Fluctuations and Higher Moments of Conserved Charges from the Lattice.

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The QCD Phase Diagram

- Second-order phase transition belonging to the $O(4)$ universality class for $(m_l, \mu_q) = (0, 0)$.
- Rich phase structure in the $(m_l, T, \mu_q)$-plane. Scaling relations near $(m_l, \mu_q) = (0, 0)$ w.r.t. all variables viz. $T$, $m_l$ and $\mu_q$.
Quark Number Susceptibilities

• Direct lattice simulations at $\mu_q \neq 0$ not possible due to the sign problem.

• Alternative: Taylor-expand the partition function around $\mu_q = 0$ viz.

$$\frac{1}{VT^3} \ln \mathcal{Z}(\mu_u, \mu_d, \mu_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}}{i! \, j! \, k!} \left( \frac{\mu_u}{T} \right)^i \left( \frac{\mu_d}{T} \right)^j \left( \frac{\mu_s}{T} \right)^k.$$ 

• Straightforward interpretation as cumulants of quark number distributions. The off-diagonals measure correlations between flavors.

• Directly related to quark degrees of freedom, hence good probes of deconfinement.
Staggered Fermions


- **Inexpensive to simulate:** possible to work with light pions and at small lattice spacings.

- **Four-flavor theory!** Sixteen pions instead of one, $O(\alpha_s a^2)$ mass splittings.

- Our action greatly reduces this splitting between tastes. Nevertheless, we are still far from the continuum as we shall see...
A First Look at Susceptibilities: $\chi^u, s_2$

- **Smøother transition.** Reduced taste-breaking $\implies$ More light degrees of freedom. Pion too is lighter (160 MeV compared to earlier 220 MeV).

- The inflection point $\sim T_c$ is at a lower temperature.

- The approach to the Stefan-Boltzmann (SB) limit too is slower.

Interactions significant even at $\sim 2.5T_c$. 
Fluctuations of Conserved Charges

- QCD conserves baryon number ($B$), electric charge ($Q$), isospin ($I$), strangeness ($S$), etc.

- Three independent quantities for $N_f = 3$; we shall choose $B$, $Q$ and $S$.

- The corresponding QNS are related to the $u$, $d$ and $s$ ones via

  \[
  \frac{\partial}{\partial \mu_B} = \left( \frac{1}{3} \frac{\partial}{\partial \mu_u} + \frac{1}{3} \frac{\partial}{\partial \mu_d} + \frac{1}{3} \frac{\partial}{\partial \mu_s} \right),
  \]

  \[
  \frac{\partial}{\partial \mu_Q} = \left( \frac{2}{3} \frac{\partial}{\partial \mu_u} - \frac{1}{3} \frac{\partial}{\partial \mu_d} - \frac{1}{3} \frac{\partial}{\partial \mu_s} \right), \quad \text{and}
  \]

  \[
  \frac{\partial}{\partial \mu_S} = - \frac{\partial}{\partial \mu_s}.
  \]

- Good quantum numbers i.e. particles carry definite values of $B$, $Q$ and $S$. Hence, experimentally these are the more relevant fluctuations.
• Qualitatively similar to the light and strange quark fluctuations.

• Cutoff effects are smaller than for the earlier p4 action, but still significant.

• More subtle cutoff effects: $T$-determination varies by a few % depending on observable used. From now on, we shall use $f_K$ to set the scale.
Hadron Resonance Gas models

- **Hadron Resonance Gas** (HRG) models assume an ideal gas of mesons and baryons with masses smaller than some cutoff (typically 2-2.5 GeV) viz.

  \[
  \ln Z = \sum_{\pi,K,...} \ln Z_{B-E}^{id}(m_i, \mu_i) + \sum_{p,n,\Lambda,...} \ln Z_{F-D}^{id}(m_i, \mu_i).
  \]

- HRG models provide a simple and intuitive picture for fluctuations.
  - The contribution of a particular meson/baryon \( X \) is inversely related to its mass \( \exp(-m_X/T) \).
  - Not all mesons contribute; only those carrying the relevant quantum number.
  - For e.g. the dominant contribution to \( \chi^Q_2 \) comes from pions, but to \( \chi^B_2 \) from protons.
Agreement with HRG is better for $\chi^B_2$ than for $\chi^Q_2$.

This is because taste violations are larger for pions than for baryons and heavier mesons.
Correlations Among Conserved Charges: Off-Diagonal Coefficients

- For $T < T_c$ the dominant contributions to $\chi_{QS}$ and $\chi_{BQ}$ come from kaons and protons respectively.

- At high temperatures only the strange quark contributes to $\chi_{BS}$, thus $\chi_{BS} \rightarrow -1/3$.

- All quarks contribute to $\chi_{BQ}$ but the net charge is zero. Hence $\chi_{BQ} \approx 0$ for $T \gg T_c$. 
Higher-order coefficients more sensitive to the presence of the critical point (see talk by F. Karsch)

\[ \chi_{2n} \sim t^{2-\alpha-n} \quad \text{where} \quad t \equiv \text{scaling variable} := \left| \frac{T - T_c}{T_c} + \frac{\mu_q^2}{T_c^2} \right|. \]

- \( \partial^n/\partial T^n = \partial^{2n}/\partial \mu_q^{2n} \) at \( T = T_c \): Inflection point in \( \chi_2 \implies \text{peak in } \chi_4 \implies \chi_6 = 0. \)

- \( \alpha < 0 \) for \( O(4) \) models. Thus the peak in \( \chi_4 \) should vanish, and \( \chi_6 \) should diverge (in the chiral limit).

Higher cumulants are more difficult to measure viz.

\[
\chi_4 = \frac{1}{Z} \frac{\partial^4 Z}{\partial \mu^4} - 3 \left( \frac{1}{Z} \frac{\partial^2 Z}{\partial \mu^2} \right)^2,
\]

\[
\chi_6 = \frac{1}{Z} \frac{\partial^6 Z}{\partial \mu^6} - 15 \left( \frac{1}{Z} \frac{\partial^2 Z}{\partial \mu^2} \right) \left( \frac{1}{Z} \frac{\partial^2 Z}{\partial \mu^2} \right) + 30 \left( \frac{1}{Z} \frac{\partial^2 Z}{\partial \mu^2} \right)^3, \quad \text{etc.}
\]

Increasingly delicate cancellations require increasingly more statistics.
Conclusions

- Quark Number Susceptibilities are directly related to quark degrees of freedom, hence they are good probes of deconfinement.

- Sensitive to the light degrees of freedom. Our simulations, with $m_\pi \approx 160$ MeV, are the closest so far to the physical pion mass.

- Our results are consistent with the picture of a gas of hadrons changing to a (strongly-coupled) quark-gluon plasma.

- Correlations are found to persist up to temperatures $T \gtrsim 2T_c$.

- Higher-order coefficients are consistent with scaling expectations. More statistics are required for a clearer picture.

- We need better control over systematics, notably discretization errors.