Model Dependence of the Deuteron Electric Dipole Moment

B. F. Gibson
Nuclear and Particle Physics, Astrophysics and Cosmology Group, T-2
Theoretical Division
Los Alamos National Laboratory

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Collaborator: Iraj R. Afnan, Flinders University

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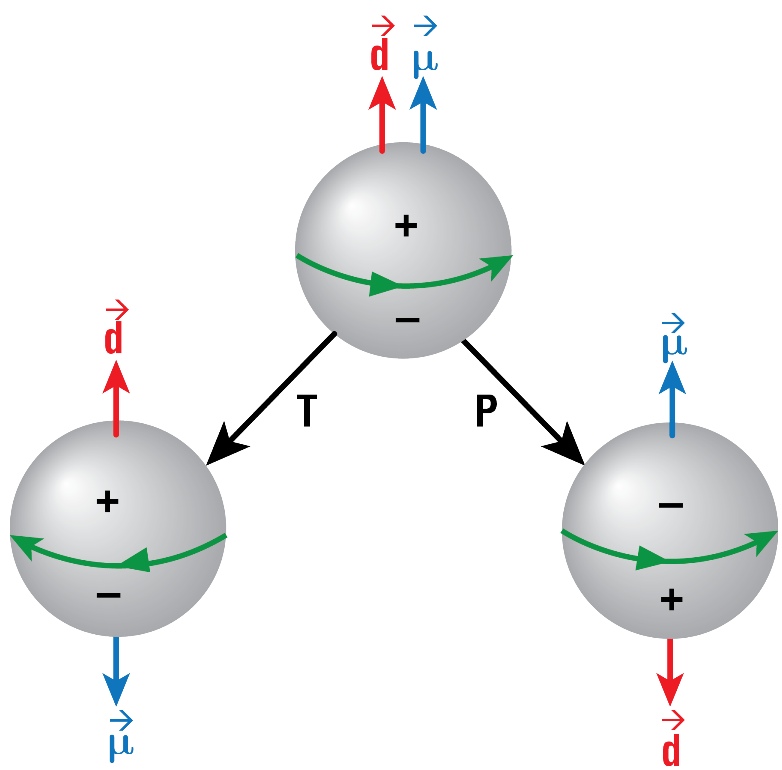
Electric Dipole Moments

Parity and Time-Reversal Violation

- The electric dipole moment (EDM) must vanish if there is a symmetry under the parity transformation (P) for which $\vec{r} \to -\vec{r}$ or under the time-reversal transformation (T) for which $t \to -t$.
- Consider a particle whose orientation is specified by its magnetic moment $\vec{\mu}$; its dipole moment $\vec{d}$ and $\vec{\mu}$ must change signs the same way under P and T if $\vec{d}$ is to have a nonzero value.
- However, $\vec{d}$ changes sign under P whereas $\vec{\mu}$ does not, and $\vec{\mu}$ changes sign under T whereas $\vec{d}$ does not.
- Thus, $\vec{d}$ must vanish if P is conserved or if T is conserved.



Electric Dipole Moments

CP/T Violation

- With discovery of parity violation by Wu *et al.*, it was recognized that charge conjugation and parity (CP) non invariance imply that EDMs should be non zero.
- The *Standard Model* predicts EDMs smaller than are experimentally detectable; finite and non-zero.
- Thus, an unambiguous observation of a non zero EDM with current technology would imply an undiscovered source of CP violation.
- New physics could arise in the strong interaction sector (*i.e.*, the θ term), OR in the weak interaction sector (*e.g.*, via left-right symmetric models OR via supersymmetry OR ???.)

Electric Dipole Moments

New Physics

- The strong interaction θ term is a problem; it provides an isoscalar EDM *but* one which must be fine tuned to avoid overtly large EDMs. We will not consider such models, the Pecci-Quinn transformation to obtain a zero θ term and the resulting axion, *etc.*
- In the weak interaction sector both PT violating potentials and P conserving/T violating potentials may give rise to an EDM.
- One-pion exchange contributes only to the former
- We focus on PT invariance violation effects in the nuclear potential

General Observations

The Optimist's View

- The strong interaction θ term, which leads to a pure isoscalar EDM, is not the answer.
- The weak interaction leads to EDMs that are measurable at a size well above that coming from the *Standard Model*, $\sim 10^{-31}e$ cm.
- The possibility of weak baryogenisis remains an option for explaining the observed matter-antimatter asymmetry in the universe.
- Storage Ring EDM measurements are imperative!

The ^{2}H Electric Dipole Moment

Why?

- Direct measurement the EDM of a charged ion in a storage ring appears to be a real possibility.
- EDM measurements for multiple few-nucleon systems are essential to fully understand an observed effect.
- The Deuteron is the simplest nucleus and is well understood theoretically.
- The one-body contributions of the neutron and proton tend to cancel in the isospin 0 deuteron, emphasizing isovector effects.
- A PT violating interaction can induce a small P-state admixture in the deuteron wave function – producing a non-vanishing two-body M.E. of the charge dipole operator $\tau_-^z e\vec{r}$.

Conventional Approach

Traditional Model

- Nuclei consist only of nucleons – other degrees of freedom are suppressed
- Nucleons move slowly within nuclei – nonrelativistic dynamics
- Nucleons interact via pairwise forces

$$H = \sum_i \frac{p_i^2}{2\,m} + \sum_{i<j} V_{ij}$$

The One-body vs. Two-body EDM Contributions

One-body Terms

- The one-body contribution to the deuteron EDM is
  $d_D^{(1)} = d_n + d_p$
- Neutron and proton EDMs can arise from many sources
- For their meson exchange model Liu & Timmermans* estimate

$$d_D^{(1)} \simeq 0.22 \times 10^{-2} \bar{G}_\pi^{(1)} + O(\bar{G}_\pi^{(0,2)}, \bar{G}_{\rho,\omega,\eta}) \; ;$$

  $\bar{G}_X^{(i)}$ is the product of the strong coupling constant g_{XNN} and the associated PT violating meson-nucleon coupling constant $\bar{g}_X^{(i)}$
- Note: The theoretical uncertainty is sizeable

*C-P. Liu and R. G. E. Timmermans, *Phys. Rev. C* **70**, 055501 (2004).

Two-body Terms

- The corresponding two-body deuteron EDM contribution is

$$d_D^{(2)} \simeq 1.5 \times 10^{-2} \bar{G}_\pi^{(1)} + O(\bar{G}_\pi^{(0)}, \bar{G}_{\rho,\omega,\eta})^{(1)}) \; ,$$

- Thus, the nuclear physics $d_D^{(2)}$ contribution dominates
- Even a 50% uncertainty in $d^{(1)}$ makes but a minor contribution

Nuclear model aspects of $d_D^{(2)}$ are our primary concern.

Prior Results

Avishai, 1985

- $d_D^{(2)}$ estimated using Mongan separable p-wave potentials.
- Value of $-0.91\, e\, fm$ reported for a physical pion mass of the exchanged meson.

Khriplovich & Korkin, 2000

- $d_D^{(2)}$ estimated using a zero-range approximation.
- Value of $-0.92\, e\, fm$ reported.

Liu & Timmermans, 2004

- $d_D^{(2)}$ estimated using contemporary potential models Av_{18}, Reid93, and Nijm II.
- Within the range of uncertainty defined by the three potentials the value is calculated to be $-0.73 \pm .01\, e\, fm$.

Details

The PT-violating interaction is a standard isovector OPE model

$$V = -A\left[(\vec{\sigma}^{(-)} \cdot \hat{r})\, \tau_z^{(+)} + (\vec{\sigma}^{(+)} \cdot \tau_z^{(-)} \right] f(r)$$

where the radial dependence is given by

$$f(r) = -\frac{1}{m_\pi} \frac{d}{dr} \left(\frac{e^{-m_\pi r}}{r} \right) \ ,$$

and A is the constant

$$A \equiv \frac{\bar{g}^{(1)}_{\pi NN}\, g_{\pi NN}}{16\pi} \ .$$

The two-body deuteron EDM is given in terms of the ground state wave function $|\Psi\rangle = |\Psi_L\rangle + |\Psi_S\rangle$ as

$$d_D^{(2)} = \langle \Psi |\, O_d \,| \Psi \rangle = \langle \Psi_L |\, O_d \,| \Psi_S \rangle + \langle \Psi_S |\, O_d \,| \Psi_L \rangle \ ,$$

where in terms of the large (parity conserving) L and small (parity non-conserving) S wave function components, one has

$$\begin{aligned} \langle \Psi_L |\, O_d \,| \Psi_S \rangle &= \langle \Psi_L |\, O_d\, G_0(E)\, V \,| \Psi_L \rangle \\ &+ \langle \Psi_L |\, O_d\, G_0(E)\, t(E)\, G_0(E)\, V \,| \Psi_L \rangle \ , \\ &\equiv \frac{e}{2} \left[d_{PW} + d_{MS} \right] A. \end{aligned}$$

The first term involves a complete set of intermediate plane wave states and corresponds to the plane-wave contribution. The second term involves multiple-scattering via $t(E)$ and is the multiple-scattering contribution.

Strong Interaction

$^3S_1 -^3 D_1$ Potentials

The strong interaction defines the deuteron. We opt for a separable representation to simplify the calculation. We compare Yamaguchi & Yamaguchi potentials with a 4% and 7% D-state probability (YY4 and YY7) but no short range repulsion with a unitary pole approximation (UPA) to the original Reid soft-core potential (RSC68) and the Nijmegen modified Reid potential (RSC93). The UPA wave function is by definition identical to that of the original potential, which provided an optimum fit to the available data at the time the potential was generated.

Table 1: Comparison of the deuteron properties for the original potential and the UPA potential for both RSC68 and RSC93. Tabulated are the binding energy ϵ_D, the asymptotic S-wave normalization A_S, the ratio of the asymptotic D-wave to S-wave η, the quadrupole moment Q_D, and the D-state probability P_D. Also included are the scattering length a_t and effective range r_t.

	RSC68		RSC93	
	UPA	Original	UPA	Original
ϵ_D	2.2246	2.2246	2.2246	2.2246
A_S	0.87893	0.87758	0.8863	0.8853
$\eta = A_D/A_S$	0.026556	0.026223	0.02565	0.0251
Q_D	0.2800	0.27964	0.2709	0.2703
P_D	6.4691	6.4696	5.699	5.699
a_t	5.408	5.390	5.445	5.422
r_t	1.752	1.720	1.799	1.755

Strong Interaction

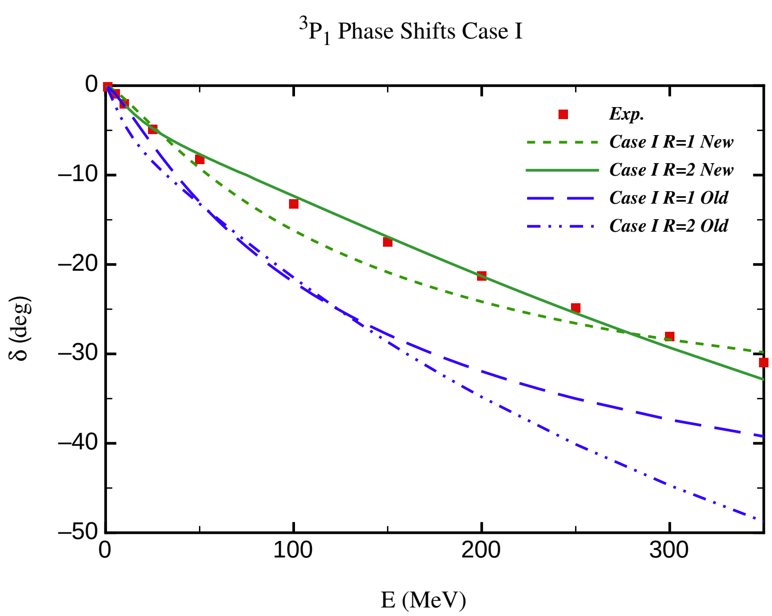
Mongan $^3P_1$ Potentials

To examine the importance of multiple scattering in determining the deuteron EDM, we need to introduce a 3P1 interaction to calculate d_{MS}. To simplify the evaluation of d_{MS}, we chose to use separable potentials with different form factors; the Mongan potentials come with different form factors and, therefore, different off-shell properties. They are either rank one or rank two to optimize the fit to the data.

Table 2: The parameters of our rank-one and rank-two potentials with the different Mongan form factors. The parameters are adjusted to minimize χ^2 taking the experimental phases from the latest (1993) Nijmegen np phase shift analysis. The form factor for Case III is written in terms of $Q_1(\xi)$ the Legendre function of the second kind. The λ_i are the potential strength functions.

Potential	form factor $g_i(k)$	Rank	β_1	λ_1	β_2	λ_2	χ^2
Case I	$k/(k^2+\beta_i^2)$	1	1.725	0.95	-	-	0.62
		2	0.90	0.059	3.58	-2.0	0.02
Case II	$k/(k^2+\beta_i^2)^{3/2}$	1	2.38	9.35	-	-	0.81
Case III	$\left[\frac{1}{k^2\pi}Q_1(1+\frac{\beta_i^2}{2k^2})\right]^{1/2}$	1	1.68	60.0	-	-	0.19
		2	1.20	120.0	4.4	-2.3	0.12
Case IV	$k/(k^2+\beta_i^2)^2$	1	2.715	147.0	-	-	0.78

Figure 1: Comparison of the ^{3}P$_1$ phase shifts for the original Mongan potentials (Old) with Case I form factor and rank one (R=1) and rank two (R=2) with our fit (New) and the experimental (*Exp.*) Nijmegen np data.



The $d_D^{(2)}$ EDM

The D-State dependence

We consider first the sensitivity of the two-body deuteron EDM $d_D^{(2)}$ to the deuteron D-State probability. For the $^3P_1$ interaction we use the Mongan rank 2, case I potential, which fits the Nijmegen phase shifts well. We include in the Table all four of the $^3S_1 -^3 D_1$ interactions mentioned before. We also include in the Table the result from Khriplovich and Korkin.

Table 3: Variation of the two-body EDM with D-state probability of the deuteron. For the ^{3}P$_1$ interaction we use our rank 2, Case I potential. Also included are the results of Khriplovich and Korkin.

^{3}S$_1$-^{3}D$_1$	P_d	$d_{PW}(A\,e\,fm)$
YY 4%	4%	-1.035
RSC93	5.7%	-0.9715
RSC68	6.5%	-0.9620
YY 7%	7%	-1.083
K & K	0%	-0.92

Plane Wave Results

The plane-wave approximation d_{PW} varies little with P_D. The short range repulsion in each of the RSC potentials provides no more than a 10% reduction. The results are effectively consistent with the zero range (chiral limit) approximation of Khriplovich and Korkin. The D-state dependence of the two YY model results suggests that a YY0 model would approach their result.

The $d_D^{(2)}$ EDM

The D-State dependence

We summarize results for the two-body deuteron EDM $d_D^{(2)}$ for deuteron models with different D-State probabilities. For the $^3P_1$ interaction we use the Mongan rank 2, case I potential, which fits the Nijmegen phase shifts well.

Table 4: Variation of the two-body EDM with D-state probability of the deuteron. For the $^3\mathrm{P}_1$ interaction we use our rank 2, Case I potential.

$^3\mathrm{S}_1$-$^3\mathrm{D}_1$	P_d	$d_{PW}(A\,e\,fm)$	$d_{MS}(A\,e\,fm)$
YY 4%	4%	-1.035	0.4115
RSC93	5.7%	-0.9715	0.2009
RSC68	6.5%	-0.9620	0.1718
YY 7%	7%	-1.083	0.4271

Multiple Scattering Contribution

The multiple scattering contribution d_{MS}, which is of opposite sign to d_{PW}, varies considerably with the short range character of the potential. The two RSC potentials have different P_D values but are quite similar in their d_{MS}. Strong short range repulsion at short distance in realistic nucleon-nucleon potentials reduces the effect of multiple scattering in the EDM matrix element to the extent that the multiple-scattering contribution d_{MS} is only about 20% of the plane wave contribution d_{PW}. The dependence upon P_D is weak.

The $d_D^{(2)}$ EDM

The D-State dependence

We summarize results for the two-body deuteron EDM $d_D^{(2)}$ for deuteron models with different D-State probabilies. For the ${}^3P_1$ interaction we use the Mongan rank 2, case I potential, which fits the Nijmegen phase shifts well. We also include in the Table the result from Khriplovich and Korkin.

Table 5: Variation of the two-body EDM with D-state probability of the deuteron. For the ${}^3\mathrm{P}_1$ interaction we use our rank 2 Case I potential. Also included are the results of Khriplovich and Korkin.

${}^3\mathrm{S}_1$-${}^3\mathrm{D}_1$	P_d	$d_{PW}(A\,e\,fm)$	$d_{MS}(A\,e\,fm)$	$d_D^{(2)}(A\,e\,fm)$
YY 4%	4%	-1.035	0.4115	-0.6234
RSC93	5.7%	-0.9715	0.2009	-0.7706
RSC68	6.5%	-0.9620	0.1718	-0.7902
YY 7%	7%	-1.083	0.4271	-0.6564
K & K	0%	-0.92		

EDM Analysis

The RSC and YY $d_D^{(2)}$ results suggest that multiple-scattering is suppressed by strong short range repulsion, which implies that inclusion of multiple scattering requires a more realistic treatment of the deuteron than employed by Khriplovich and Korkin. For the RSC93 model, multiple-scattering contributions are only about 20% of the plane-wave contribution, or 25% of $d_D^{(2)}$, in contrast to effects for the YY models that are a factor of two greater.

Results for our UPA to the Reid 1993 potential

Liu & Timmermans used potentials that reproduce the 1993 Nijmegen phase shift analysis, one of which is the updated Reid 1993 potential (RSC93). We have generated a UPA for that deuteron and calculated results to compare with the UPA for the Reid 1968 potential (RSC68) and the YY4 model in Table 7, for several of our new Mongan potentials fitted to the 1993 phase shift data.

Table 6: The deuteron EDM values for three rank-one deuteron $^3S_1$–$^3D_1$ potentials and several different rank-one and rank-two $^3P_1$ potentials. The $^3P_1$ potentials are of the Mongan type with parameters adjusted to fit the 1993 Nijmegen phase shifts.

$^3S_1$-$^3D_1$	YY4		RSC68		RSC93	
	$d_{PW} = -1.04$		$d_{PW} = -0.96$		$d_{PW} = -0.97$	
Case/rank	d_{MS}	$d_D^{(2)}$	d_{MS}	$d_D^{(2)}$	d_{MS}	$d_D^{(2)}$
I/1	0.57	-0.47	0.21	-0.75	0.26	-0.71
I/2	0.41	-0.62	0.17	-0.79	0.20	-0.77
II/1	0.38	-0.65			0.22	-0.75
III/1	0.77	-0.27	0.25	-0.71	0.31	-0.66
IV/1	0.33	-0.71			0.22	-0.76

With the exception of Case III there is little variation in the EDM with the differing choice of form factors. The suggestion is that d_{MS} may be sensitive to the off-shell behavior of the $^3P_1$ amplitude; the scattering function for case III can be seen to differ markedly from that of the other $^3P_1$ models.

Liu & Timmermans obtained:

$$d_D^{(2)} = -0.73 \pm .01\, e\, fm,$$

so that our separable potential results differ by of the order of 10%.

Conclusions and Summary

Conclusions

- The fit to the improved phase shifts reduces the values for d_{MS} considerably, suggesting that the new ${}^3\mathrm{P}_1$ potentials are weaker
- It may prove possible to treat the ${}^3\mathrm{P}_1$ interaction perturbativly when calculating the ^{3}He EDM
- The d_{MS} term is smaller for the UPA (RSC) calculation, resulting in $d_D^{(2)}$ being closer to the result of L&T
- The dependence of the UPA (RSC) result on a particular ${}^3\mathrm{P}_1$ potential is reduced to the point that $d_D^{(2)}$ is almost independent of that interaction
- The more recent UPA approximation to the Reid 1993 potential shows that $d_D^{(2)}$ may depend upon the off-shell properties of the ${}^3\mathrm{P}_1$ potential

Summary

Until the precision of ^{2}H EDM measurements is considerably enhanced ($<$ 10%), a separable potential approach to calculating such a quantity should be more than adequate.