The Lattice QCD study of the Three Nucleon Force

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arXiv:1106.2276 [hep-lat]
Importance of Three Nucleon Force (3NF)

- 3NF $\leftarrow$ potential among 3N which cannot be reduced to pair-wise 2N potential

\[
\pi \pi \Delta
\]

Fujita-Miyazawa(1957)
Importance of Three Nucleon Force (3NF)

- Precise few-body calc: NN force cannot reproduce B.E.
  \[ \delta \text{B.E.} = 0.5-1 \text{MeV for } ^3\text{H} \]
  \[ \delta \text{B.E.} = 2-4 \text{ MeV for } ^4\text{He} \]
  - Attractive 3NF necessary

- Saturation density/energy of nuclear matter also requires 3NF
  - EOS of neutron star
  - Flavor universal 3BF (repulsive) ?

- Repulsive 3NF also necessary

\[ E/A \]

Nogga et al., PRL85(2000)944
A.Akmal et al., PRC58(1998)1804
Takatsuka et al., PTPS174(2008)80

Demorest et al. (2010)
Freire (2009)

\[ 1.97 M_\odot \] (J 1614-2230)
\[ 1.67 M_\odot \] (J 1903+0327)
\[ 1.44 M_\odot \]

\[ 3\text{NF?} \]

\[ \rho \]

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Nishizaki et al., PTP108(2002)703
Importance of Three Nucleon Force (3NF)

- The effect on the nuclear chart
  - Anomaly in drip line and nontrivial magic number in neutron rich nuclei by 3NF

**drip line:** $^{28}\text{O} \rightarrow ^{24}\text{O}$

**nontrivial magic number**

$N=28$ for $^{20}\text{Ca}$

T. Otsuka et al., PRL105(2010)032501
J. D. Holt et al., arXiv:1009.5984
Lattice QCD as 1st principle calc

- well-defined statistical system (finite a and L)
- gauge invariant
- fully non-perturbative

Monte-Carlo simulations

Quenched QCD: neglects creation-annihilation of quark-antiquark pair
Full QCD: includes creation-annihilation of quark-antiquark pair
Nuclear Force from Lattice QCD

[HAL QCD strategy]

- Potential is constructed so as to reproduce the NN phase shift (or, S-matrix)

- Nambu-Bethe-Salpeter (NBS) wave function
  \[ \psi(\vec{r}) = \langle 0 | N(\vec{x} + \vec{r}, t) N(\vec{x}, t) | 2N \rangle \]
  - Key concept: asymptotic region \( \leftrightarrow \) phase shift
  \[ (\nabla^2 + k_0^2) \psi(\vec{r}) = 0, \quad r > R \]
  - Luscher, NPB354(1991)531
  - C.-J.Lin et al., NPB619(2001)467
  - CP-PACS Coll., PRD71(2005)094504

- Define the potential at interaction region
  \[ (\nabla^2 + k_0^2) \psi(\vec{r}) = \int d\vec{r'} U(\vec{r}, \vec{r'}) \psi(\vec{r'}), \quad r < R \]
  - Non-local, but \textbf{E-independent} potential

- Velocity expansion
  \[ U(\vec{r}, \vec{r'}) = V_c(r) + S_{12} V_T(r) + L \cdot \vec{S} V_L S(r) + O(\nabla^2) \]
  - Okubo-Marshak(1958)
  - Aoki-Hatsuda-Ishii PTP123(2010)89
  - Truncation in expansion introduces E-dep (only practically), but we can \textbf{improve order by order}
2N potentials (parity-even) from Lattice QCD

\[ M(\pi) = 1.13\text{GeV} \]

Nf=2 (CP-PACS)
\[ 16^3\times32, \ L=2.5\text{fm} \]
\[ 1/a=1.27\text{GeV} \]
How can we tackle 3NF in Lattice QCD?

- In the case of 2N system...
  - Calc 4pt func $\Rightarrow$ NBS amp.
    $\psi(\vec{r}) = \langle 0 | N(\vec{x} + \vec{r}; t) N(\vec{x}; t) | 2N \rangle$
    $\Rightarrow \quad (E - H_0)\psi(\vec{r})$
    $= \quad [V_c(r) + S_{12}V_T(r) + \cdots]\psi(r)$

- Extension to 3N system
  - Calc 6pt func $\Rightarrow$ NBS amp. of 3N
    $\psi(\vec{r}, \vec{\rho}) = \langle 0 | N(\vec{x} + \vec{r}) N(\vec{x}) N(\vec{x} + \vec{r}/2 + \vec{\rho}) | 3N \rangle$
  - Obtain 3NF through
    $(E - H'_0 - H'_0)\psi(\vec{r}, \vec{\rho}) = \left(\sum_{i<j} V_{ij}(\vec{r}_{ij}) + V_{3NF}(\vec{r}, \vec{\rho})\right)\psi(\vec{r}, \vec{\rho})$

- Difficulty(1): volume factor
  - 2N: naïve $O(L^6)$ calc $\Rightarrow O(L^3 \log L^3)$
  - 3N: naïve $O(L^9)$ calc $\Rightarrow O(L^6 \log L^6)$

- Difficulty(2): naïve calc of quark dof grows in factorial ($\sim N_u! \cdot N_d!$)
  - 2N: $O(L^3) \times N_{\text{wick}} \times$ color/spinor loops
  - 3N: $O(L^6) \times N_{\text{wick}} \times$ color/spinor loops

3NF is exceptionally challenging problem!

C.f. pioneering lat calc of B.E. $^3$He($=^3$H), $^4$He
T.Yamazaki et al., arXiv:0912.1383
How can we tackle 3NF in Lattice QCD? (cont’d)

- Calculation for **fixed 3D-configuration** of 3N system
  - **Direct access to 3NF is possible!**
  - We can explore the various features of 3NF (spin/isospin/spacial, etc.)
  - Huge calc cost (**O(10^2-10^3)** factor compared to 2N)
  - We study **linear setup**

We consider Triton channel

- **Linear setup** with various distance “r₂”

**short “r₂” setup**

**long “r₂” setup**

Study r₂-dependence of 3NF

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Features of Linear setup for $^3\text{H}$

- Simplified coupled channel analysis possible
  - The vector to 3rd particle $\vec{\rho} = \vec{0}$
  - $L^{(1,2)}\text{-pair} = L^{\text{total}} = 0$ or $2$ only
  - Possible bases are only three, which can be labeled by $1S_0$, $3S_1$, $3D_1$ for (1,2)-pair

\textbf{Schrodinger Eq.}

\[\hat{H}_0 \begin{pmatrix} \psi^{(1S_0)} \\ \psi^{(3S_1)} \\ \psi^{(3D_1)} \end{pmatrix} + \begin{pmatrix} V \\ (V_{2N} + V_{3NF}) \end{pmatrix} \begin{pmatrix} \psi^{(1S_0)} \\ \psi^{(3S_1)} \\ \psi^{(3D_1)} \end{pmatrix} = E \begin{pmatrix} \psi^{(1S_0)} \\ \psi^{(3S_1)} \\ \psi^{(3D_1)} \end{pmatrix}\]
Parity-odd potential Issue

- However, in order to determine TNF in 3x3 coupled channel, we need information of parity-odd potential
  - Although (1,2)-pair is L=even, (3,1),(2,3)-pair have L=odd components

- Parity-odd potential from lattice QCD (still) in progress
  - $\Rightarrow$ 3X3 channel, but unknown $V_C^{I,S=0,0}, V_C^{I,S=1,1}, V_T^{I,S=1,1}, \text{TNF}(s)$

\[
\hat{H}_0 \begin{pmatrix}
\psi(1S_0) \\
\psi(3S_1) \\
\psi(3D_1)
\end{pmatrix} + \begin{pmatrix}
V \\
(V_{2N} + V_{3NF})
\end{pmatrix} \begin{pmatrix}
\psi(1S_0) \\
\psi(3S_1) \\
\psi(3D_1)
\end{pmatrix} = E \begin{pmatrix}
\psi(1S_0) \\
\psi(3S_1) \\
\psi(3D_1)
\end{pmatrix}
\]

$V_C^{I,S=1,0}, V_C^{I,S=0,1}, V_T^{I,S=0,1}$ : (P = even)

$V_C^{I,S=0,0}, V_C^{I,S=1,1}, V_T^{I,S=1,1}$ : (P = odd)

Target to be determined
Solution using “symmetric” wave function

- Rotate the basis
  \[ |\psi_S\rangle = 1/\sqrt{2} (-|\psi_{1S_0}\rangle + |\psi_{3S_1}\rangle) \]
  \[ |\psi_M\rangle = 1/\sqrt{2} (+|\psi_{1S_0}\rangle + |\psi_{3S_1}\rangle) \]

- We can construct the wave function in which any 2N pair is spin/isospin anti-symmetric
  \[ \vec{\rho} = 0 \]

- \( L=\text{even for any 2N pair} \) automatically guaranteed

\[
\hat{H}_0 \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{3D_1} \end{pmatrix} + \begin{pmatrix} V_{2N} & & \\
& & \\
& & \\
\end{pmatrix} \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{3D_1} \end{pmatrix} + \hat{V}_{3NF} \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{3D_1} \end{pmatrix} = E \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{3D_1} \end{pmatrix}
\]
Solution using "symmetric" wave function

- We can construct the wave function in which any 2N pair is spin/isospin anti-symmetric
  - \( \Rightarrow L=\text{even for any 2N pair} \) automatically guaranteed
- 3x3 coupled channel is reduced to
  - one channel with only 3NF unknown
  - two channels with \( V_C^{I,S=0,0}, V_C^{I,S=1,1}, V_T^{I,S=1,1} \), (3NF) unknown

\[
\begin{pmatrix}
H_0 & V_{2N} & V_{3NF} \\
V_M & \psi_M & \psi_{D_1} \\
V_M & \psi_M & \psi_{D_1}
\end{pmatrix}
= E
\begin{pmatrix}
\psi_S \\
\psi_M \\
\psi_{D_1}
\end{pmatrix}
\]

- \( \Rightarrow \) Even without parity-odd \( V \), we can determine one 3NF
  - This methodology works for any fixed 3D-conf other than linear

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Repulsive 3NF (3NR)

- We determine 3NF effectively represented by a scalar/isoscalar functional form
  - c.f. phenomenological 3NF to reproduce saturation point of nuclear matter, etc.

\[ V_{3NF} = V_{2\pi E} + (V_{3\pi R}) + V_{3NR} \]

\[ V_{3NR} = U_0 \sum_{cyc} T^2(r_{12})T^2(r_{13}) \]

\[ T(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}\right) \frac{e^{-\mu r}}{\mu r} T_{\text{cut}}(r) \]

AdS/CFT: \[ V_{3NF} = +\text{const.} \cdot \frac{1}{r^4} \]

K. Hashimoto, N. Iizuka
JHEP 1011 (2010) 058

Plot of 3NR only: there is cancellation from 3NA

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Lattice calculation setup

- Nf=2 dynamical clover fermion + RG improved gauge configs (CP-PACS)
  - 598 configs X 32 measurements
  - beta=1.95, \( a^{-1}=1.27 \text{GeV} \), \( a=0.156 \text{fm} \)
  - \( 16^3 \times 32 \) lattice, \( L=2.5 \text{fm} \)
  - Kappa(ud)=0.13750
    - \( M(\pi) = 1.13 \text{GeV} \)
    - \( M(N) = 2.15 \text{GeV} \) \( (M_\pi L=14) \)
    - \( M(\Delta) = 2.31 \text{GeV} \)
- Techniques
  - **Automatic Wick contraction code** to handle 4 up- and 5 down-quarks
  - **Non-rela limit op** is used to create 3N state at source
  
  \[
  N_{\text{src}} = \epsilon_{abc}(u_a^T C\gamma_5 \frac{1+\gamma_4}{2} d_b) \frac{1+\gamma_4}{2} u_c
  \]

  \( \Rightarrow \) Factor of \( 2^3=8 \) faster


BGL@KEK

T2K@Tsukuba
Results for wave functions

\[ \Psi_S \text{ overwhelms the wave function:} \]

\[ \Rightarrow \text{Indication of the dominance of all S-wave component, higher waves suppressed} \]
Genuine Three Nucleon Force

T.D. et al. (HAL QCD Coll.)
arXiv:1106.2276 [hep-lat]

Huge Impact on physics of high density matters, EoS, Neutron Star, SuperNova, ...

short-range repulsive 3NF!

M(\pi) = 1.13GeV

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Check on sink time dependence

Huge Impact on physics of high density matters, EoS, Neutron Star, SuperNova, ...

Consistent \( \rightarrow \) Saturated to G.S.

\[ V_{3NF}(r_2) \text{ [MeV]} \]

\[ (t-t_0)/a=8 \]
\[ (t-t_0)/a=9 \]

\( M(\pi) = 1.13 \text{GeV} \)

T.D. et al. (HAL QCD Coll.)
arXiv:1106.2276 [hep-lat]

(07/28/2011) (further improvement in progress)
Studies on discretization error

Discretization error in Laplacian op. is small

Comparison with Improved Laplacian op.

Discretization error in Laplacian op. is small

\[ \nabla^2_{\text{std}} f(\vec{x}) = \frac{1}{a^2} \sum_{i} [f(\vec{x} + a_i) + f(\vec{x} - a_i) - 2f(\vec{x})] \]

\[ \nabla^2_{\text{imp}} f(\vec{x}) = \frac{1}{12a^2} \sum_{i} [-f(\vec{x} + 2a_i) + f(\vec{x} - 2a_i) + 16f(\vec{x} + a_i) + f(\vec{x} - a_i) - 30f(\vec{x})] = \nabla^2 f(\vec{x}) + \mathcal{O}(a^4) \]

\[ = \nabla^2 f(\vec{x}) + \mathcal{O}(a^2) \]

Used for Kinetic energy
Summary/Outlook

- We have performed the **Lattice QCD** study of the **Genuine Three Nucleon Force (3NF)**
  - Wave function with anti-symmetric in spin/isospin for any 2N pair
    - 2N subtraction is possible using only parity-even potentials
  - We have calculated **linear setup** of 3N ($^3\text{H}$) system
    - System is reduced to 3X3 coupled channel
  - Nf=2 dynamical clover fermion at $m_\pi = 1.13$ GeV
    - **Repulsive 3NF at short distance**, further studies ongoing

- Outlook
  - Finer lattices and lighter masses, larger volumes
  - More independent 3NFs using parity-odd potential $\Rightarrow$ FM, chEFT
  - Other 3D-conf of 3N, such as triangle $\Rightarrow$ spacial information of 3NF
  - In future: other channel, I=3/2 [hard to access by scatt. exp]
    Extend the flavor space SU(2) $\Rightarrow$ SU(3) : Astrophysics (e.g., Neutron Star)
Backup Slides
Parity-even 2N potentials (input)

\[ M(\pi) = 1.13 \text{GeV} \]
Effective 2N potential in 3N

\[ M(\pi) = 1.13 \text{GeV} \]
Solution using “symmetric” wave function

- Rotate the basis

\[ |\psi_S\rangle = \frac{1}{\sqrt{2}} (-|\psi_{S_0}\rangle + |\psi_{S_1}\rangle) \quad |\psi_M\rangle = \frac{1}{\sqrt{2}} (+|\psi_{S_0}\rangle + |\psi_{S_1}\rangle) \]

- We can construct the wave function in which any 2N pair is spin/isospin anti-symmetric

\[
|\psi_S\rangle = 1/\sqrt{6} \left[ - (p_{\uparrow}\downarrow n_{\uparrow} - n_{\uparrow} p_{\uparrow}) n_{\downarrow} \\
- (n_{\uparrow} n_{\downarrow} - n_{\downarrow} n_{\uparrow}) p_{\uparrow} \\
+ 1/2 (p_{\uparrow}\downarrow n_{\uparrow} + n_{\uparrow} p_{\downarrow} - p_{\downarrow} n_{\uparrow} - n_{\downarrow} p_{\uparrow}) n_{\uparrow} \\
+ 1/2 (p_{\uparrow}\downarrow n_{\downarrow} - n_{\uparrow} p_{\downarrow} + p_{\downarrow} n_{\downarrow} - n_{\uparrow} p_{\downarrow}) n_{\uparrow} \right] 
\]

\( \leftrightarrow I = 0, S = 1 \)

\( \leftrightarrow I = 1, S = 0 \)

\( \leftrightarrow I = 1, S = 0 \)

\( \leftrightarrow I = 0, S = 1 \)

\( \rightarrow L=\text{even for any 2N pair} \) automatically guaranteed
Explicit formula for the potential matrix

The potential matrix for the 2N part in 3x3 coupled channel in linear setup can be written as:

\[
V_{2N} = \begin{pmatrix}
+V_C^{10}(r) + V_C^{01}(r) & +\frac{1}{2}V_C^{10}(r) - \frac{1}{2}V_C^{01}(r) & -2V_T^{01}(r) \\
+\frac{1}{2}V_C^{10}(2r) + \frac{1}{2}V_C^{01}(2r) & -\frac{1}{2}V_C^{10}(2r) + \frac{1}{2}V_C^{01}(2r) & +2V_T^{01}(2r) \\
\end{pmatrix} \\
\begin{pmatrix}
+\frac{1}{2}V_C^{10}(r) - \frac{1}{2}V_C^{01}(r) & +\frac{3}{4}V_T^{01}(r) + \frac{1}{4}V_C^{10}(r) + \frac{1}{4}V_C^{01}(r) + V_T^{11}(r) \\
-\frac{1}{2}V_C^{10}(2r) + \frac{1}{2}V_C^{01}(2r) & +\frac{1}{2}V_C^{10}(2r) + \frac{1}{2}V_C^{01}(2r) \\
\end{pmatrix}
\]

\[
V_T^{11}(r) - 3V_T^{11}(r) \\
+V_T^{01}(r) - 3V_T^{11}(r) \\
+2V_T^{01}(2r) \\
+V_T^{01}(r) - 3V_T^{11}(r) \\
+2V_T^{01}(2r) \\
+2V_T^{01}(2r)
\]

\[
V_C^{I,S=1,0}, V_C^{I,S=0,1}, V_T^{I,S=0,1} : (P = \text{even}) \\
V_C^{I,S=0,0}, V_C^{I,S=1,1}, V_T^{I,S=1,1} : (P = \text{odd})
\]

(LO-terms) 07/28/2011 PANIC11 @ MIT (r → 2r convention) 25
Importance of Three Nucleon Force (3NF)

- **3NF** $\leftrightarrow$ potential among 3N which cannot be reduced to pair-wise 2N potential
  - Important role in B.E. of light-nuclei
  - Nucleus-nucleus scattering, Ay puzzle?
  - Saturation point of nuclear matter
  - **EoS** of high density matter $\Rightarrow$ Neutron Star, SuperNova
  - Properties of neutron rich nuclei $\Rightarrow$ Nucleosynthesis

Fujita-Miyazawa (1957)
Importance of Three Nucleon Force (3NF)

- Precise few-body calc: NN force cannot reproduce B.E.
  \[ \delta B.E. = 0.5-1 \text{MeV for } ^3\text{H} \]
  \[ \delta B.E. = 2-4 \text{ MeV for } ^4\text{He} \]
  - Attractive 3NF necessary

- Saturation density/energy of nuclear matter also requires 3NF
  - EOS of neutron star
    - Flavor universal 3NF (repulsive) ?
  - Repulsive 3NF also necessary
    - A.Akmal et al., PRC58(1998)1804
    - Takatsuka et al., PTPS174(2008)80

- The effect on the nuclear chart
  - anomaly in drip line and magic numbers by 3NF
  - Ay puzzle in N-d, N-A scatt., etc.
    - (3NF may worsen the situation)
    - T.Otsuka et al., PRL105(2010)032501
Importance of Three Nucleon Force (3NF)

- **Precise few-body calc:**
  - e.g. benchmark calc of $^4\text{He}$ by 7 methods (NN only)
  
  ![Graph showing precision for binding energy](Image)

  \[ \delta B.E. = \begin{array}{c}
  0.5-1 \text{MeV for } ^3\text{H} \\
  2-4 \text{ MeV for } ^4\text{He}
  \end{array} \]

  ![Graph showing missing binding energy](Image)

  H. Kamada et al., PRC64(2001)044001

- **2N force cannot reproduce B.E.**
  - Attractive 3NF necessary

  ![Graph showing 3NF and binding energy](Image)

  Nogga et al., PRL85(2000)944
Attractive 3NF necessary!
Three Nucleon Force (3NF)

- It is natural to expect the existence of 3NF
- It is very nontrivial to determine 3NF from QCD
- $2\pi E$-3NF Fujita-Miyazawa, PTP17(1957)360
  - Off-energy-shell $\pi N$ scatt
- EFT expansion $\Rightarrow$ 3NF appears at NNLO order
- Phenomenological short-range repulsion is necessary
- $2\pi E$-3NF too attractive, often suppressed (artificially) by form factor
- NB: the combination of (2NF,3NF) $\Rightarrow$ observables

U.v.Kolck, PRC49(1994)2932
Epelbaum, Prog.Part.Nucl.Phys.57(06)654
Japan’s next gen computer

- K computer at Kobe, Japan
- 10PFlops (2012)

K (Kei) = $10^{16} = 10$ Peta