Fluctuations and Higher Moments of Conserved Charges from the Lattice.

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(for the HotQCD and
RBC-Bielefeld collaborations)

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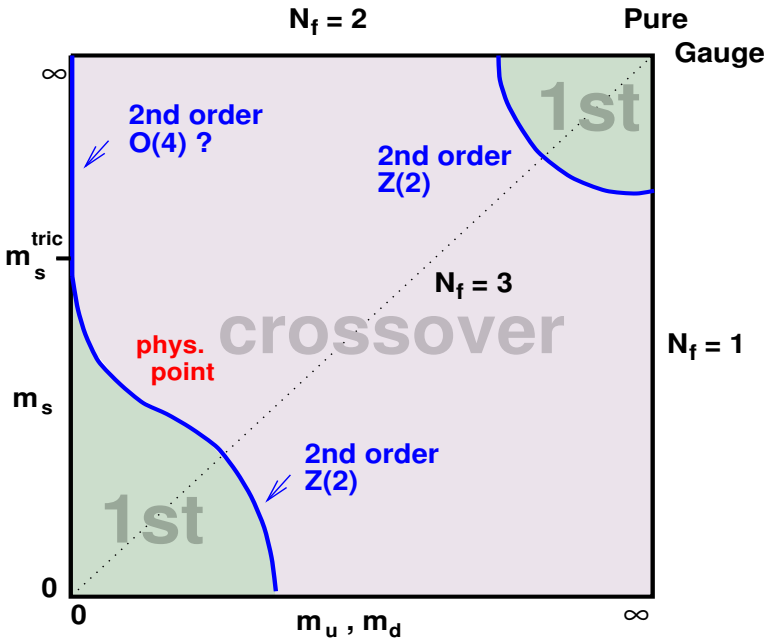
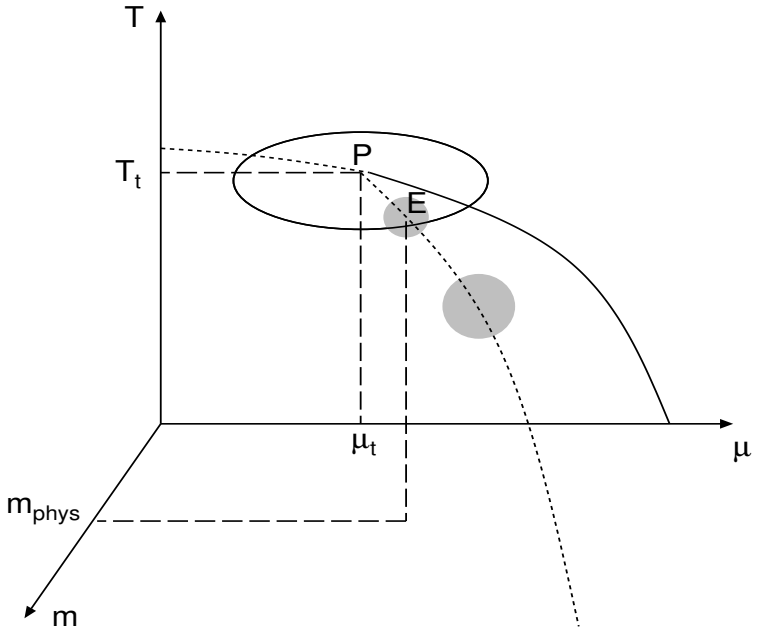
The QCD Phase Diagram





Y. Hatta,T. Ikeda; **PRD27**,014028 (2003)

- Second-order phase transition belonging to the $O(4)$ universality class for $(m_l, \mu_q) = (0, 0)$.
- Rich phase structure in the (m_l, T, μ_q)-plane. Scaling relations near $(m_l, \mu_q) = (0, 0)$ w.r.t. all variables viz. T, m_l and μ_q.

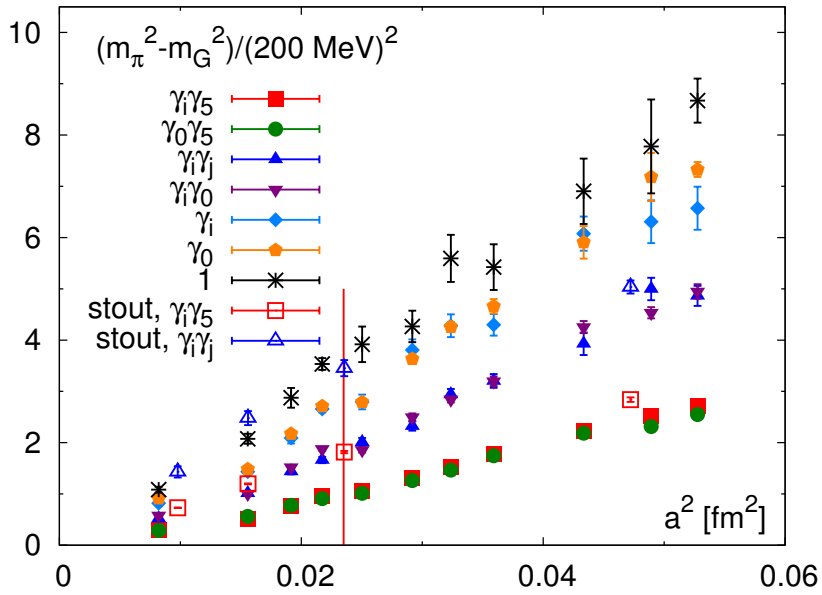
Quark Number Susceptibilities

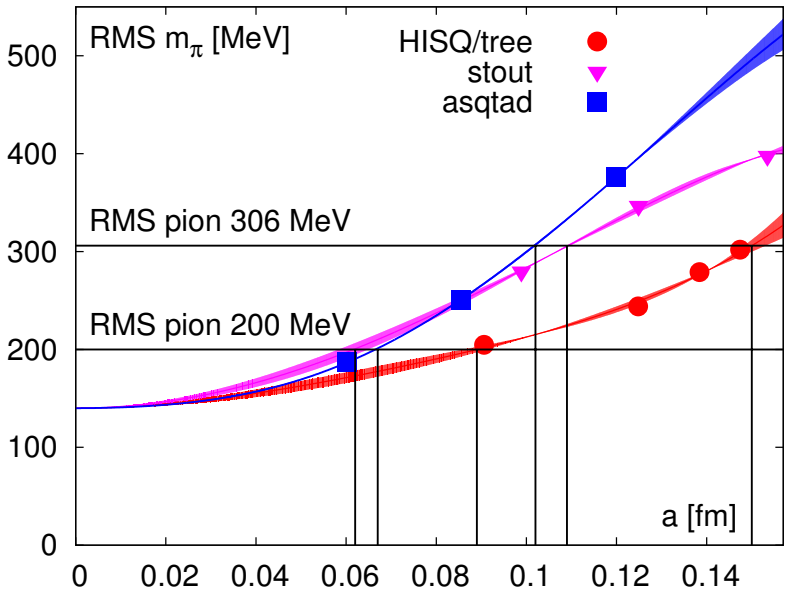
- Direct lattice simulations at $\mu_q \neq 0$ not possible due to the sign problem.
- Alternative: Taylor-expand the partition function around $\mu_q = 0$ viz.

$$\frac{1}{VT^3} \ln \mathcal{Z}\left(\mu_u, \mu_d, \mu_s\right) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}}{i!\, j!\, k!} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k .$$

- Straightforward interpretation as cumulants of quark number distributions. The off-diagonals measure correlations between flavors.
- Directly related to quark degrees of freedom, hence good probes of deconfinement.

Staggered Fermions

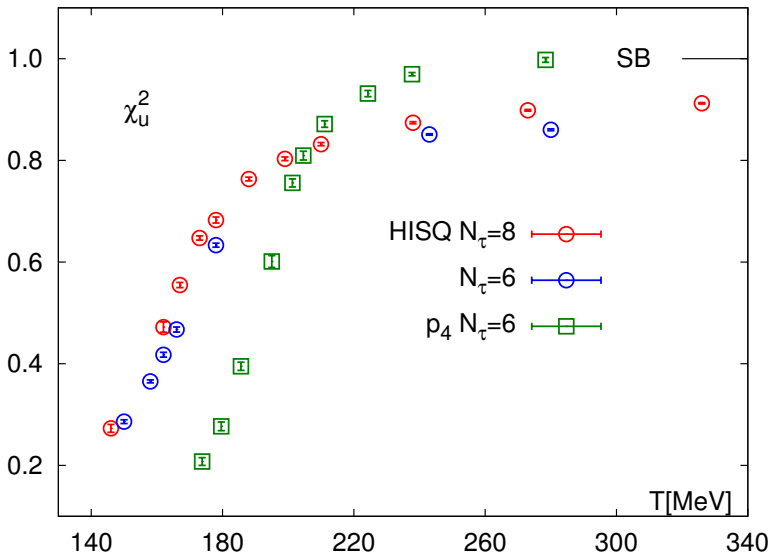


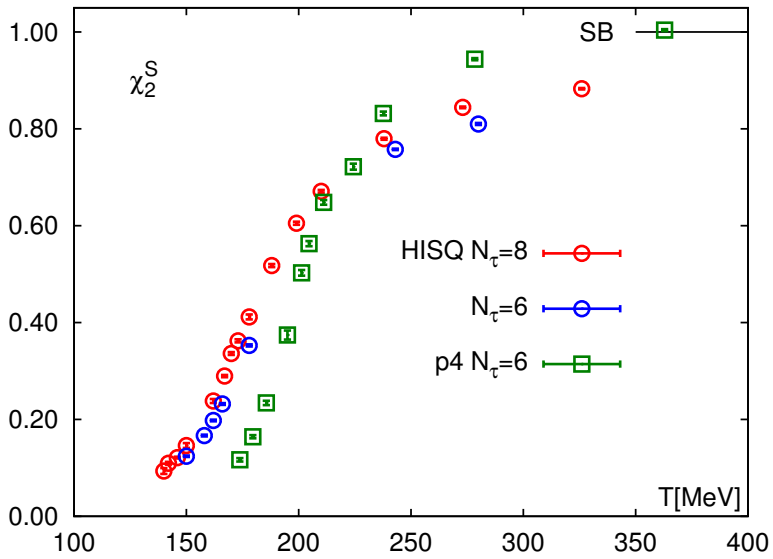


A. Bazavov and P. Petreczky, PoS (Lattice 2010) 169.

- Inexpensive to simulate; possible to work with light pions and at small lattice spacings.
- Four-flavor theory! Sixteen pions instead of one, $\mathcal{O}(\alpha_s a^2)$ mass splittings.
- Our action greatly reduces this splitting between tastes. Nevertheless, we are still far from the continuum as we shall see...

A First Look at Susceptibilities: $\chi_2^{u,s}$





- Smoother transition. Reduced taste-breaking $\Longrightarrow$ More light degrees of freedom. Pion too is lighter (160 MeV compared to earlier 220 MeV).
- The inflection point $\sim T_c$ is at a lower temperature.
- The approach to the Stefan-Boltzmann (SB) limit too is slower Interactions significant even at $\sim 2.5T_c$.

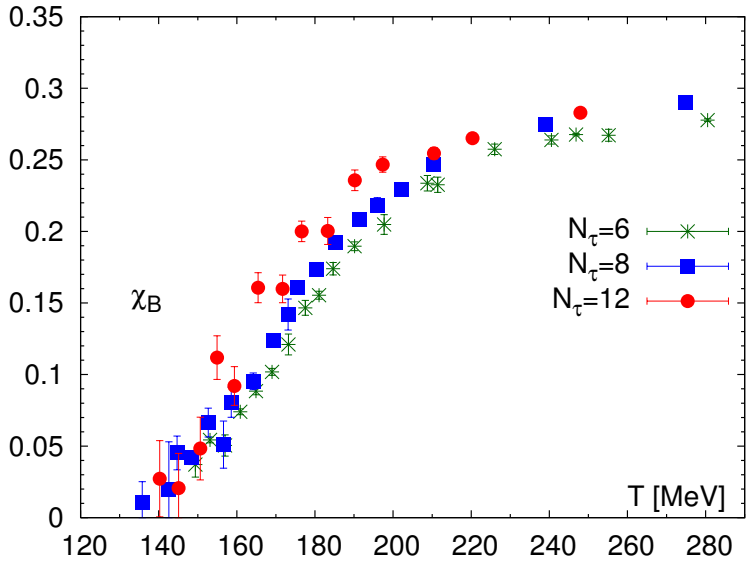
Fluctuations of Conserved Charges

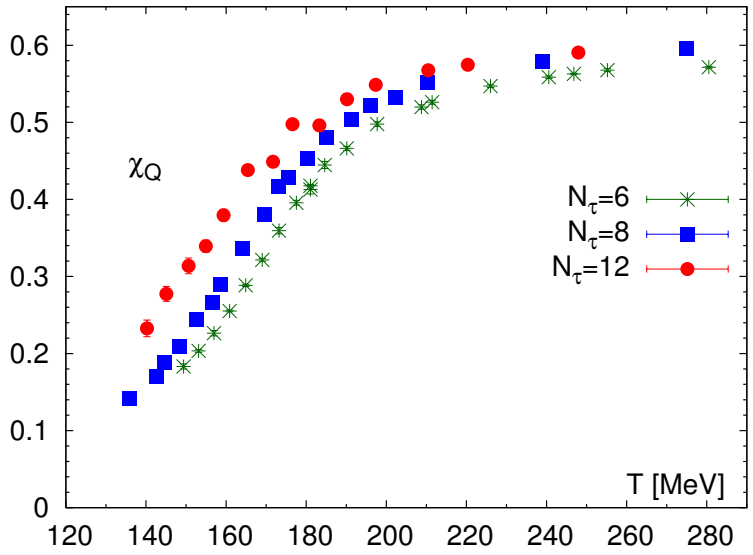
- QCD conserves baryon number (B), electric charge (Q), isospin (I), strangeness (S), etc.
- Three independent quantities for $N_f = 3$; we shall choose B, Q and S.
- The corresponding QNS are related to the u, d and s ones via

$$\frac{\partial}{\partial \mu_B} = \left(\frac{1}{3}\frac{\partial}{\partial \mu_u} + \frac{1}{3}\frac{\partial}{\partial \mu_d} + \frac{1}{3}\frac{\partial}{\partial \mu_s} \right),$$
$$\frac{\partial}{\partial \mu_Q} = \left(\frac{2}{3}\frac{\partial}{\partial \mu_u} - \frac{1}{3}\frac{\partial}{\partial \mu_d} - \frac{1}{3}\frac{\partial}{\partial \mu_s} \right), \quad \text{and}$$
$$\frac{\partial}{\partial \mu_S} = -\frac{\partial}{\partial \mu_s}.$$

- Good quantum numbers i.e. particles carry definite values of B, Q and S. Hence, experimentally these are the more relevant fluctuations.

Conserved Charge Fluctuations: A First Look





- Qualitatively similar to the light and strange quark fluctuations.
- Cutoff effects are smaller than for the earlier p4 action, but still significant.
- More subtle cutoff effects: T-determination varies by a few % depending on observable used. From now on, we shall use f_K to set the scale.

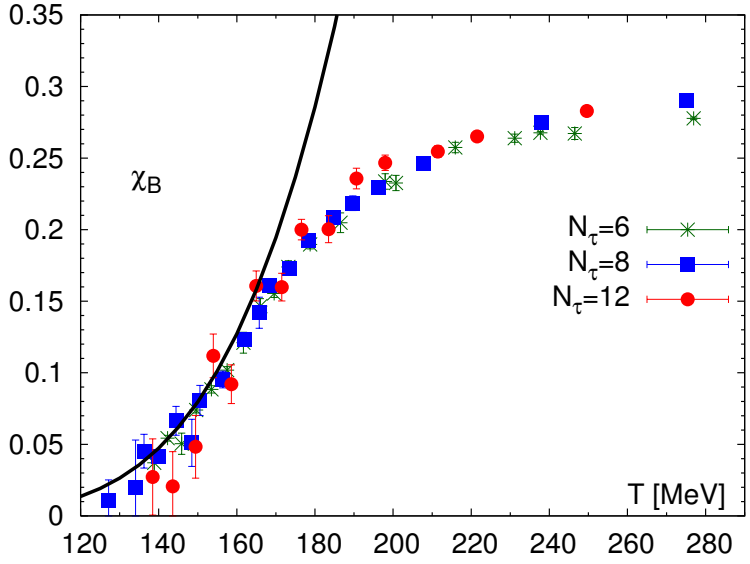
Hadron Resonance Gas models

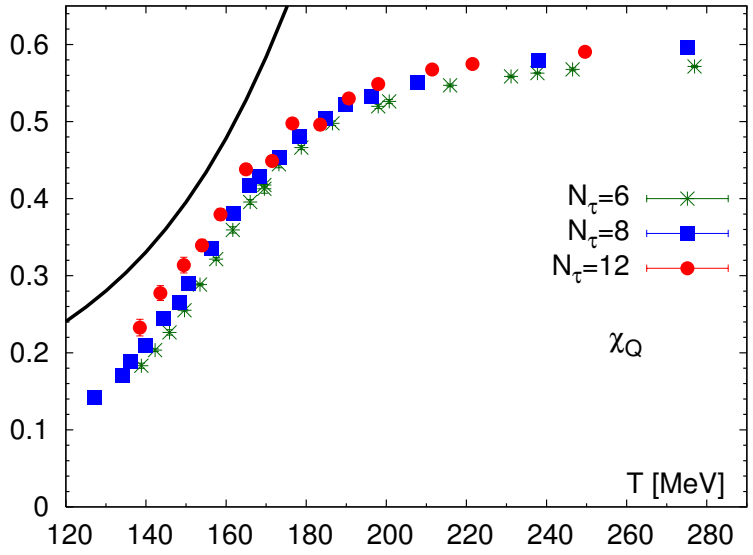
- Hadron Resonance Gas (HRG) models assume an ideal gas of mesons and baryons with masses smaller than some cutoff (typically 2-2.5 GeV) viz.

$$\ln \mathcal{Z} = \sum_{\pi,K,\ldots} \ln \mathcal{Z}^{\text{id.}}_{\text{B-E}}(m_i, \mu_i) + \sum_{p,n,\Lambda,\ldots} \ln \mathcal{Z}^{\text{id.}}_{\text{F-D}}(m_i, \mu_i).$$

- HRG models provide a simple and intuitive picture for fluctuations.
  - The contribution of a particular meson/baryon X is inversely related to its mass ($\mathrm{e}^{-m_X/T}$).
  - Not all mesons contribute; only those carrying the relevant quantum number.
  - For e.g. the dominant contribution to χ_2^Q comes from pions, but to χ_2^B from protons.

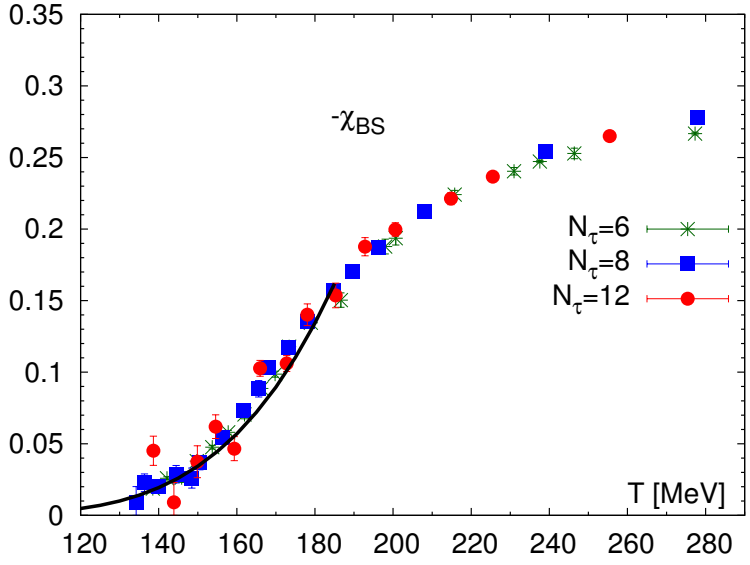
B/Q Fluctuations and HRG

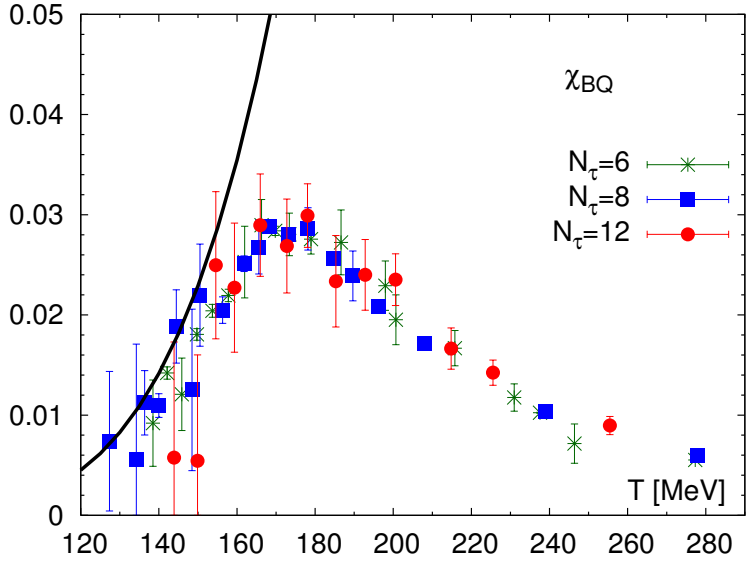




- Agreement with HRG is better for χ_2^B than for χ_2^Q.
- This is because taste violations are larger for pions than for baryons and heavier mesons.

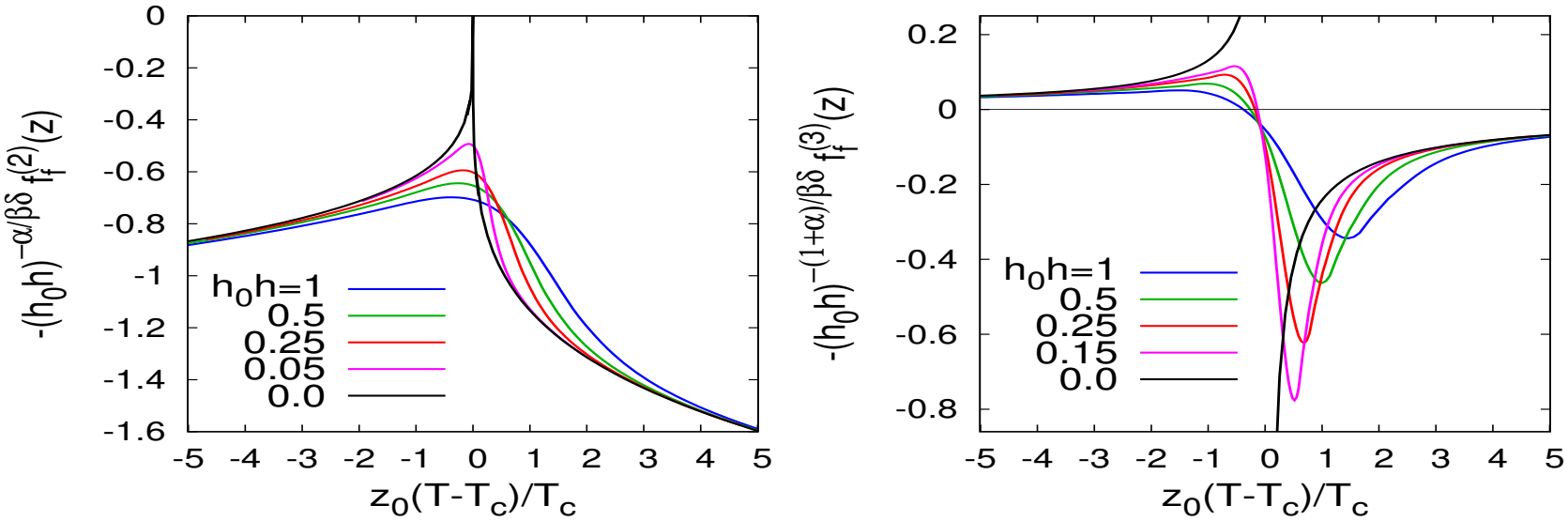
Correlations Among Conserved Charges: Off-Diagonal Coefficients





- For $T < T_c$ the dominant contributions to χ_{QS} and χ_{BQ} come from kaons and protons respectively.
- At high temperatures only the strange quark contributes to χ_{BS}, thus $\chi_{BS} \to -1/3$.
- All quarks contribute to χ_{BQ} but the net charge is zero. Hence $\chi_{BQ} \approx 0$ for $T \gg T_c$.

Higher-Order Coefficients



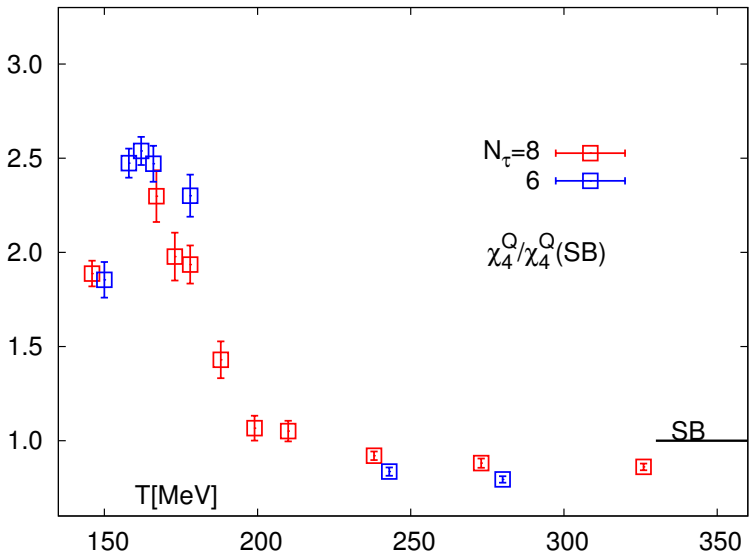
B. Friman et al. Eur. Phys. J C71 (2011) 1694.

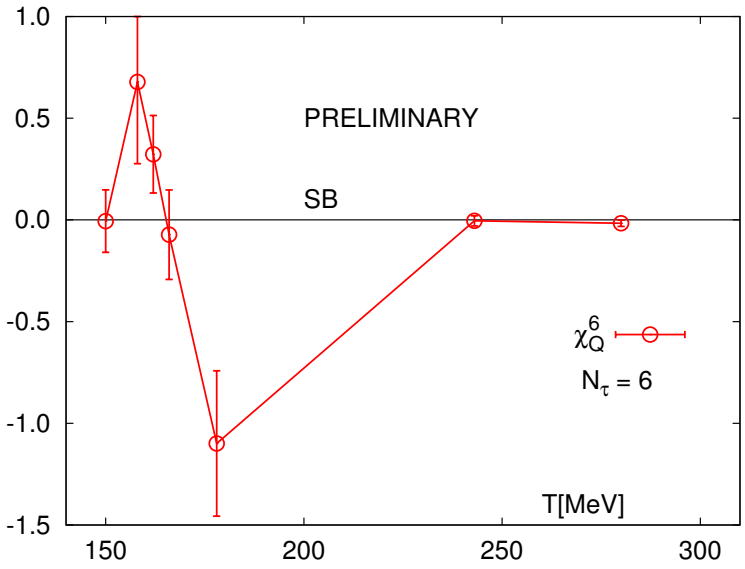
- Higher-order coefficients more sensitive to the presence of the critical point (see talk by F. Karsch)

$$\chi_{2n} \sim t^{2-\alpha-n} \quad \text{where} \quad t \equiv \text{scaling variable} := \left| \frac{T - T_c}{T_c} + \frac{\mu_q^2}{T_c^2} \right| .$$

- $\partial^n / \partial T^n \equiv \partial^{2n} / \partial \mu_q^{2n}$ at $T = T_c$: Inflection point in $\chi_2 \implies$ peak in $\chi_4 \implies \chi_6 = 0$.
- $\alpha < 0$ for $O(4)$ models. Thus the peak in χ_4 should vanish, and χ_6 should diverge (in the chiral limit).

Higher-Order Coefficients: Results





- Higher cumulants are more difficult to measure viz.

$$\chi_4 = \frac{1}{\mathcal{Z}}\frac{\partial^4 \mathcal{Z}}{\partial \mu^4} - 3\left(\frac{1}{\mathcal{Z}}\frac{\partial^2 \mathcal{Z}}{\partial \mu^2}\right)^2,$$

$$\chi_6 = \frac{1}{\mathcal{Z}}\frac{\partial^6 \mathcal{Z}}{\partial \mu^6} - 15\left(\frac{1}{\mathcal{Z}}\frac{\partial^2 \mathcal{Z}}{\partial \mu^2}\right)\left(\frac{1}{\mathcal{Z}}\frac{\partial^2 \mathcal{Z}}{\partial \mu^2}\right) + 30\left(\frac{1}{\mathcal{Z}}\frac{\partial^2 \mathcal{Z}}{\partial \mu^2}\right)^3, \text{etc.}$$

- Increasingly delicate cancellations require increasingly more statistics.

Conclusions

- Quark Number Susceptibilities are directly related to quark degrees of freedom, hence they are good probes of deconfinement.
- Sensitive to the light degrees of freedom. Our simulations, with $m_\pi \approx 160$ MeV, are the closest so far to the physical pion mass.
- Our results are consistent with the picture of a gas of hadrons changing to a (strongly-coupled) quark-gluon plasma.
- Correlations are found to persist up to temperatures $T \gtrsim 2T_c$.
- Higher-order coefficients are consistent with scaling expectations. More statistics are required for a clearer picture.
- We need better control over systematics, notably discretization errors.