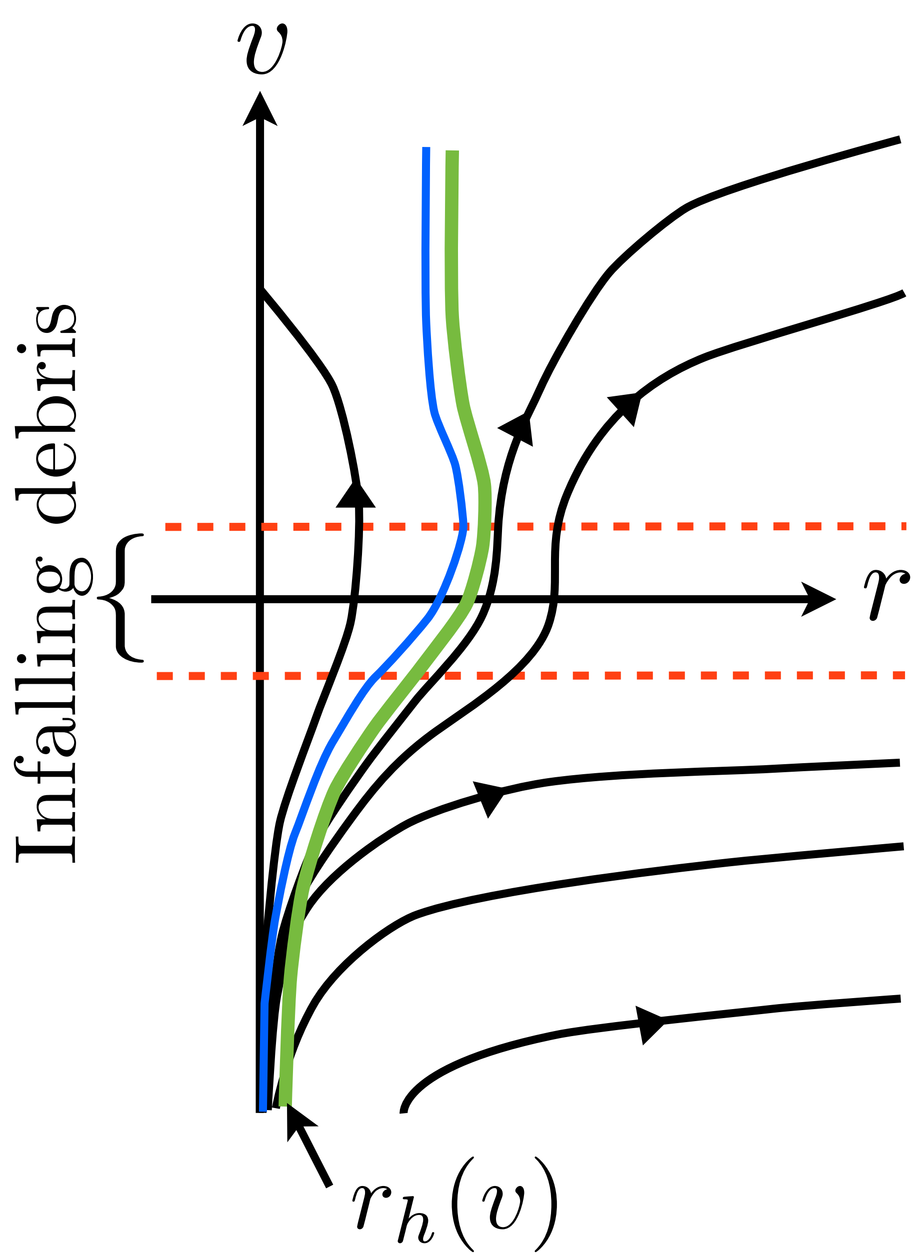

Properties of horizon correlators:

- Only depend on local surface gravity of BH and UV behavior of initial conditions.
- Encode time-dependent Hawking radiaiton.
- Satisfy FDT in equilibrium

$$G^h_{\rm sym} = -(1+2n)G^h_{\rm ret}.$$

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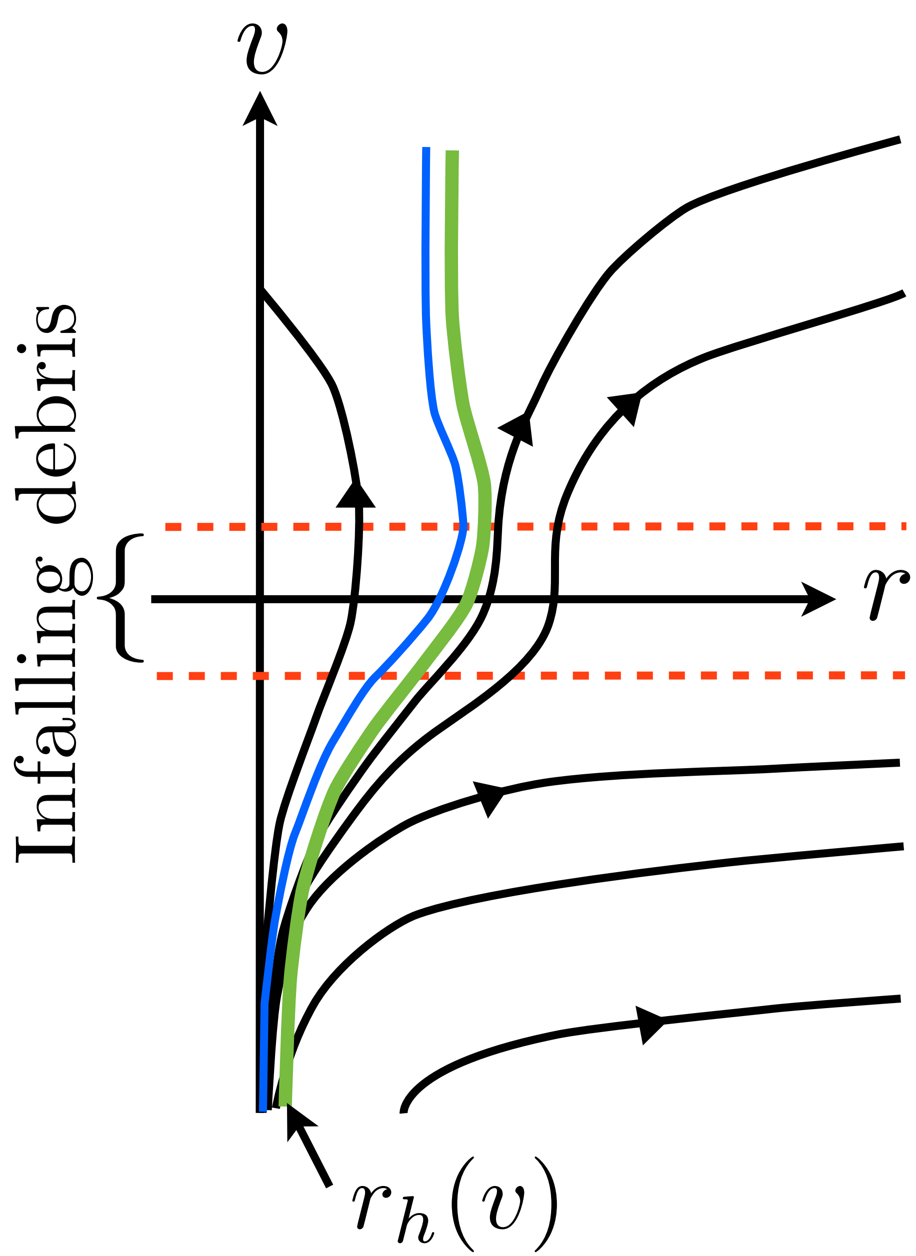

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Stay tuned for QFT correlators!

Concluding remarks

- Difficult QFT problems tractable with holography & numerical GR.
- Thermalization times < 1 fm/c are natural in strongly coupled theories.

Open questions:

- Relevance for heavy ion phenomenology?
  - Lower bound for QCD thermalization $\tau_{\text{therm}} = 0.35$ fm/c?
  - How much longer should QCD thermalization be?
- How does thermalization of correlators compare to $\langle T_{\mu\nu} \rangle$?

Future directions:

- Far-from-equilibrium jet quenching.

Comparing to boost invariant flow

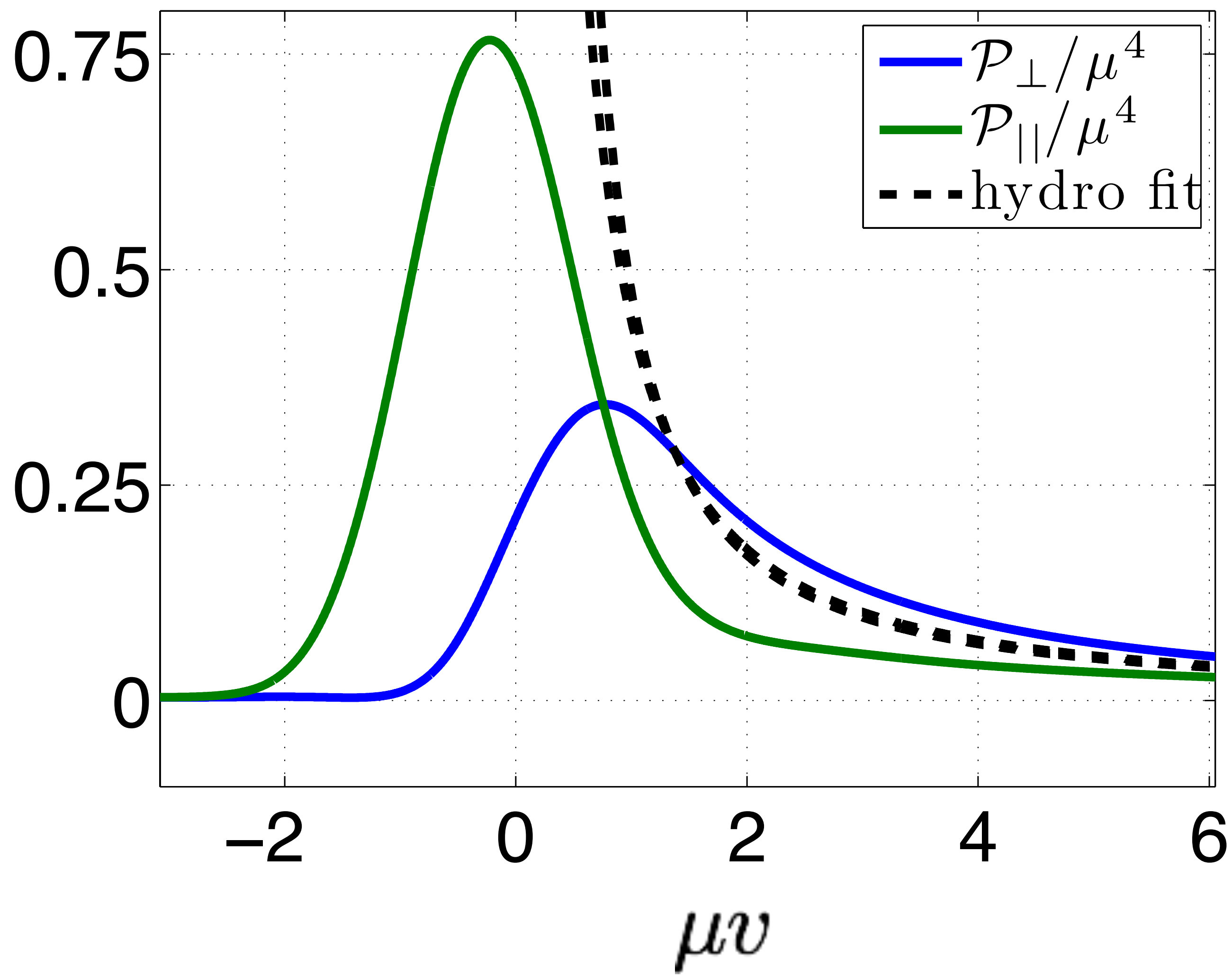
Hydro expansion:

(Janik & Peschanski: hep-th/0512162)
(Kinoshita, Mukohyama, Nakamura & Oda: 0807.3797)

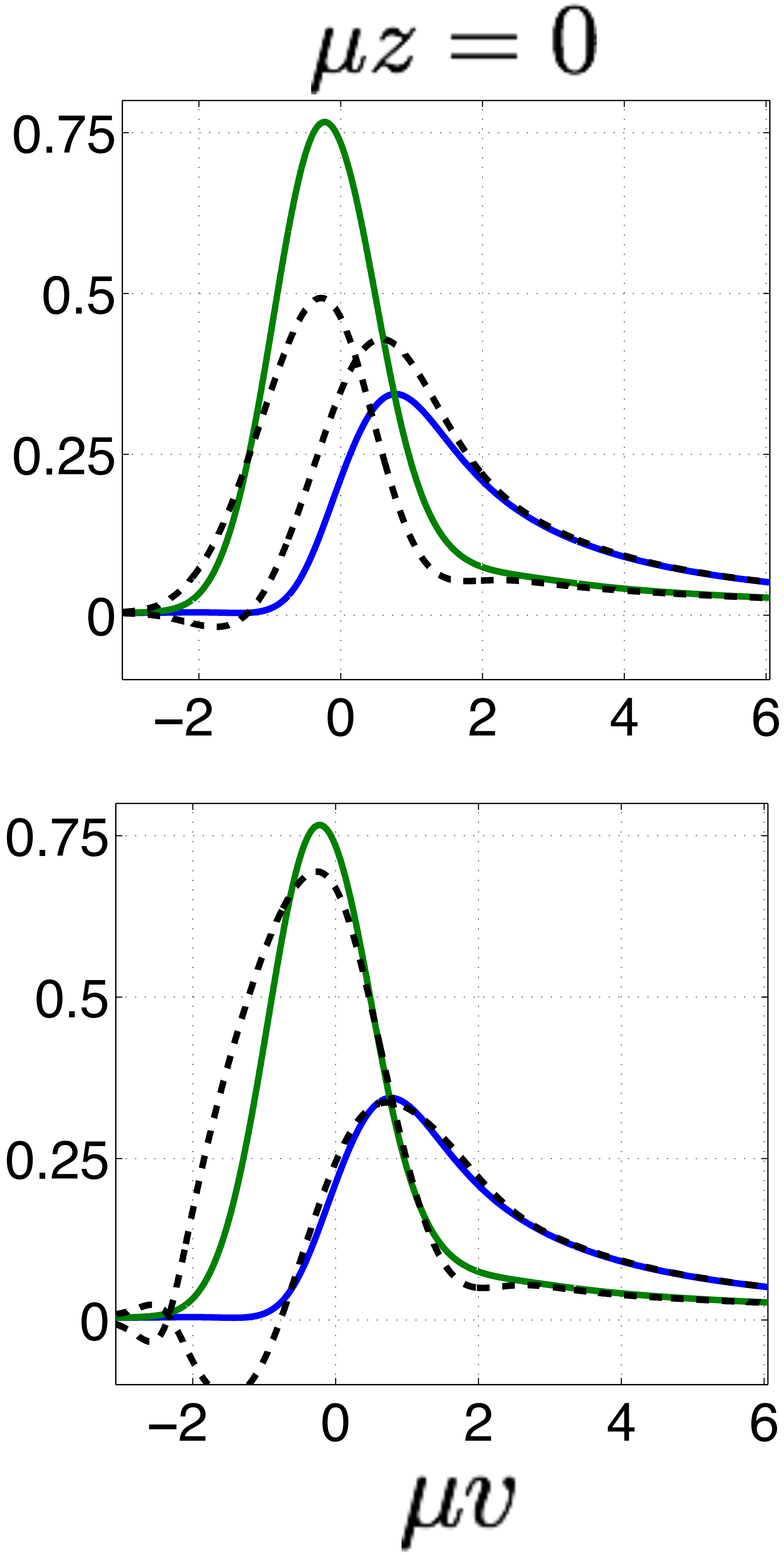
$$\mathcal{P}_\perp = \frac{\pi^4\Lambda^4}{4(\Lambda v)^{4/3}}\left[1 - \frac{C_2}{3(\Lambda v)^{4/3}} + \mathcal{O}(v^{-2})\right],$$

$$\mathcal{P}_{||} = \frac{\pi^4\Lambda^4}{4(\Lambda v)^{4/3}}\left[1 - \frac{2C_1}{(\Lambda v)^{2/3}} + \frac{5C_2}{3(\Lambda v)^{4/3}} + \mathcal{O}(v^{-2})\right].$$

$$C_1 = \frac{1}{3\pi} \propto \frac{\eta}{s}, \quad C_2 = \frac{2+\ln 2}{18\pi^2}$$



Comparing to 2^{nd} order hydrodynamics



1^{st} order

2^{nd} order

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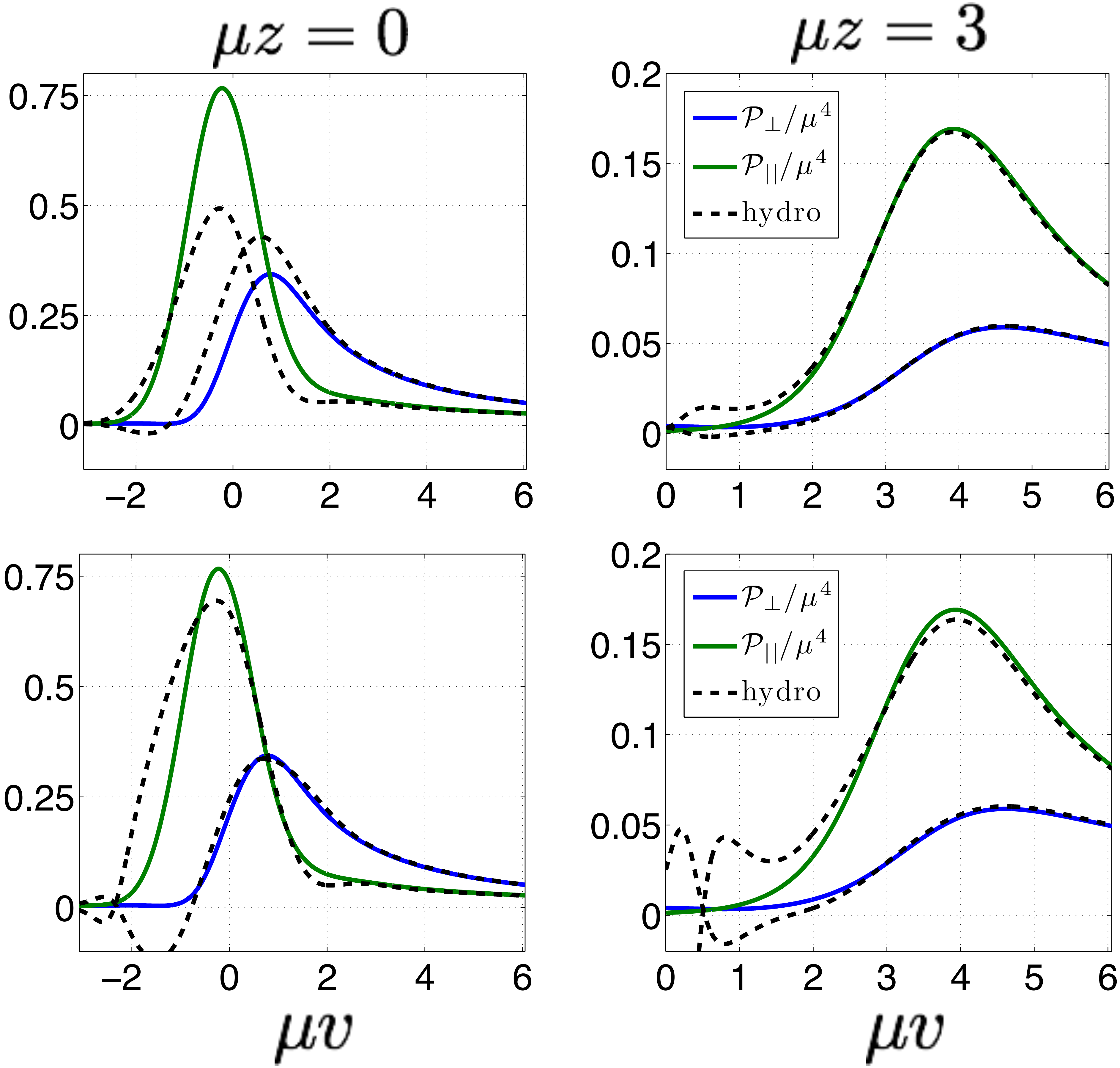
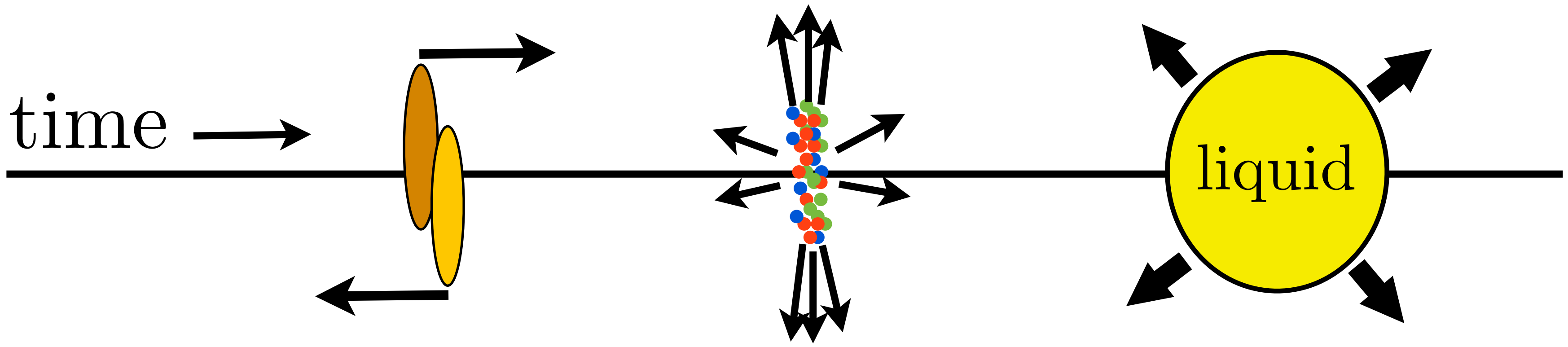


1^{st} order

2^{nd} order

The destination: collision of particles in $5d$



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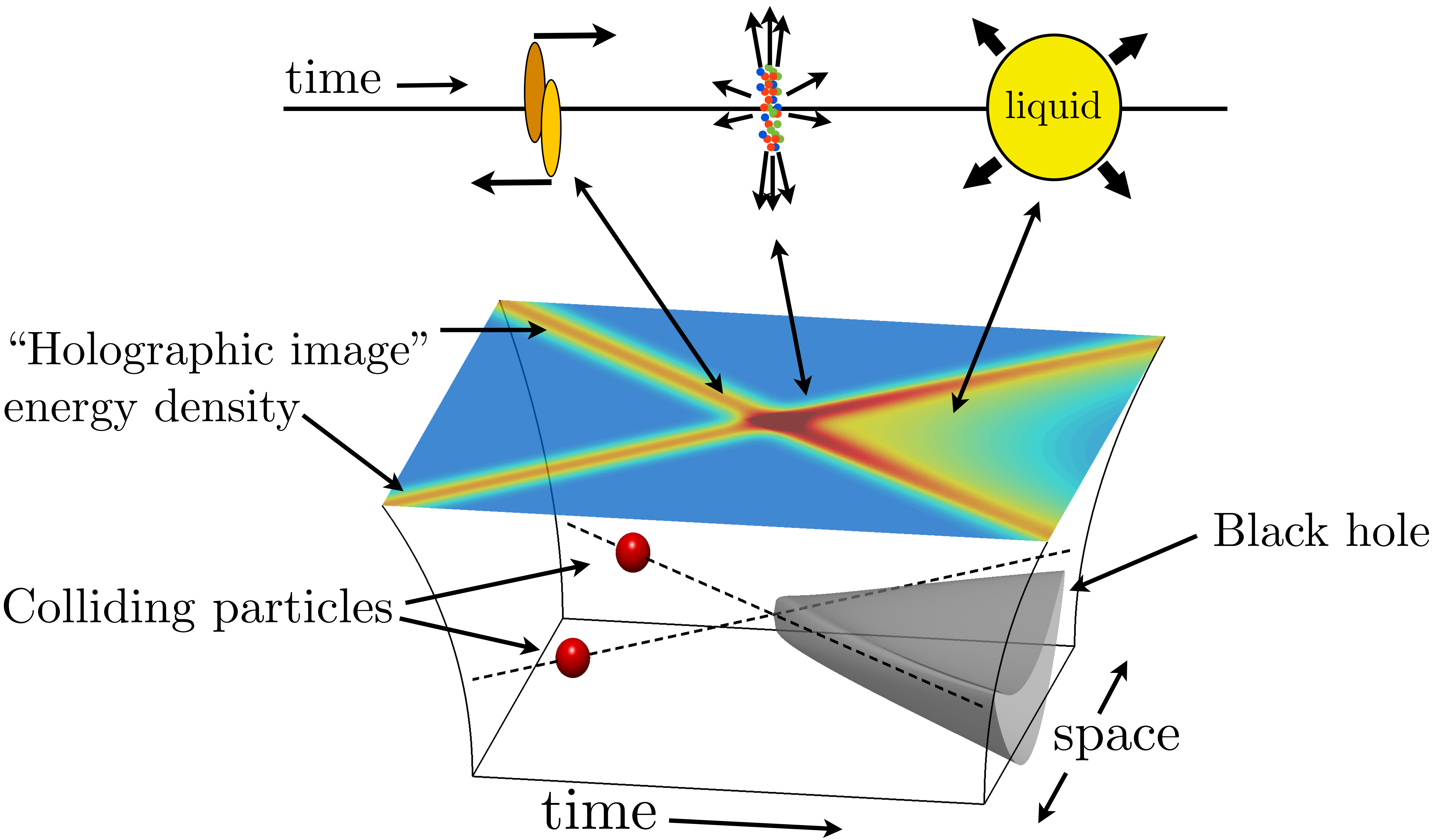

Einstein's equations

$$0 = \Sigma'' + \tfrac{1}{2}(B')^2\,\Sigma\,,$$

$$0 = \Sigma^2\left[F'' - 2(d_3B)' - 3B'd_3B\right] + 4\Sigma' d_3\Sigma\,, -\Sigma\left[3\Sigma' F' + 4(d_3\Sigma)' + 6B'd_3\Sigma\right],$$

$$0 = 2\Sigma^2 A'' + 6\Sigma^4 B'd_+B - 24\Sigma^2\Sigma' d_+\Sigma + 8\Sigma^4 + e^{2B}\Big\{\Sigma^2\left[(F')^2 - 7(d_3B)^2 - 4d_3^2B\right]$$
$$+\, 4(d_3\Sigma)^2 - 8\Sigma\left[2(d_3B)d_3\Sigma + d_3^2\Sigma\right]\Big\}\,,$$

$$0 = 6\Sigma^3(d_+\Sigma)' + 12\Sigma^2(\Sigma' d_+\Sigma - \Sigma^2) - e^{2B}\Big\{2(d_3\Sigma)^2 + \Sigma^2\left[\tfrac{1}{2}(F')^2 + (d_3F)' + 2F'd_3B - \tfrac{7}{2}(d_3B)^2 - 2d_3^2B\right]$$
$$+\, \Sigma\left[(F' - 8d_3B)\,d_3\Sigma - 4d_3^2\Sigma\right]\Big\}\,.$$

$$0 = 6\Sigma^4(d_+B)' + 9\Sigma^3(\Sigma' d_+B + B'd_+\Sigma) + e^{2B}\Big\{\Sigma^2[(F')^2 + 2(d_3F)' + F'd_3B - (d_3B)^2 - d_3^2B]$$
$$+\, 4(d_3\Sigma)^2 - \Sigma\left[(4F' + d_3B)\,d_3\Sigma + 2d_3^2\Sigma\right]\Big\}\,,$$

$$0 = 6\Sigma^2 d_+^2\Sigma - 3\Sigma^2 A'd_+\Sigma + 3\Sigma^3(d_+B)^2 - e^{2B}\Big\{(d_3\Sigma + 2\Sigma d_3B)(2d_+F + d_3A)$$
$$+\, \Sigma\left[2d_3(d_+F) + d_3^2A\right]\Big\}\,,$$

$$0 = \Sigma\left[2d_+(d_3\Sigma) + 2d_3(d_+\Sigma) + 3F'd_+\Sigma\right] + \Sigma^2\left[d_+(F') + d_3(A') + 4d_3(d_+B) - 2d_+(d_3B)\right]$$
$$+\, 3\Sigma\,(\Sigma d_3B + 2d_3\Sigma)\,d_+B - 4(d_3\Sigma)d_+\Sigma\,,$$

where $h' \equiv \partial_r h$, $d_+h \equiv \partial_v h + \frac{1}{2}A\,\partial_r h$, $d_3h \equiv \partial_z h - F\,\partial_r h\,.$

What are the required boundary conditions?

Solve Einstein with series expansion in r:

$$A = r^2\left[1 + \frac{2\xi}{r} + \frac{\xi^2 - 2\partial_v\xi}{r^2} + \frac{a_4}{r^4} + O(r^{-5})\right],$$

$$F = \partial_z\xi + \frac{f_2}{r^2} + O(r^{-3}).$$

$$B = \frac{b_4}{r^4} + O(r^{-5}),$$

$$\Sigma = r + \xi + O(r^{-7}),$$

$$\partial_v a_4 = -\tfrac{4}{3}\,\partial_z f_2\,, \quad \partial_v f_2 = -\partial_z(\tfrac{1}{4}a_4 + 2b_4)\,.$$

Field theory stress:

$$\mathcal{E} = -\tfrac{3}{4}a_4\,, \qquad \mathcal{P}_\perp = -\tfrac{1}{4}a_4 + b_4\,,$$

$$\mathcal{S} = -f_2\,, \qquad \mathcal{P}_\| = -\tfrac{1}{4}a_4 - 2b_4\,.$$

Required initial data:

B, a_4 f_2 at $v =$ constant and ξ at all v.

Problems and solutions

Challenges:

i. Singular point in Einstein at $r = \infty$.

ii. Coord trans to EF coordinates:

$$ds^2 = r^2 \left[-dx_+ dx_- + dx_\perp^2 + \frac{1}{r^4}\varphi(x_+) dx_+^2 + \frac{1}{r^4}\varphi(x_-) dx_-^2 \right] + \frac{dr^2}{r^2}.$$

$$\Downarrow$$

$$ds^2 = -A dv^2 + \Sigma^2 \left[e^B dx_\perp^2 + e^{-2B} dz^2 \right] + 2 dr dv + 2F dz dv.$$

iii. $1/r^4$ sickness $\Rightarrow$ A & F get very large near horizon.

Solutions:

i. Use spectral methods.

ii. Introduce IR cuttoff = small background energy density.