Nucleation of Antikaon Condensed Matter in Hot Neutron Stars

Sarmistha Banik

Saha Institute of Nuclear Physics, Kolkata, India

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Plan of the Talk

- Motivation
- Nucleation of antikaon droplets
- Shear Viscosity
- Model EoS
- Results
- Summary

Motivation

First order phase transition from nuclear to antikaon condensed matter is triggered by the thermal nucleation of a critical size droplet of antikaon condensed matter.

Our main focus is to investigate the role of shear viscosity on the thermal nucleation rate.

Shear viscosity plays important roles in neutron star physics

- Damping gravitational wave driven instabilities (for instance r-modes).
- Essential in understanding pulsar glitches.

The Nucleation of Antikaon Bubbles:Introduction

- A first order phase transition is driven by nucleation of droplets
- Nucleation may be due to quantum and thermal fluctuation.
- Nucleation of Quark Matter droplets in NS matter
  (I. Bombaci et. al .Phys. Lett. **B680**, 448 (2009)
  B. W. Mintz et. al. Phy. Rev **D 81**, 123012

  E. Flowers and N. Itoh, Astrophys. J. **230**, 847 (1979))
- Nucleation in NS matter with a first order hadron to kaon condensed phase transition (T. Norsen, Phys. Rev.**C 65**, 045805)

The Nucleation of Antikaon Bubbles:Langer Theory

- The modern theory of homogeneous nucleation via thermal activation pioneered by Langer, yields the nucleation per unit time per unit volume

$$I = \frac{\kappa}{2\pi}\Omega_0 \exp(-\frac{\Delta F(R_c)}{T}),$$

where ΔF is excess free energy of the system required to activate the formation of the critical bubble.

- The statistical prefactor $\Omega_0 = \frac{2}{3\sqrt{3}}\left(\frac{\sigma}{T}\right)^{1.5}\left(\frac{R}{\xi}\right)^4$

measures available phase space volume near saddle point region of ΔF,

κ is the dynamical prefactor $\kappa = \frac{2\sigma}{R_c^3(\Delta w)^2}\left[\lambda T + 2\left(\frac{4}{3}\eta + \zeta\right)\right]$

where Δw is the difference of the enthalpy of the 2 phases,
λ is the thermal conductivity,η and ζ are the shear and bulk viscosity respectively,
σ is the surface tension for the surface separating the 2 phases.

- Dominant contribution to κ comes from shear viscosity term.

The Dynamical Prefactor

Kotchine conditions for a spherically growing bubble

$$
\begin{aligned}
[nU_R] &= [n]\frac{dR}{dt}, \\
[P] &= -\frac{2\sigma}{R}, \\
[\mu] &= 0, \\
\frac{ln_a n_b\,[U_R]}{[n]} &= -\lambda\,(\nabla T)_R - \left(\frac{4}{3}\eta + \zeta\right) U_R \left(\frac{dU}{dr}\right)_R .
\end{aligned}
$$

U_R is the velocity of matter through the interface, dR/dt velocity of the bubble wall, l is the latent heat/particle

- •non-linear effects are ignored, •R > correlation length (ξ)
- •heat dissipation is slow, •1st order PT-releasing enough LH.

$$R - R_c = \exp(\kappa t)$$

Venugopalan & Vischer, PRD49, 5849 (1994), Turski & Langer, PRA 22, 2189 (1980)

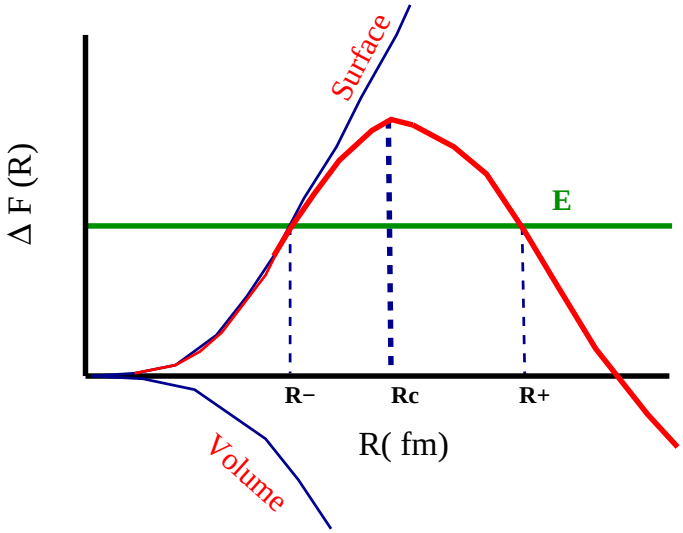
Free Energy

The change in free energy of the system is

$$\Delta F = -\frac{4\pi}{3}\left(P^K - P^N\right)R^3 + 4\pi\sigma R^2$$

where R is the radius of the droplet.

- For subcritical droplets $R < R_c$,surface energy dominates, bubble shrinks & collapses.
- For supercritical droplets $R > R_c$, volume energy dominates & bubble grows.
- The free energy is maximum at this critical radius R_c. $R_c(T) = \frac{2\sigma}{\left(P^K - P^N\right)}$.



Nucleation time

- The thermal nucleation time can be written as

$$\tau_{th} = (V_{nuc} I)^{-1}$$

where the $V_{nuc} = \frac{4\pi}{3} R_{nuc}^3$ is the volume of the NS core. Here P, ϵ, n_b, T remain constant.

Thermal nucleation of the antikaon condensed phase is possible if τ_{th} < cooling time of PNS.

Calculation of Shear Viscosity in nuclear matter

- Shear viscosity relates to the momentum transfer.
- The microscopic mechanism for momentum transfer is through collisions of different species.
- In Neutron Star core n, p, e, μ contribute to the total shear viscosity. (Strong and Electromagnetic(EM) interactions.)

  R. Nandi, S. Banik and D.Bandyopadhyay, PRD80, 2009
- Neutrinos contribute dominantly to the shear viscosity in lepton-trapped Proto Neutron Star. (Weak interaction)

  Goodwin, B.T.,& Pethick, C.J. ApJ 253, 816, 1982

Shear Viscosity

- Shear Viscosity is defined in terms of stress energy momentum tensor T_{ij} of a fluid.
- Close to equilibrium it can be expanded in terms of flow velocity V_i.

$$T_{ij} = (P + \varepsilon)V_i V_j - P\delta_{ij} + \delta T ij$$

$$\begin{aligned} \delta T_{ij} &= -\eta(\nabla_i V_j + \nabla_j V_i - \frac{2}{3}\delta_{ij}\nabla \cdot V) + \zeta\delta_{ij}\nabla \cdot V \\ &= -\eta V_{ij} + \zeta\delta_{ij}\nabla \cdot V \end{aligned}$$

Using kinetic theory,

$$\delta T_{ij} = \int \frac{d^3p}{(2\pi)^3}\frac{p_i p_j}{E_p}\delta f_c$$

- f_c is the distribution function which slightly deviate from the equilibrium FD distribution $f_c^{(0)}$ due to the presence of a small hydrodynamic velocity field V :

$$
\begin{aligned}
f_c &= f_c^{(0)} - \Phi_c \frac{\partial f_c^{(0)}}{\partial \varepsilon_c} \\
&= f_c^{(0)} + \frac{f_{p_c}^{(0)}(1 - f_{p_c}^{(0)})}{k_B T} \Phi_c
\end{aligned}
$$

Φ measures deviation from equilibrium

Boltzmann equation

- The shear viscosity is calculated from a system of coupled Boltzmann kinetic equation:

$$\frac{\mathrm{d}f_c}{\mathrm{d}t} = \frac{\partial f_c}{\partial t} + \frac{\boldsymbol{p}}{\boldsymbol{E}} \cdot \boldsymbol{\nabla} \boldsymbol{f_c} + \boldsymbol{F} \cdot \boldsymbol{\nabla_p} \boldsymbol{f_c} = \sum_i I_{ci}$$

In absence of external force $\boldsymbol{F} = 0$ the LHS of the kinetic equation can be further simplified.

$$\frac{p}{E}.\nabla f_c = I_{ci}(f_c)$$

c=n,p,e, μ for NS, ν for PNS

Lepton-trapped PNS

-

$$
\begin{aligned}
I_c &= \sum_{\sigma_{1'}\sigma_2\sigma_{2'}} \int \frac{\mathrm{d}\boldsymbol{p}_2\,\mathrm{d}\boldsymbol{p}_{1'}\,\mathrm{d}\boldsymbol{p}_{2'}}{(2\pi)^9}\, W(12|1'2')_c \\
&\quad [f_{1'}f_{2'}(1-f_1)(1-f_2) - f_1 f_2 (1-f_{1'})(1-f_{2'})]\,.
\end{aligned}
$$

$$
\begin{aligned}
W(12|1'2')_c &= (2\pi)^4 < |M|^2 >_c\ \delta(\mathbf{p_1} + \mathbf{p_2} - \mathbf{p'_1} - \mathbf{p'_2}) \\
&\quad \delta(E_1 + E_2 - E'_1 - E'_2) \\
\left\langle |M|^2 \right\rangle_c &= \frac{16G^2}{E_1 E_2 E'_1 E'_2}\Big[(C_{V_c} + C_{A_c})^2 (P_1.P_2)(P_{1'}.P_{2'}) \\
&* \ (C_{V_c} - C_{A_c})^2 (P_1.P_{2'})(P_{1'}.P_2) \\
&+ \ (C^2_{A_c} - C^2_{V_c}) m_2 m_{2'} (P_1.P_{1'})\Big]
\end{aligned}
$$

Lepton-trapped PNS

- Scattering of neutrinos by nucleons and electrons:

$$\nu_e + n \rightarrow \nu_e + n$$
$$\nu_e + p \rightarrow \nu_e + p$$
$$\nu_e + e \rightarrow \nu_e + e$$

- Shear viscosity is obtained solving kinetic equation and is given by

$$\eta = \frac{1}{5} n_\nu p_\nu \tau \left[\frac{\pi^2}{12} + \lambda \sum_{k=odd} \frac{2(2k+1)}{k^2(k+1)^2[k(k+1)-2\lambda]} \right]$$

$$\frac{1}{\tau} = \frac{1}{\tau_n} + \frac{1}{\tau_p} + \frac{1}{\tau_e},$$
$$\lambda = \tau \left[\frac{\lambda_n}{\tau_n} + \frac{\lambda_p}{\tau_p} + \frac{\lambda_e}{\tau_e} \right]$$

Lepton-trapped PNS

- τ's are the relaxation times given as

$$\tau_\alpha = \frac{16\pi^4}{(k_B T)^2 E_{F_\alpha}^2 < Z >_\alpha}$$

- where

$$< Z >_\alpha = \int d\Omega_2 d\Omega_1' d\Omega_2' < |M|^2 >_\alpha \delta(\mathbf{p_1} + \mathbf{p_2} - \mathbf{p_1'} - \mathbf{p_2'})$$

$$\lambda_\alpha = \frac{1}{< W >_\alpha} \int d\Omega_2 d\Omega_1' d\Omega_2' < |M|^2 >_\alpha \frac{1}{2} \left[3(\hat{p_1}.\hat{p_{1'}})^2 - 1\right]$$

Goodwin, B.T.,& Pethick, C.J. ApJ 253, 816, 1982

NS

- Shear viscosity of c for NS matter

$$\eta_c = \frac{n_c p_{Fc}^2 \tau_c}{5 m_c^*}$$

where

$$\begin{aligned} n_c &= \text{number density} \\ p_{Fc} &= \text{Fermi momentum} \\ m_c^* &= \text{effective mass} \\ \tau_c &= \text{relaxation time} \end{aligned}$$

- The total shear viscosity is

$$\eta = \sum_c \eta_c$$

where $c = e, \mu, p, n$

Model for EOS

- To calculate the shear viscosity, we need to know the effective masses and Fermi momenta of particles as a function of density (EoS).
- We use the relativistic mean field model. The interaction between baryons is mediated by the exchange of scalar (σ) and vector (ω, ρ) mesons.This picture is consistently extended to include the kaons.
- The Lagrangian density for baryons is given by

$$\begin{aligned}\mathcal{L}_B &= \sum_{B=n,p} \bar{\Psi}_B \left(i\gamma_\mu \partial^\mu - m_B^* - g_{\omega B}\gamma_\mu \omega^\mu - g_{\rho B}\gamma_\mu \mathrm{t}_B \cdot \boldsymbol{\rho}^\mu\right) \Psi_B \\ &+ \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2\right) - U(\sigma) \\ &- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu \,.\end{aligned}$$

where $m_B^* = m_B - g_\sigma \sigma$ is the baryon effective mass.

- The Lagrangian density for (anti)kaons in the minimal coupling scheme is

$$\mathcal{L}_K = D^*_\mu \bar{K} D^\mu K - m_K^{*2} \bar{K} K \ ,$$

where the vector fields are coupled via the standard form $D_\mu = \partial_\mu + i g_{\omega K} \omega_\mu + i g_{\rho K} \mathrm{t}_K \cdot \rho_\mu$ and the effective mass of (anti)kaons is $m_K^* = m_K - g_{\sigma K} \sigma$.

- The equation of motion for (anti)kaons is

$$(D_\mu D^\mu + m_K^*) K = 0$$

Solving the EoM within mean field approximation we get:

$$\omega_K^- = m_K^* - g_{\omega K} \omega_0 - \frac{1}{2} g_{\rho K} \rho_{03}.$$

- Threshold condition for (anti) kaon condensation

$$\omega_K^- = \mu_{K^-} = \mu_e$$

(N.K. Glendenning, J. Schaffner-Bielich,Phys. Rev. **C60**, 025803 (1999) , S.B., D. Bandyopadhyay, Phys. Rev. **C64** (2001) 055805.)

EoS: Presure & Energy density

The thermodynamic potential per unit volume for nucleons is given by

$$\frac{\Omega_N}{V} = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 - \frac{1}{2}m_\omega^2\omega_0^2 - \frac{1}{2}m_\rho^2\rho_{03}^2$$
$$-2T\sum_{i=n,p}\int\frac{d^3k}{(2\pi)^3}[ln(1+e^{-\beta(E^*-\nu_i)}) + ln(1+e^{-\beta(E^*+\nu_i)})] \,.$$

Here, $\beta = 1/T$ and $E^* = \sqrt{(k^2 + m_N^{*2})}$.

$$P_N = -\Omega_N/V \,.$$

The energy density is given by,

$$\epsilon_N = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2$$
$$+2\sum_{i=n,p}\int\frac{d^3k}{(2\pi)^3}E^*\left(\frac{1}{e^{\beta(E^*-\nu_i)}+1} + \frac{1}{e^{\beta(E^*+\nu_i)}+1}\right) \,.$$

EoS: Presure & Energy density :antikaon phase

The pressure of thermal (anti)kaon is

$$P_K = -\Omega_K / V .$$

$$\frac{\Omega_K}{V} = T \int \frac{d^3p}{(2\pi)^3} [ln(1 - e^{-\beta(\omega_{K^-} - \mu)}) + ln(1 - e^{-\beta(\omega_{K^+} + \mu)})] .$$

The net (anti)kaon number density is given by

$$n_K = n_K^C + n_K^T ,$$

where n_K^C gives number density of K^- mesons in the condensates and n_K^T represents the number density of thermal (anti)kaons.

(S. Banik, W. Greiner, D. Bandyopadhyay Phys.Rev.C78:065804,2008)

The Mixed Phase

Phase equilibrium conditions

$$P^H - P^K = -2\sigma/R,$$
$$\mu_B^H = \mu_B^K,$$
$$T^H = T^K.$$

Conditions of global charge neutrality

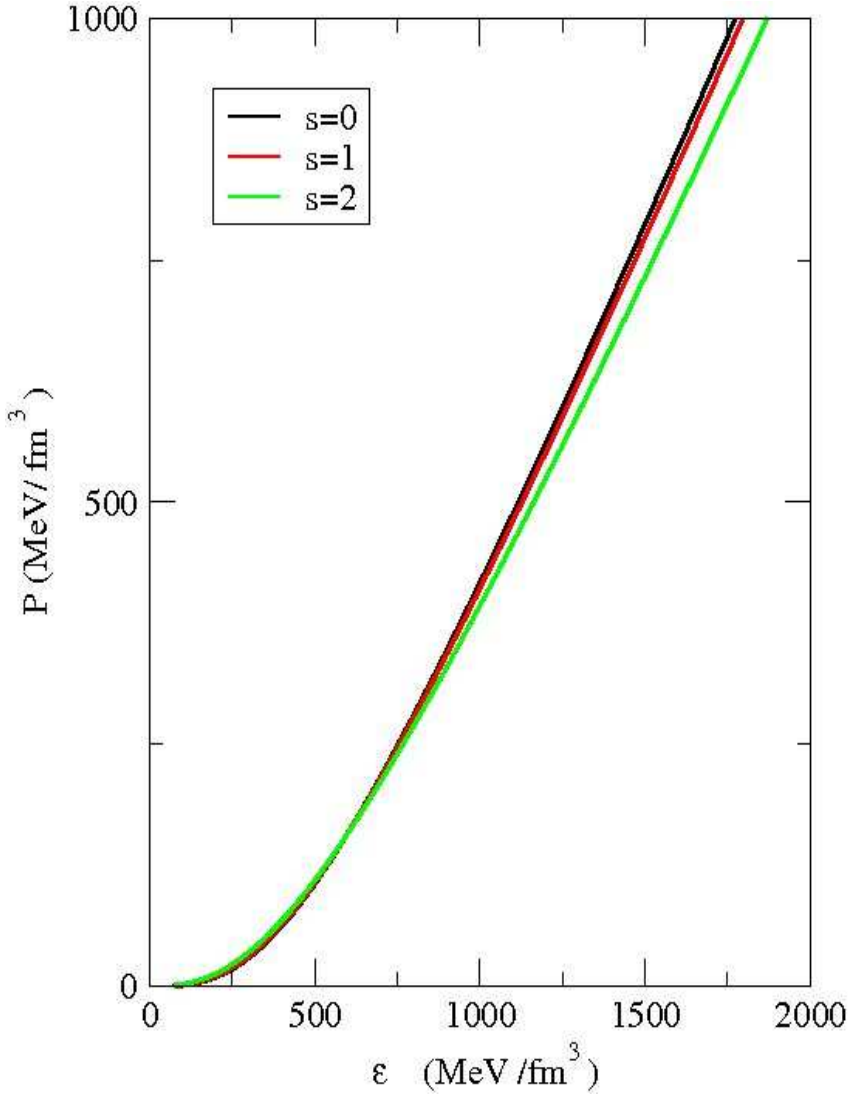
$$(1-\chi)Q^H + \chi Q^K = 0,$$

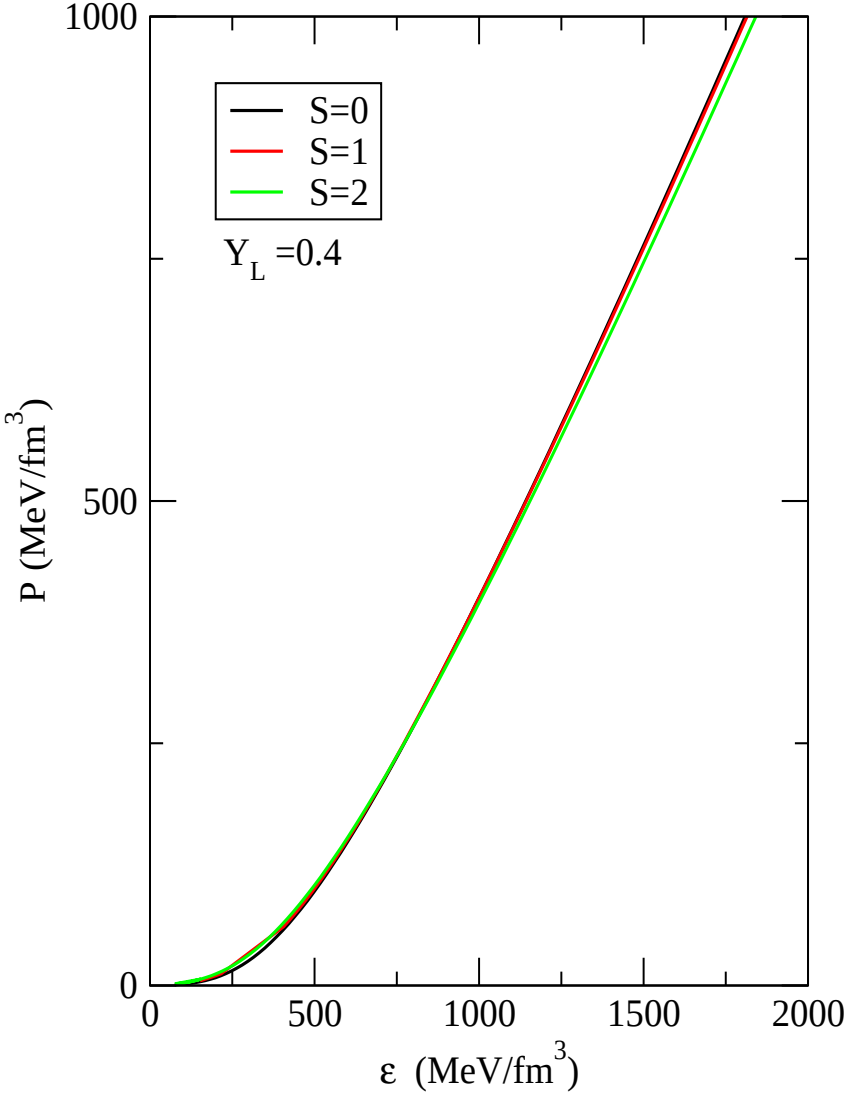
Baryon number conservation

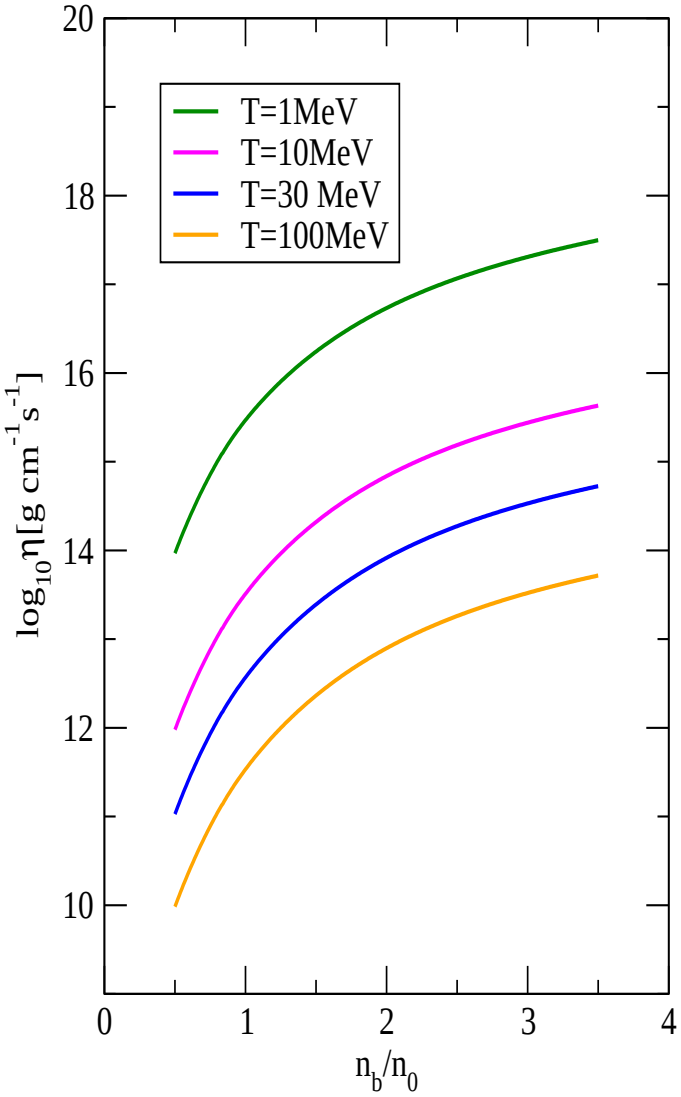
$$n_B = (1-\chi)n_B^H + \chi n_B^K$$

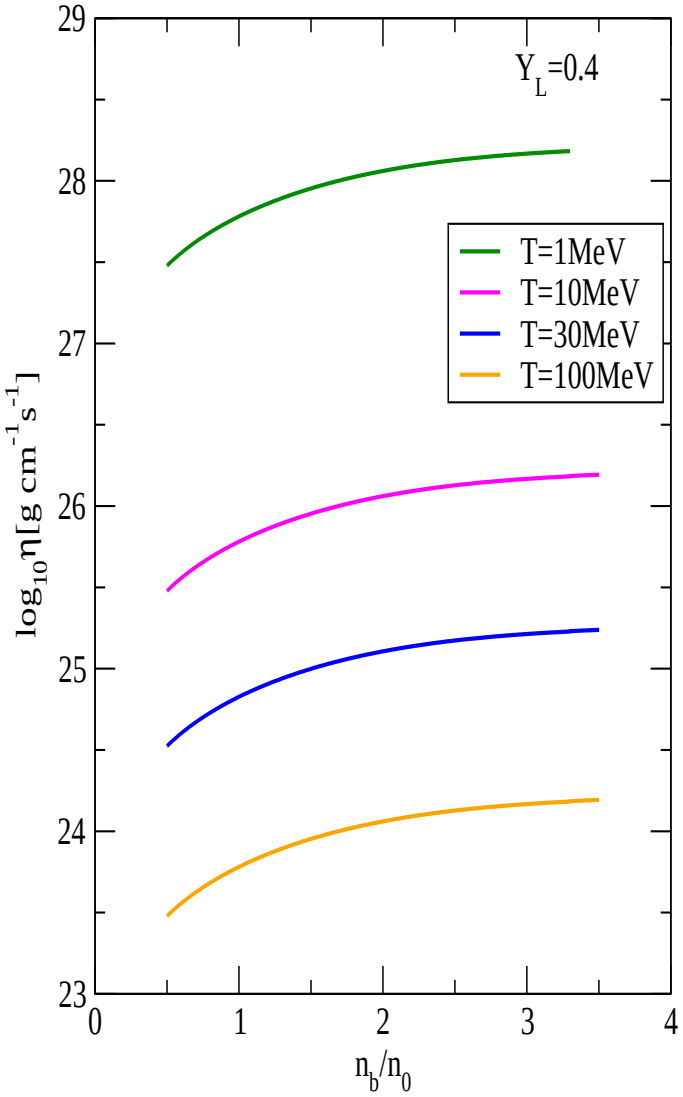
Total energy density

$$\epsilon = (1-\chi)\epsilon^H + \chi\epsilon^K \ .$$


1000
500
0
0
500
1000
1500
2000
s=0
s=1
s=2
P (MeV/ fm 3)
ε (MeV /fm 3)

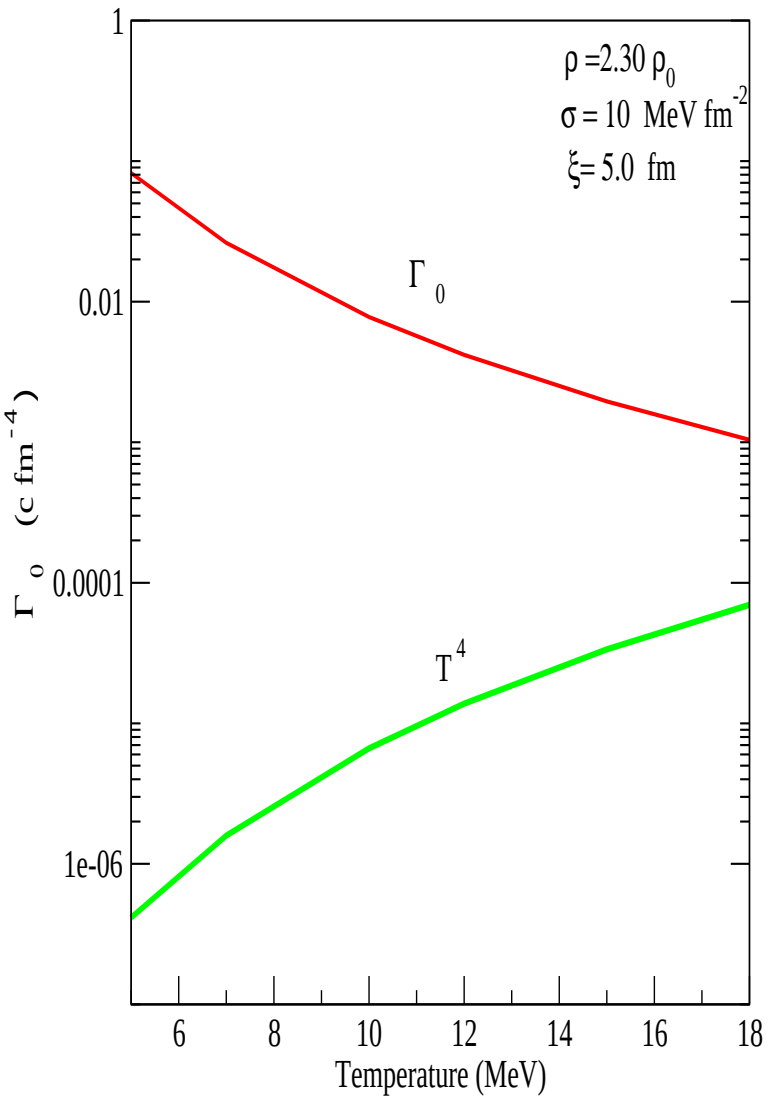

1000
500
0
0
500
1000
1500
2000
S=0
S=1
S=2
Y L =0.4
P (MeV/fm 3)
ε (MeV/fm 3)

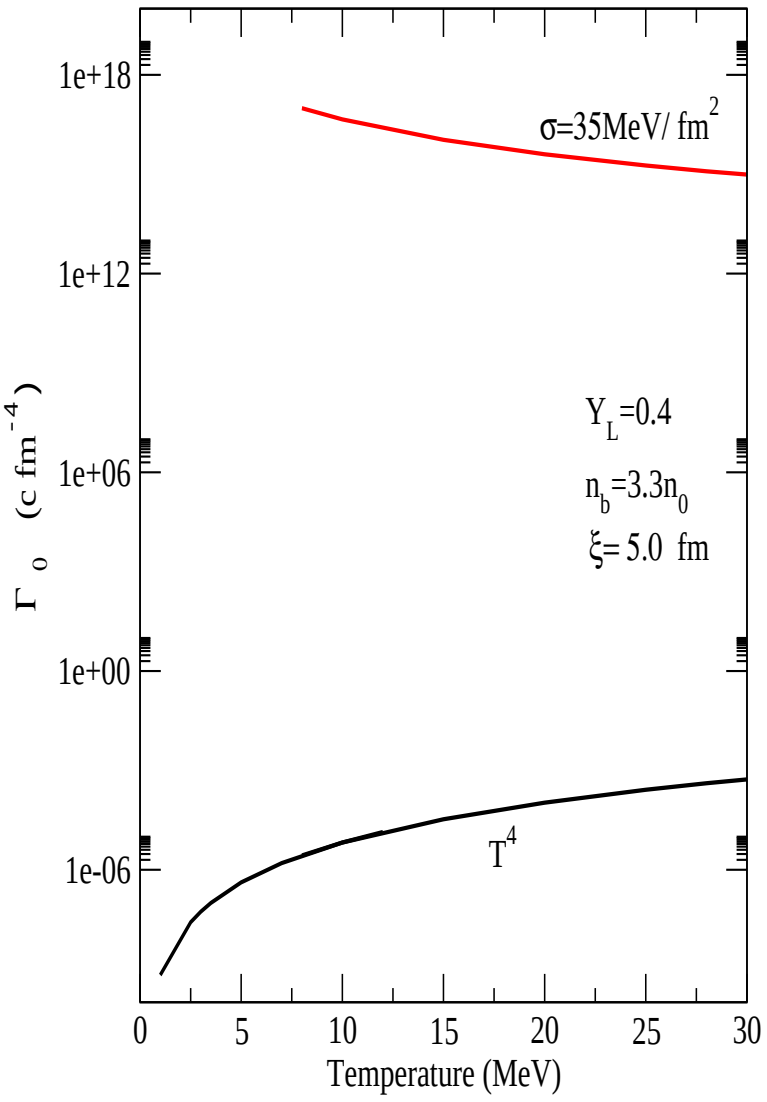


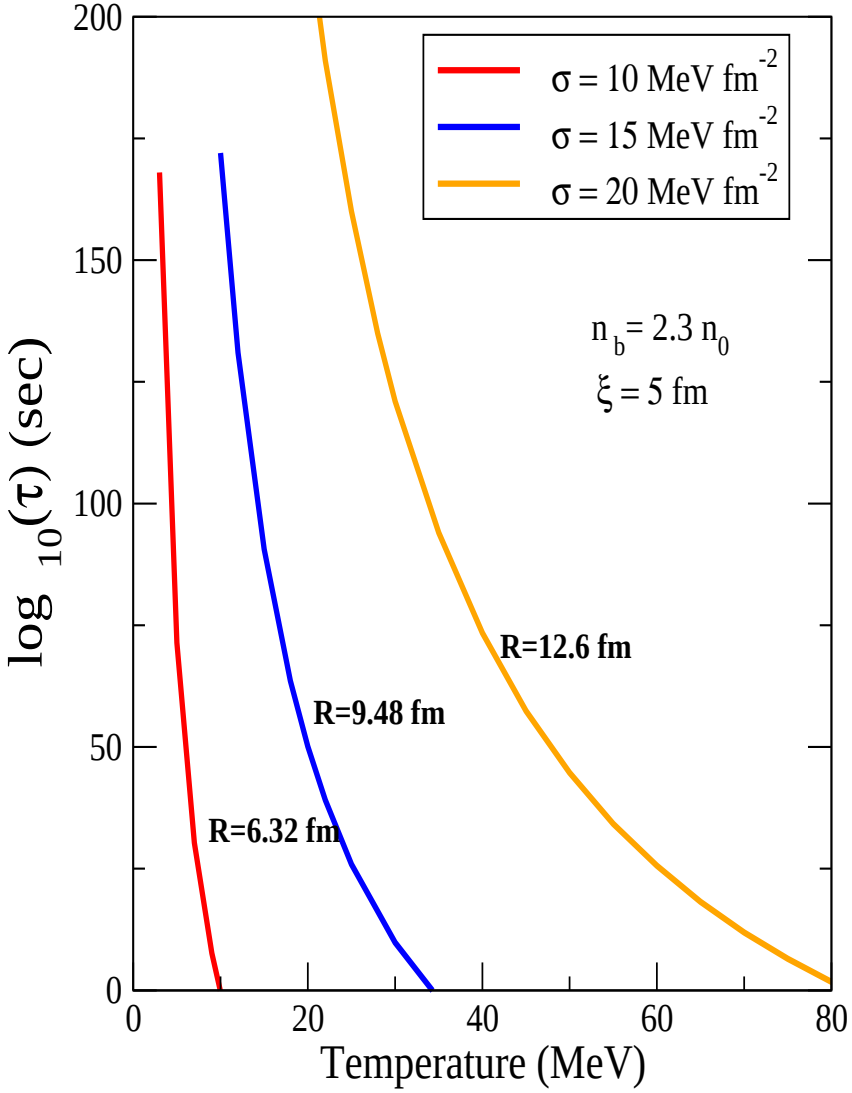


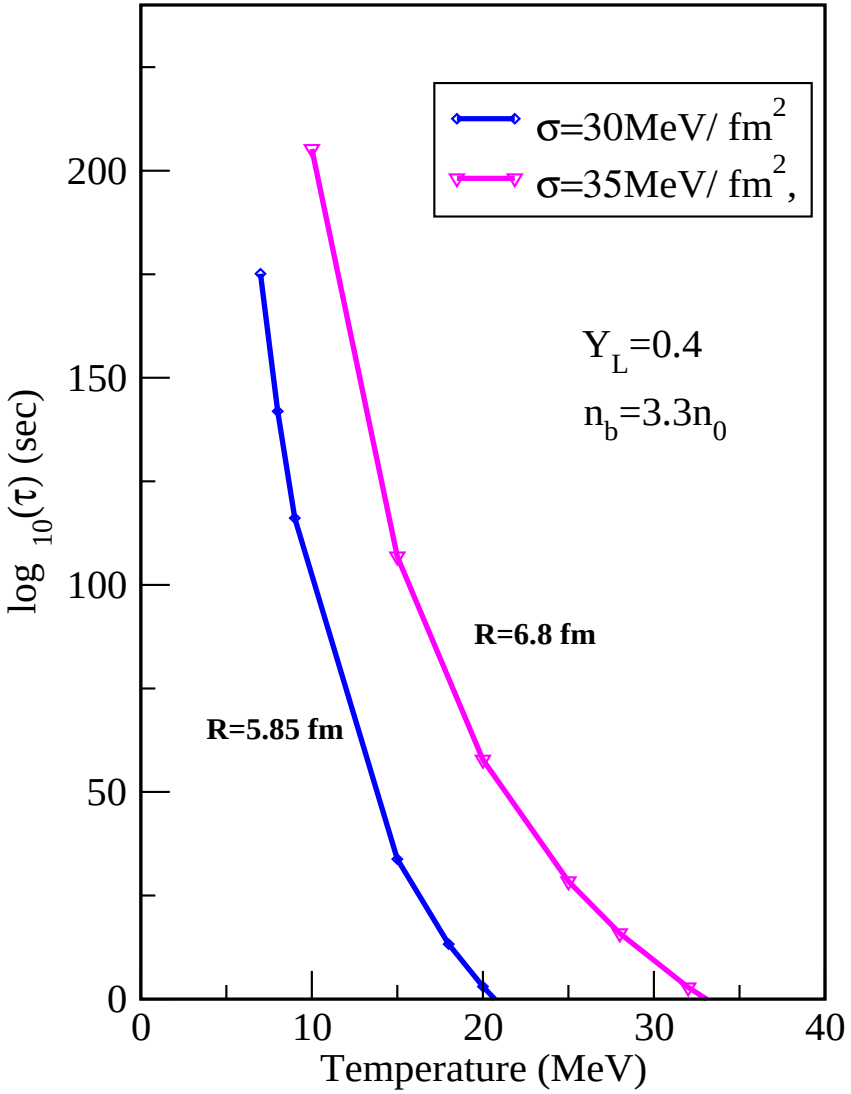
$$\eta = \eta_n + \eta_p + \eta_e + \eta_\mu$$

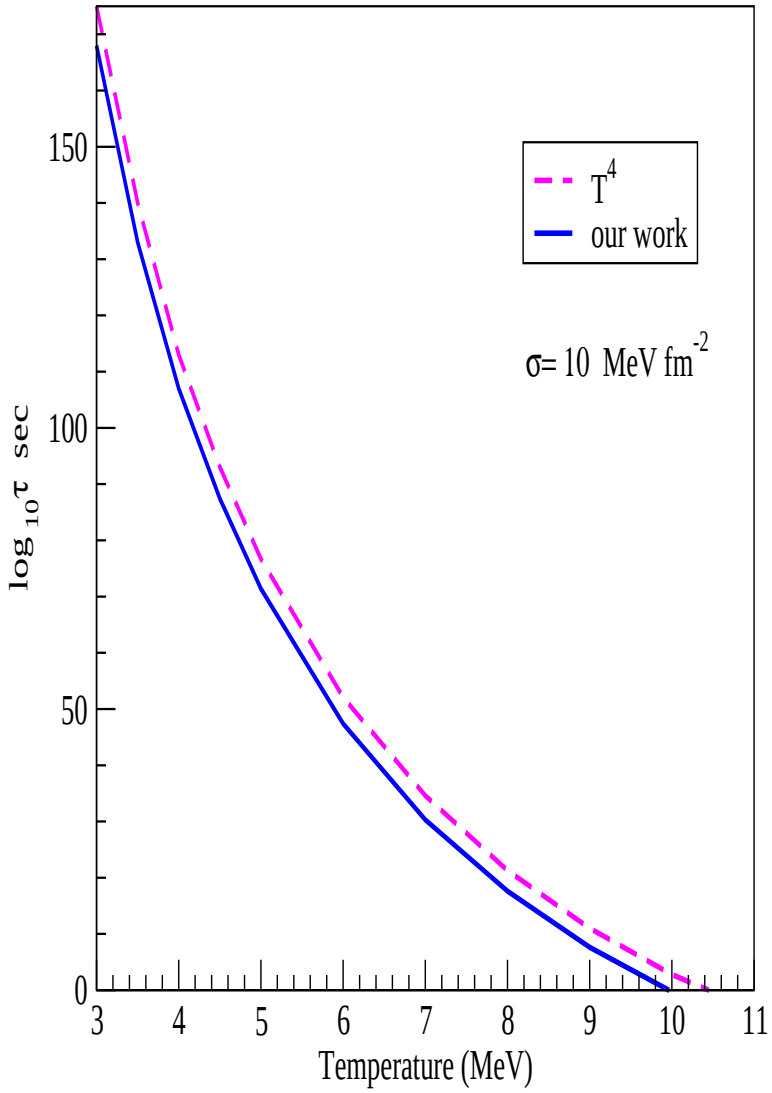
$$\eta = \eta_\nu$$

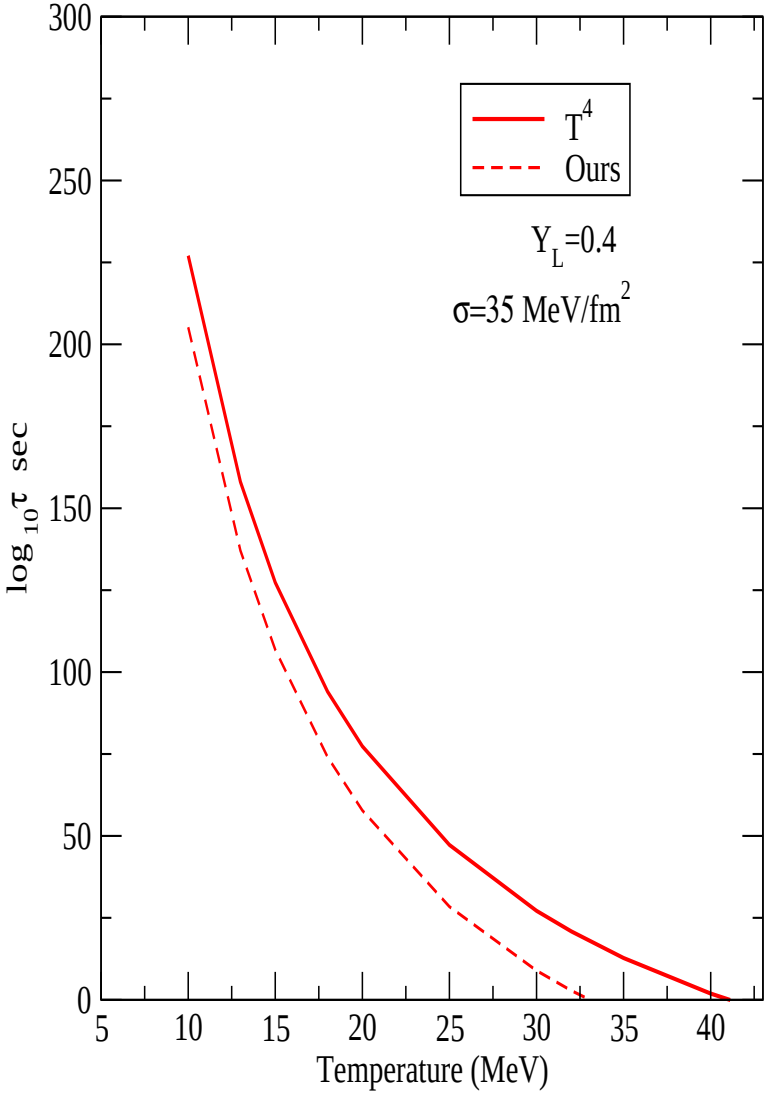

ρ =2.30 ρ0
σ = 10 MeV fm-2
ξ= 5.0 fm
Γ 0
T4
Γ 0 (c fm-4)
1
0.01
0.0001
1e-06
6
8
10
12
14
16
18
Temperature (MeV)


σ=35MeV/ fm2
YL=0.4
nb=3.3n0
ξ= 5.0 fm
T4
Γ 0 (c fm-4)
1e+18
1e+12
1e+06
1e+00
1e-06
0
5
10
15
20
25
30
Temperature (MeV)


σ = 10 MeV fm-2
σ = 15 MeV fm-2
σ = 20 MeV fm-2
n b= 2.3 n0
ξ = 5 fm
R=12.6 fm
R=9.48 fm
R=6.32 fm
log 10(τ) (sec)
Temperature (MeV)
0
50
100
150
200
0
20
40
60
80


σ=30MeV/ fm2
σ=35MeV/ fm2,
YL=0.4
nb=3.3n0
R=6.8 fm
R=5.85 fm
log 10(τ) (sec)
Temperature (MeV)
0
50
100
150
200
0
10
20
30
40


T^4
our work
σ= 10 MeV fm^-2
log 10 τ sec
Temperature (MeV)
0
50
100
150
3
4
5
6
7
8
9
10
11


T^4
Ours
Y_L=0.4
σ=35 MeV/fm^2
log 10 τ sec
Temperature (MeV)
0
50
100
150
200
250
300
5
10
15
20
25
30
35
40

Summary

- Prefactor in the nucleation rate is enhanced by several orders of magnitude due to shear viscosity compared with the T^4 approximation.
- The thermal nucleation time in the T^4 approximation overestimates our result.
- Shear viscosity affects the nucleation of antikaon droplets in hot neutron stars.
- Thermal nucleation is possible at a temperature of $\sim$ a few MeV in deleptonised matter, when nucleation time < cooling time ($\sim$ 100 sec).
- Formation of droplets of exotic matter is possible in a deleptonised protoneutron star (PNS) at a tempreature of a few 10s of MeV.

Collaborators:**Rana Nandi & Debades Bandyopadhyay**
Saha institute of Nuclear Physics,
Kolkata, India.

Thank you.....