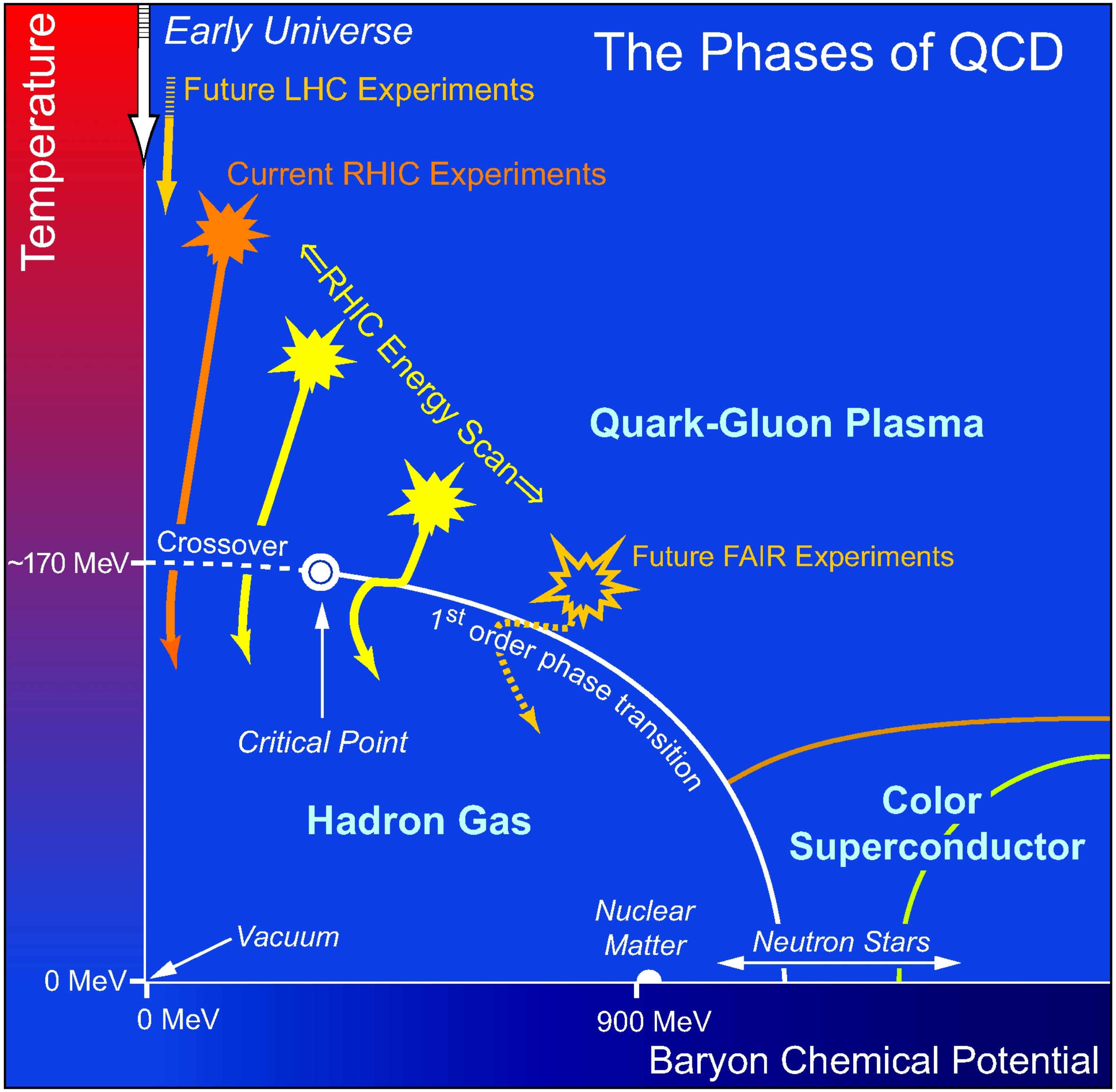
O(4) scaling, pseudo-critical temperatures and freeze-out in Heavy Ion Collisions

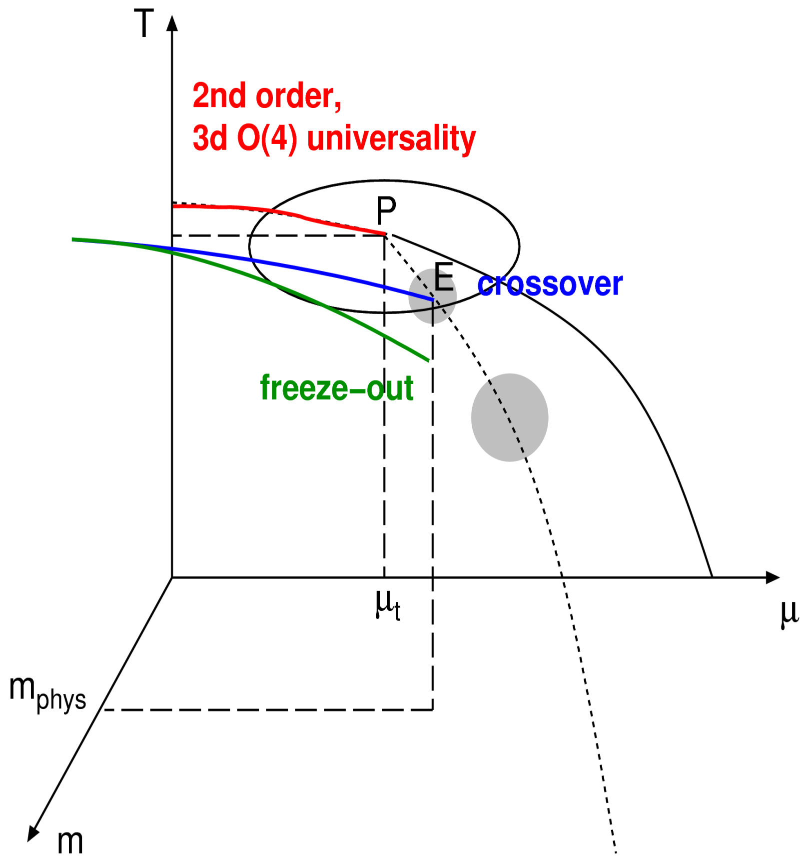
Frithjof Karsch, BNL&Bielefeld



OUTLINE:

- QCD phase diagram close to the chiral limit
- QCD close to the chiral limit, O(N) scaling
- *moments of charge fluctuations as probe for for proximity to criticality*

Phase diagram for $\mu_B \geq 0,\ m_q > 0$



based on:
Y. Hatta, T. Ikeda, PRD27,014028 (2003)

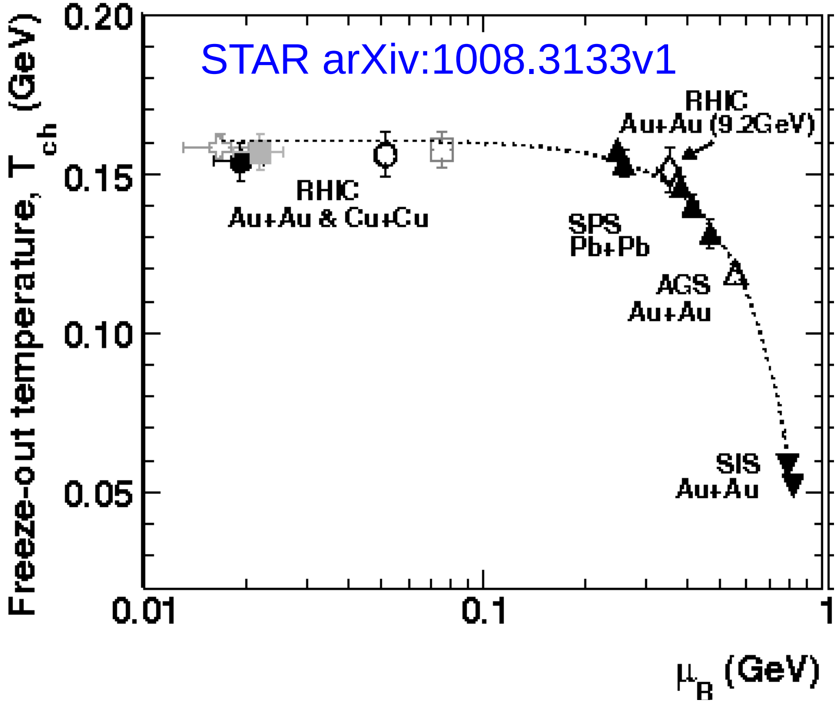
- critical line at m_q=0

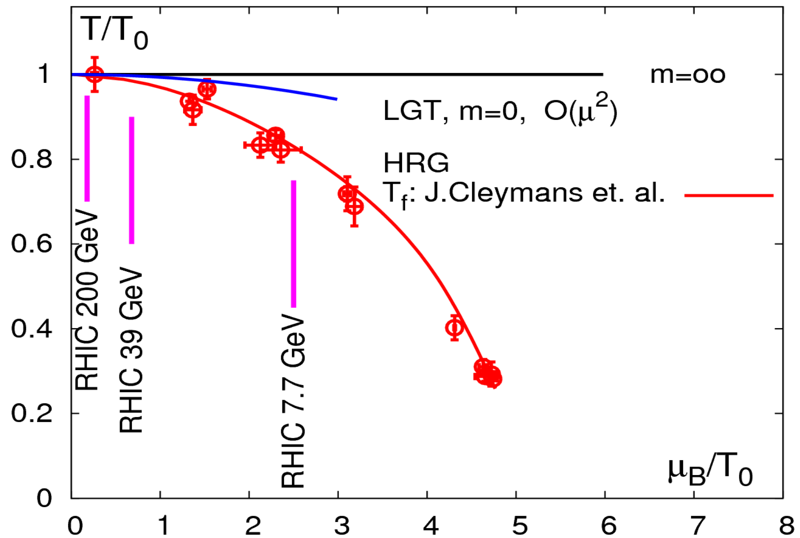
$$\frac{T_c(\mu_q)}{T_c} = 1 - \kappa_q \left(\frac{\mu_q}{T}\right)^2 - \mathcal{O}(\mu_q^4)$$

- crossover line
  physics on crossover line controlled by universal scaling relations ?

- freeze-out line
  Is the crossover line related to the experimentally determined freeze-out curve?

The RHIC low energy runs





Moments of charge fluctuations

susceptibilities = cumulants of charge fluctuations

$$\delta N_B = N_B - \langle N_B \rangle$$

$$\chi_{B,\mu}^{(2)} = \langle (\delta N_B)^2 \rangle$$

$$\chi_{B,\mu}^{(4)} = \langle (\delta N_B)^4 \rangle - 3\langle (\delta N_B)^2 \rangle^2$$

....

freeze-out line and chiral phase transition

J. Cleymans et al, PRC73, 034905 (2006)

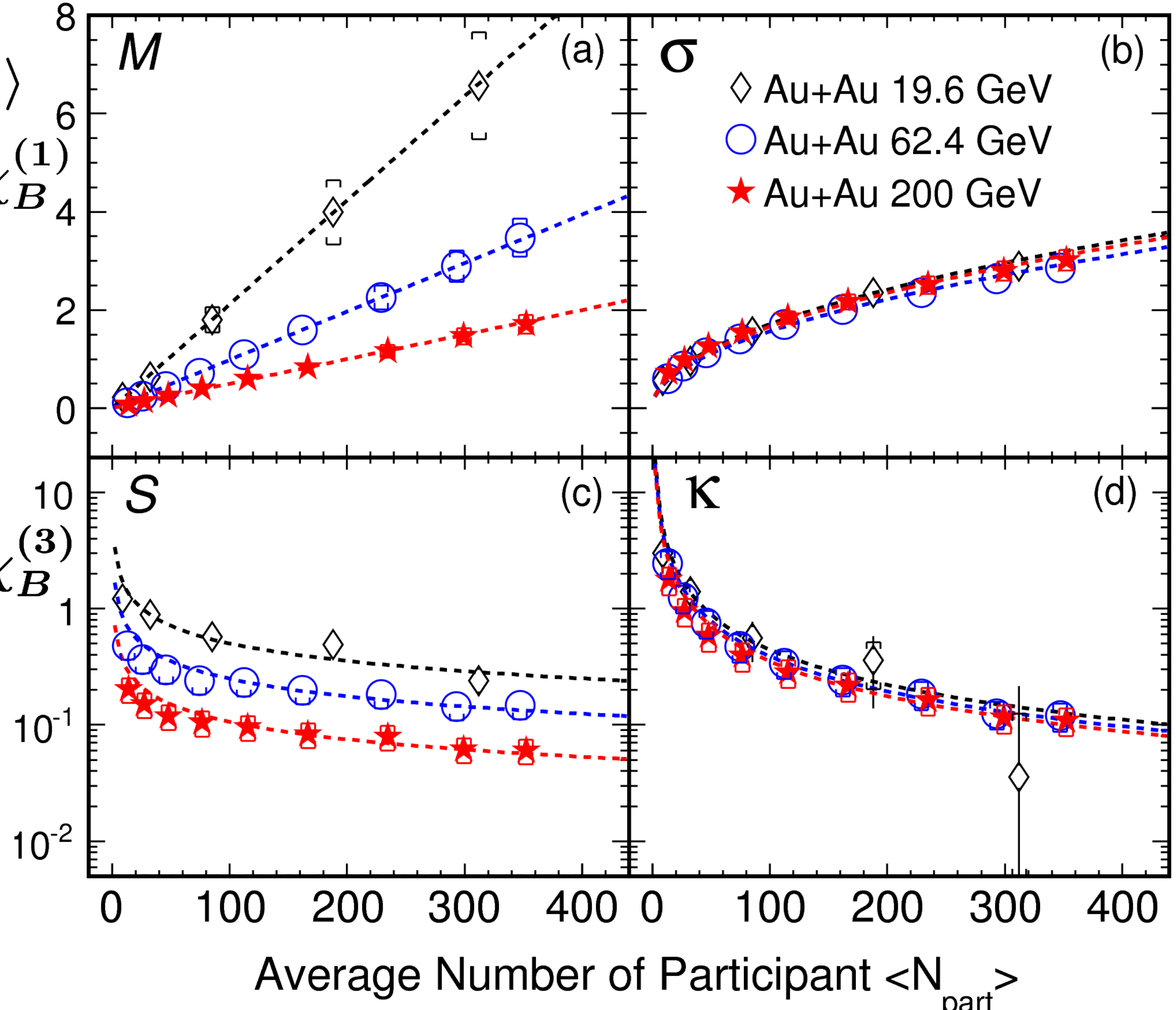
BNL-Bielefeld-GSI, arXiv:1011.3130

Mean, variance, skewness & kurtosis

STAR results from RHIC low energy run

$$M = \langle \delta N_B \rangle = VT^3 \chi_B^{(1)}$$

$$\sigma^2 = \langle (\delta N_B)^2 \rangle = VT^3 \chi_B^{(2)}$$



$$S\sigma^3 = VT^3 \chi_B^{(3)}$$

$$\kappa\sigma^4 = VT^3 \chi_B^{(4)}$$

M. M. Aggarwal et al. (STAR Collaboration), Phys. Rev. Lett. 105 (2010) 22302

Baryon number susceptibilities in the Hadron Resonance Gas

- the pressure in the HRG model

$$\frac{P}{T^4} = \frac{1}{\pi^2} \sum_i d_i \left(\frac{m_i}{T}\right)^2 K_2\left(\frac{m_i}{T}\right) \cosh[(B_i\mu_B + S_i\mu_S + Q_i\mu_Q)/T]$$

$$\mu_S = \mu_Q = 0 :$$

$$\kappa_B \sigma_B^2 = 1$$

$$S_B \sigma_B = \tanh(\mu_B/T)$$

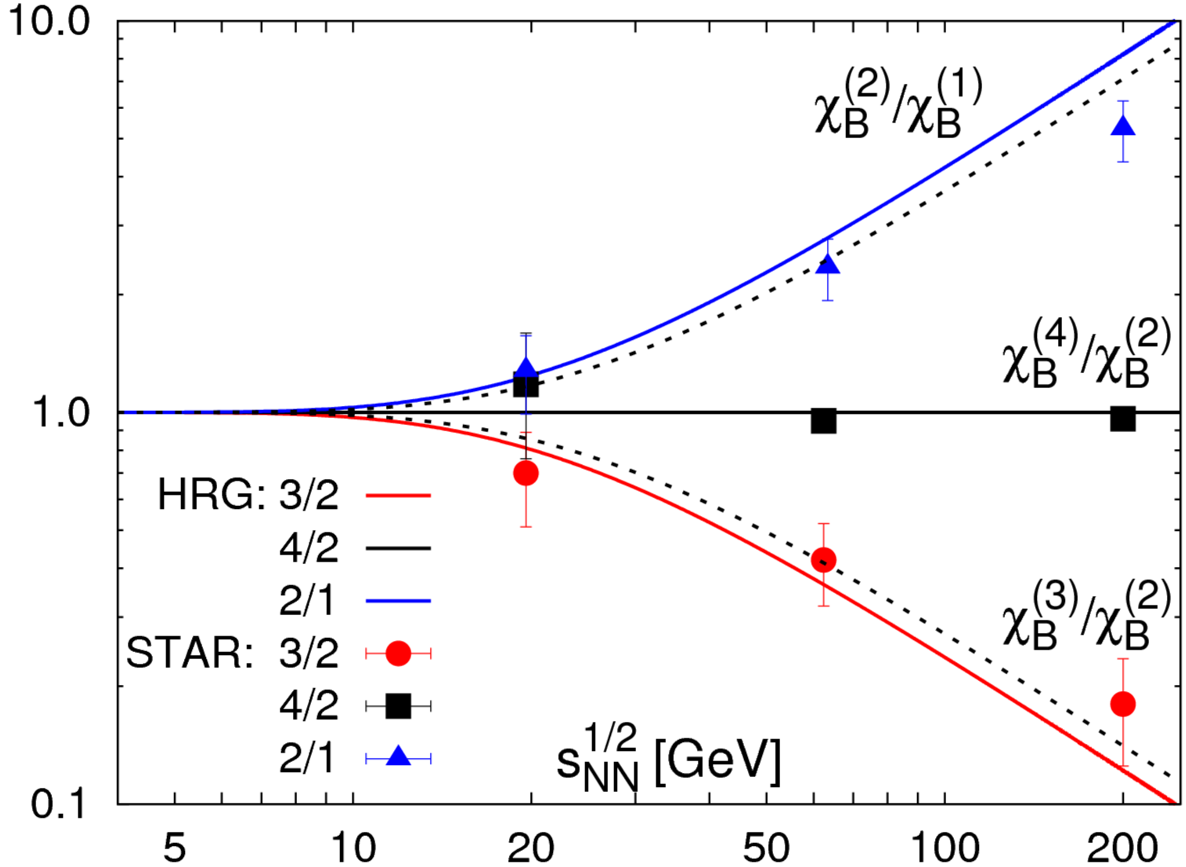
$$\frac{\sigma_B^2}{M_B} = \frac{1}{S_B \sigma_B}$$

FK, K. Redlich, arXiv:1007.2581

other HRG consequence:

$$\frac{S_B \sigma_B^3}{M_B} \equiv \frac{\chi_B^{(3)}}{\chi_B^{(1)}} = 1 \, , \qquad \frac{\kappa_B \sigma_B}{S_B} \equiv \frac{\chi_B^{(4)}}{\chi_B^{(3)}} = \coth(\mu_B/T)$$

Cumulants of net-baryon fluctuations



dashed:

$$\mu_Q = \mu_S = 0$$

solid:

$$\mu_Q(\mu_B),\ \mu_S(\mu_B)$$

on freeze-out curve

$\chi_B^{(1)}$ ~ input for V

FK, K Redlich, arXiv:1007.2581
data from STAR: arXiv:1004.4959

charge fluctuations at freeze-out agree 'almost' with HRG model predictions

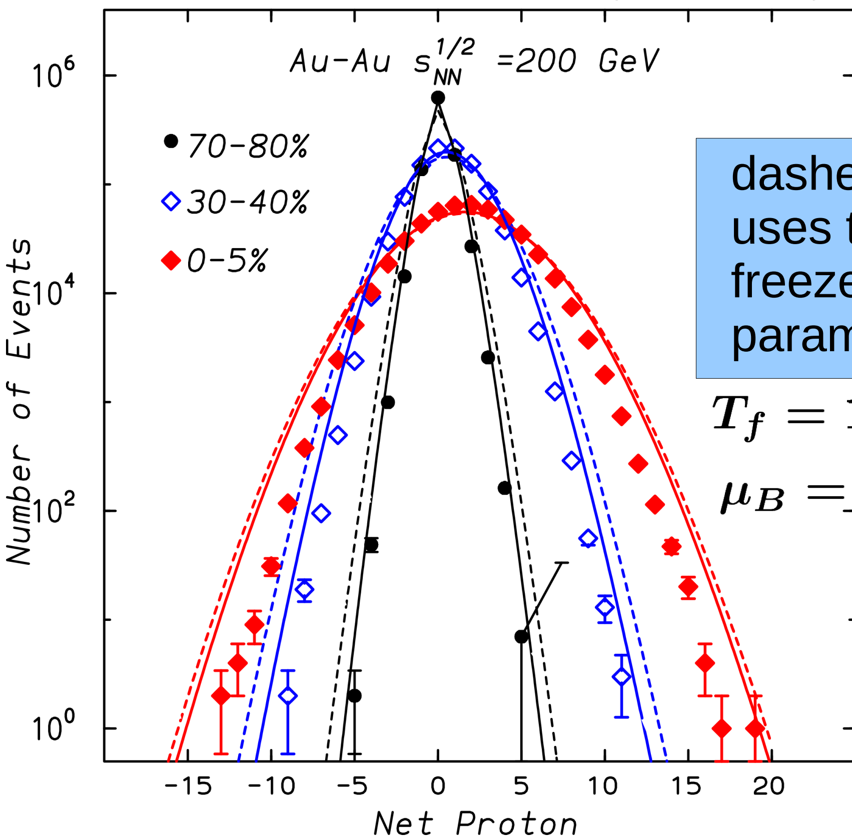
significant deviations between HRG model and data for the variance ($\chi_B^{(2)}$)?

Net-proton distribution from HRG model

probability distribution P(N) for net-proton numbers within the HRG model:

entirely given in terms of (measurable) proton (b) and anti-proton ($\bar{b}$) numbers

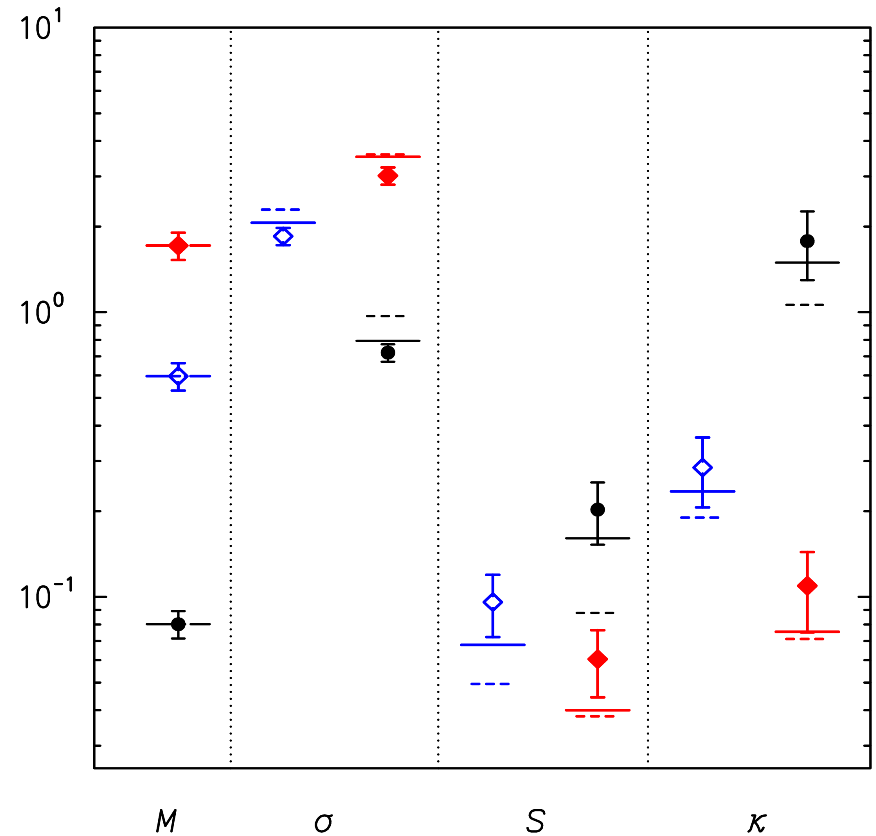
$$P(N) = \mathrm{e}^{N\mu/T}\,\frac{Z_N(V,T)}{Z(V,T,\mu)} = \left(\frac{b}{\bar{b}}\right)^{N/2} I_N(2\sqrt{b\bar{b}}\,)\,\exp[-(b+\bar{b})]$$



dashed line uses thermal freeze-out parameter

$$T_f = 159.3\mathrm{MeV}$$

$$\mu_B = 21.9\mathrm{MeV}$$



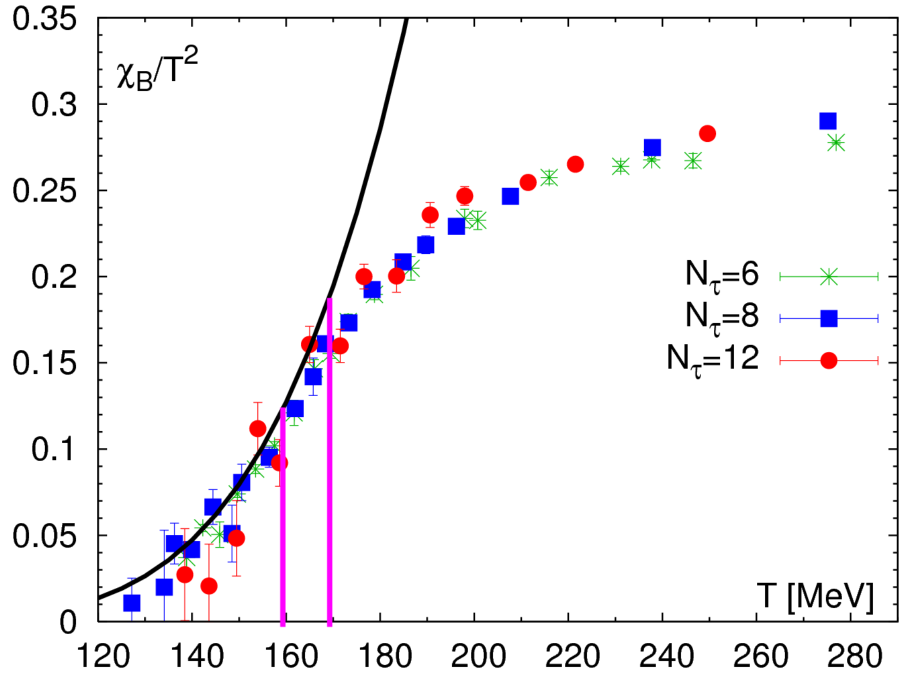
distribution is narrower than HRG model predicts

P. Braun-Munziger et al., arXiv:1107.4267

Net-baryon number susceptibility

P(N) may be expressed in terms of a cumulant expansion; leading order corresponds to Gaussian approximation:

$$P(N) \sim \exp[-N^2/(2VT^3\chi_B^{(2)})]$$



HISQ action:
hotQCD, preliminary

in the transition region the probability distribution derived in leading order from QCD may become narrower than the HRG model result

➡ higher order cumulants are more sensitive to critical behavior

O(N) scaling and chiral transition

- close to the chiral limit thermodynamics in the vicinity of the QCD transition is controlled by O(4) scaling functions:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = h^{1+1/\delta} f_s(t/h^{1/\beta\delta}) + f_r(V,T,\vec{\mu})$$

- critical behavior controlled by two relevant fields: t, h

  - all couplings that do not explicitly break chiral symmetry contribute in leading order only to 't'

$$t = \frac{1}{t_0}\left(\left(\frac{T}{T_c}-1\right) - \kappa_q \left(\frac{\mu_q}{T}\right)^2\right) \qquad h = \frac{1}{h_0}\frac{m_l}{m_s}$$

Scaling properties of higher order cumulants of net-charge fluctuations

- Taylor expansion of the pressure

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi_{B,0}^{(n)}(T) \left(\frac{\mu_B}{T} \right)^n$$

- generalized quark number susceptibilities

$$\chi_{B,0}^{(n)} = \frac{1}{VT^3} \frac{\partial^n \ln Z}{\partial(\mu_B/T)^n} \bigg|_{\mu_B=0}$$

$$\sim -h^{(2-\alpha-n/2)} f_s^{(n/2)}(z) + \frac{\partial^{2n} f_r(T,V,\vec{\mu})}{\partial(\mu_B/T)^{2n}} \bigg|_{\vec{\mu}=0}$$

susceptibilities = cumulants of charge fluctuations

$$\delta N_B = N_B - \langle N_B \rangle$$

$$\chi_{B,\mu}^{(2)} = \langle (\delta N_B)^2 \rangle$$

$$\chi_{B,\mu}^{(4)} = \langle (\delta N_B)^4 \rangle - 3 \langle (\delta N_B)^2 \rangle^2$$

....

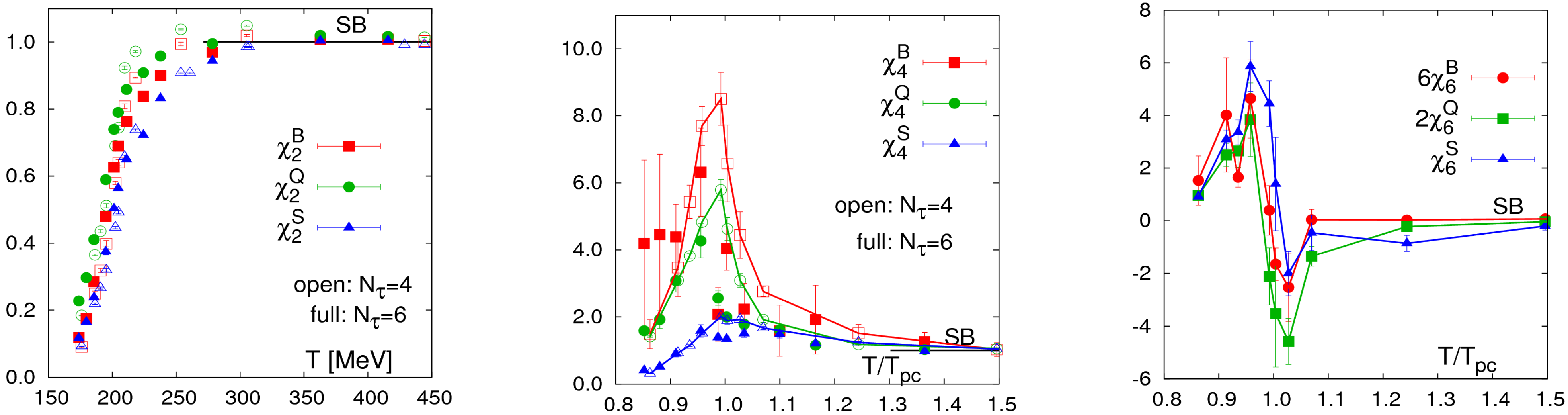
diverges at Tc for m=0 only for n≥6

$$h \sim m_q$$

3d, O(4) scaling function; derivatives of free energy scaling function

Cumulants of net-charge susceptibilities in Lattice QCD

p4-action: $16^3 \times 4,\ 24^3 \times 6$



M. Cheng et al, Phys. Rev. D79 (2009) 074505

- results from an O(a^2) improved action (p4) on coarse lattices
- currently repeated with highly improved action (HISQ) on fine lattices

(talk by Prasad Hegde)

generic features: $\chi_2^X, \chi_4^X \geq 0$ for all $T>0$

$\chi_6^X > 0$ for all $T<T_0$

structure consistent with O(4) universality

Taylor expansions of baryon number susceptibilities

$$\chi_{B,\mu}^{(n)} = \sum_{k=0}^{\infty} \frac{1}{k!} \chi_{B,0}^{(k+n)}(T) \left(\frac{\mu_B}{T}\right)^k$$

mean: $$M_B = VT^3 \chi_{B,\mu}^{(1)} = VT^3 \left(\frac{\mu_B}{T} \chi_{B,0}^{(2)} + \ldots \right)$$

variance: $$\sigma_B^2 = VT^3 \chi_{B,\mu}^{(2)} = VT^3 \left(\chi_{B,0}^{(2)} + \frac{1}{2} \left(\frac{\mu_B}{T}\right)^2 \chi_{B,0}^{(4)} + \ldots \right)$$

skewness and kurtosis and volume independent ratios of susceptibilities

$$S_B \equiv \frac{\langle (\delta N_B)^3 \rangle}{\sigma_B^3} \ , \ \kappa_B \equiv \frac{\langle (\delta N_B)^4 \rangle}{\sigma_B^4} - 3$$

$$\frac{\sigma_B^2}{M_B} = \frac{\chi_B^{(2)}}{\chi_B^{(1)}}, \qquad S_B \sigma_B = \frac{\chi_B^{(3)}}{\chi_B^{(2)}}, \qquad \kappa_B \sigma_B^2 = \frac{\chi_B^{(4)}}{\chi_B^{(2)}}$$

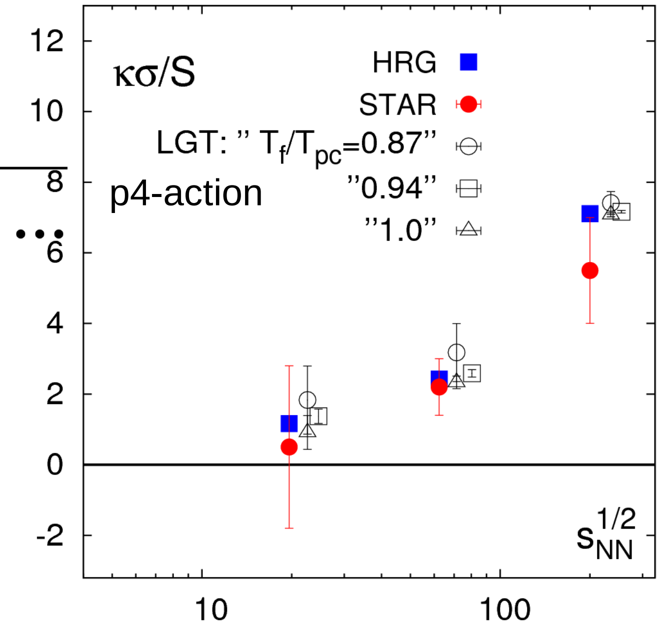
Charge fluctuations in LGT: approach to HRG as T decreases

from LGT to HRG

e.g.:

$$\frac{\chi^{(4)}_{B,\mu}}{\chi^{(3)}_{B,\mu}} = \frac{\chi^{(4)}_{B,0} + \frac{1}{2}\chi^{(6)}_{B,0}(\mu_B/T)^2 + \dots}{\chi^{(4)}_{B,0}(\mu_B/T) + \frac{1}{6}\chi^{(6)}_{B,0}(\mu_B/T)^3 + \dots}$$

$$= \frac{1 + \frac{1}{2}\frac{\chi^{(6)}_{B,0}}{\chi^{(4)}_{B,0}}(\mu_B/T)^2 + \dots}{\mu_B/T + \frac{1}{6}\frac{\chi^{(6)}_{B,0}}{\chi^{(4)}_{B,0}}(\mu_B/T)^3 + \dots}$$



$T \ll T_c$ →

$$\frac{1 + \frac{1}{2}(\mu_B/T)^2 + \dots}{\mu_B/T + \frac{1}{6}(\mu_B/T)^3 + \dots} = \frac{\cosh(\mu_B/T)}{\sinh(\mu_B/T)}$$

FK, K. Redlich, arXiv:1107.1412

expect that all even ratios of susceptibilities approach unity for $T_{freeze} \ll T_c$

so far lattice calculations of cumulants have been done on coarse lattices

Higher moments of charge fluctuations at RHIC and LHC

- higher moments (e.g. 6th order) are drastically different in QCD close to criticality and in a hadron resonance gas, e.g.

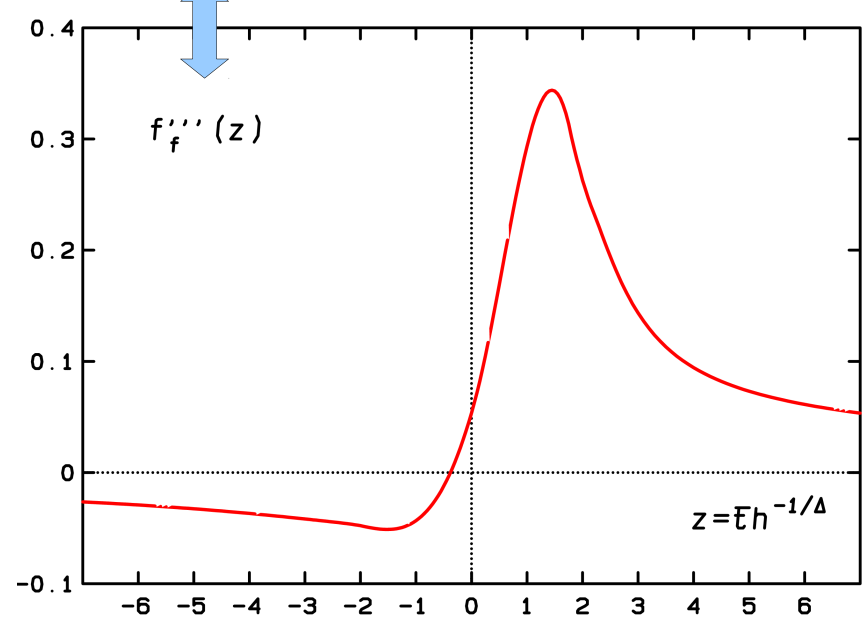
$$\frac{\chi_{B,0}^{(6)}}{\chi_{B,0}^{(2)}} = \begin{cases} = 1 & \text{, hadron resonance gas} \\ < 0 & \text{, QCD at the crossover transition} \end{cases}$$

O(N) scaling: $\chi_{B,0}^{(n)} \sim -h^{(2-\alpha-n/2)} f_s^{(n/2)}(z)$

$$\boxed{\chi_{B,0}^{(6)} \sim \pm A_{\mp} \left| \frac{T - T_c}{T_c} \right|^{-(1+\alpha)}}$$

diverges in the chiral limit: h=mq=0

B. Friman et al., arXiv:1103.3511;
J. Engels, FK, arXiv:1105.0584



Higher moments of charge fluctuations at RHIC and LHC

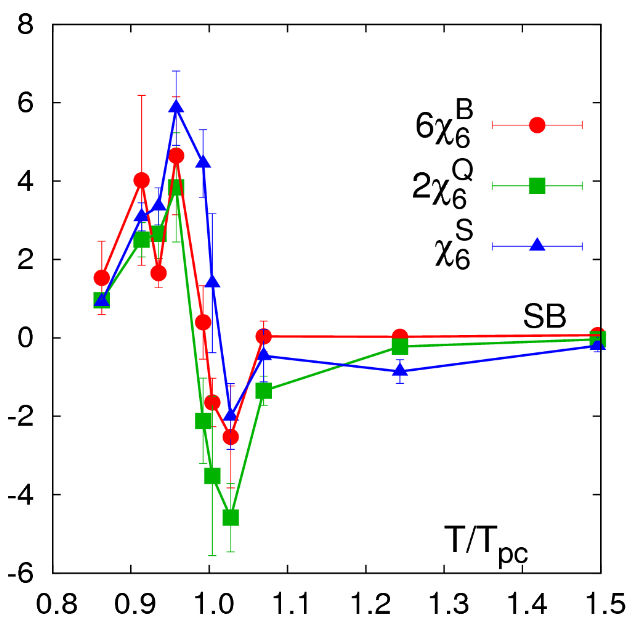
- higher moments (e.g. 6th order) are drastically different in QCD close to criticality and in a hadron resonance gas, e.g.

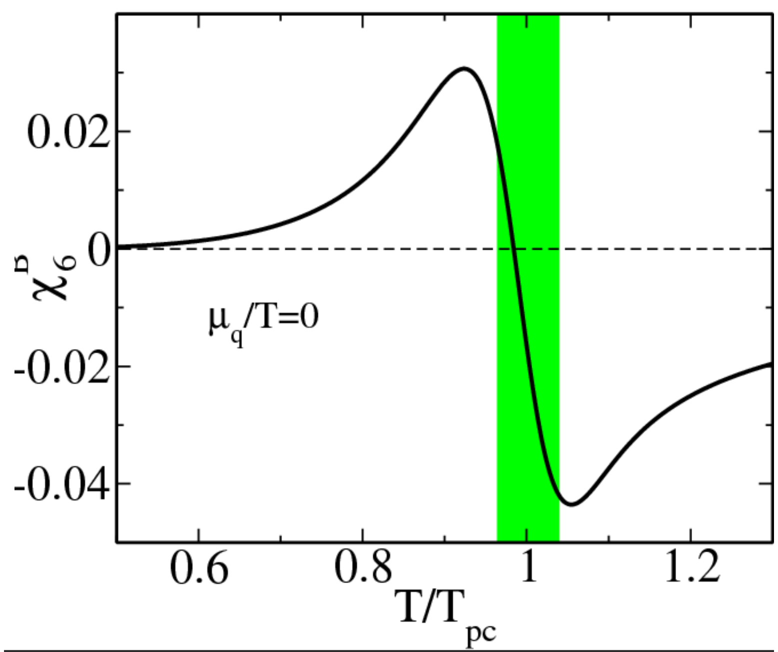
$\mu_B = 0$

$$\frac{\chi_{B,0}^{(6)}}{\chi_{B,0}^{(2)}} = \begin{cases} = 1 & \text{, hadron resonance gas} \\ < 0 & \text{, QCD at the crossover transition} \end{cases}$$

LGT: $16^3 \times 4$ (p4)

PQM model





PQM model and LGT calculations reproduce expected O(4) scaling structure

B. Friman et al, arXiv:1103.3511

Higher moments of charge fluctuations at RHIC and LHC

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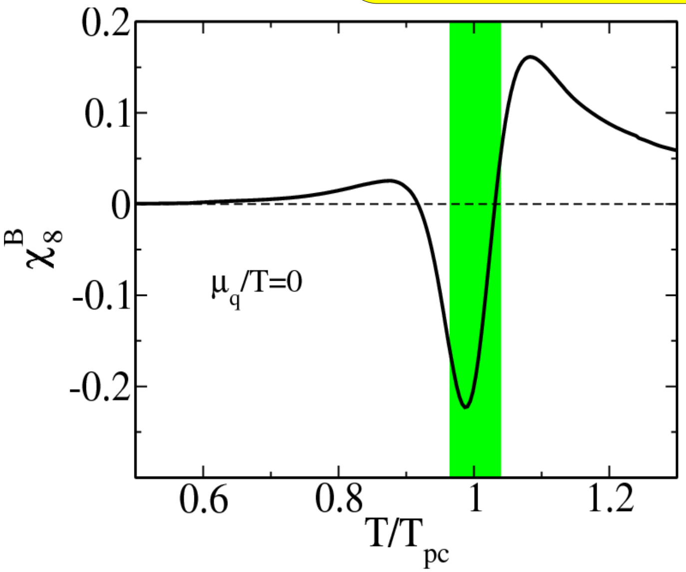
$$\frac{\chi^{(6)}_{B,\mu}}{\chi^{(2)}_{B,\mu}} = \begin{cases} = 1 & \text{, hadron resonance gas} \\ < 0 & \text{, QCD at the crossover transition} \end{cases}$$

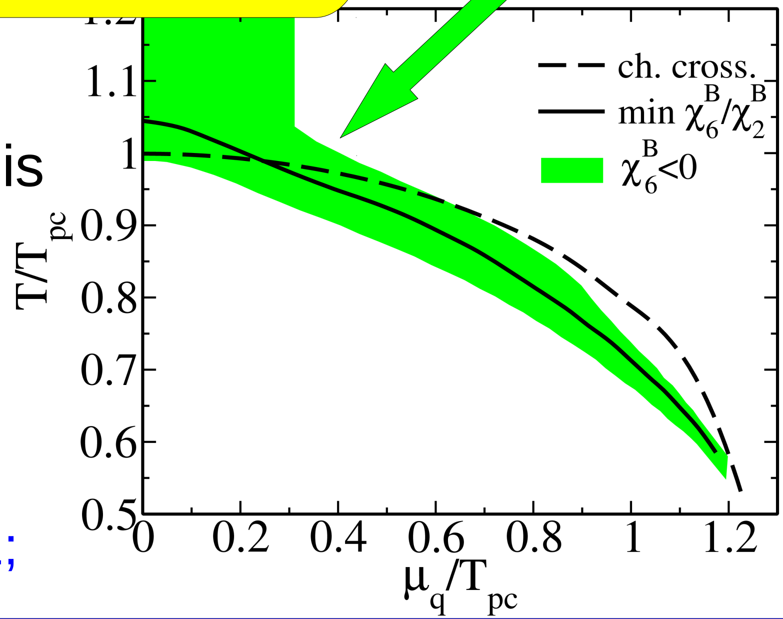
$$\chi^{(6)}_{B,\mu_B} = \chi^{(6)}_{B,0} + \frac{1}{2}\chi^{(8)}_{B,0}\left(\frac{\mu_B}{T}\right)^2 + \ldots$$

$$\chi^{(6)}_B(\mu_B) < 0$$



PQM model
FRG analysis

B. Friman et al., arXiv:1103.3511;



Conclusion

- higher moments of charge fluctuations are increasingly sensitive to critical behavior, even at $\mu_B = 0$

- experimental results on moments of B-charge fluctuations up to 4th order similar to HRG and LGT calculations, BUT significant deviations from HRG model exist already for the 2nd order cumulant (variance)

- close to the QCD transition temperature higher order moments are significantly different in HRG model and lattice QCD

- even at $\mu_B = 0$ negative 6th order moments signal the proximity of freeze out and transition temperatures

  Establishing this at $\mu_B \simeq 0$ provides an anchor point for the analysis of the entire phase diagram

Charge fluctuations in the HRG, LGT & RHIC

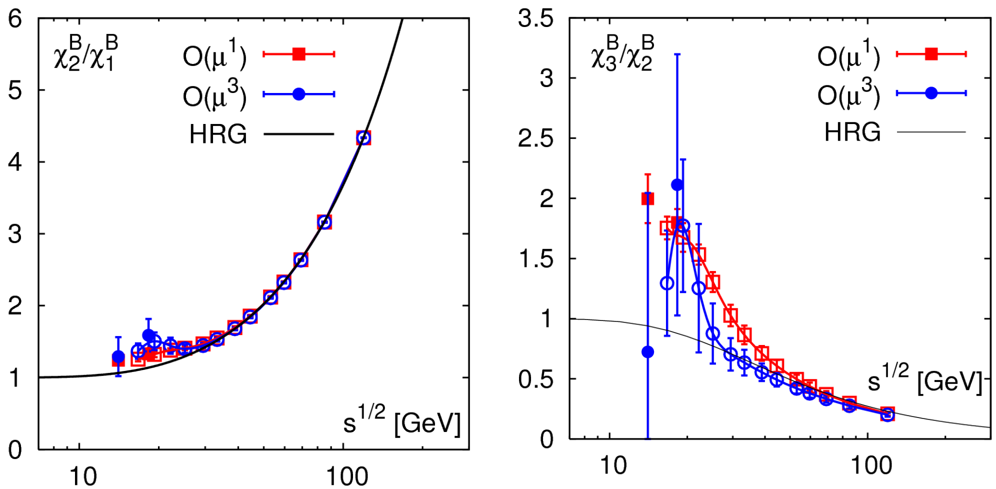
quark number susceptibilities
on the freeze-out curve:

J. Cleymans et al., PRC 73, 034905 (2006)

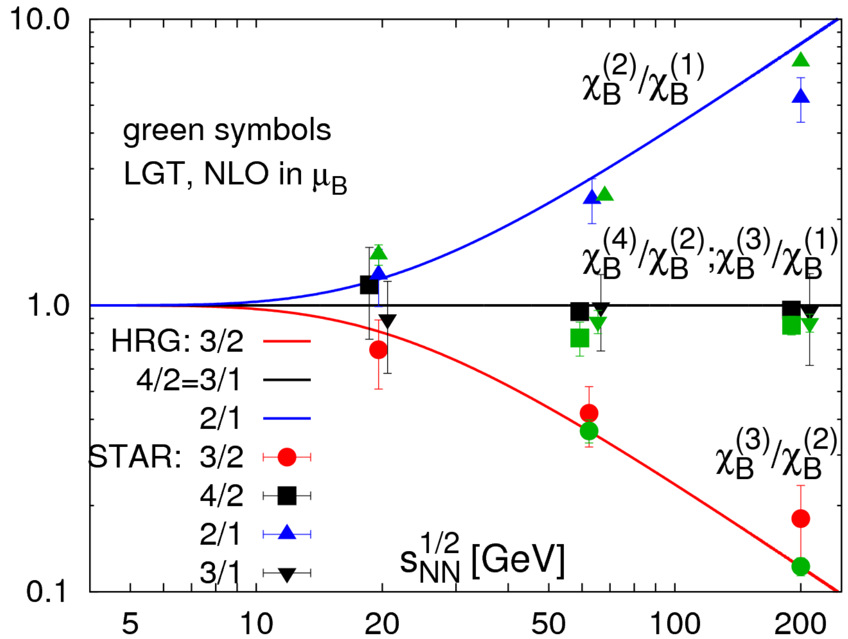
$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4$$
$$\mu_B(\sqrt{s_{NN}}) = d/(1 + e\sqrt{s_{NN}})$$

e.g.:

$$\frac{\chi^{(3)}_{B,\mu}}{\chi^{(1)}_{B,\mu}} = \frac{\chi^{(4)}_{B,0} + \frac{1}{6}\chi^{(6)}_{B,0}(\mu_B/T)^2 + \ldots}{\chi^{(2)}_{B,0} + \frac{1}{6}\chi^{(4)}_{B,0}(\mu_B/T)^2 + \ldots}$$



Moments of B-charge fluctuations



HRG(lines): FK, K Redlich, PL B695 (2011) 136
lattice (green): C. Schmidt, arXiv:1007.5164
Experiment: STAR: arXiv:1004.4959

B-charge fluctuations at freeze-out are consistent with HRG model predictions and lattice calculations

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J. Cleymans et al., PRC 73, 034905 (2006)

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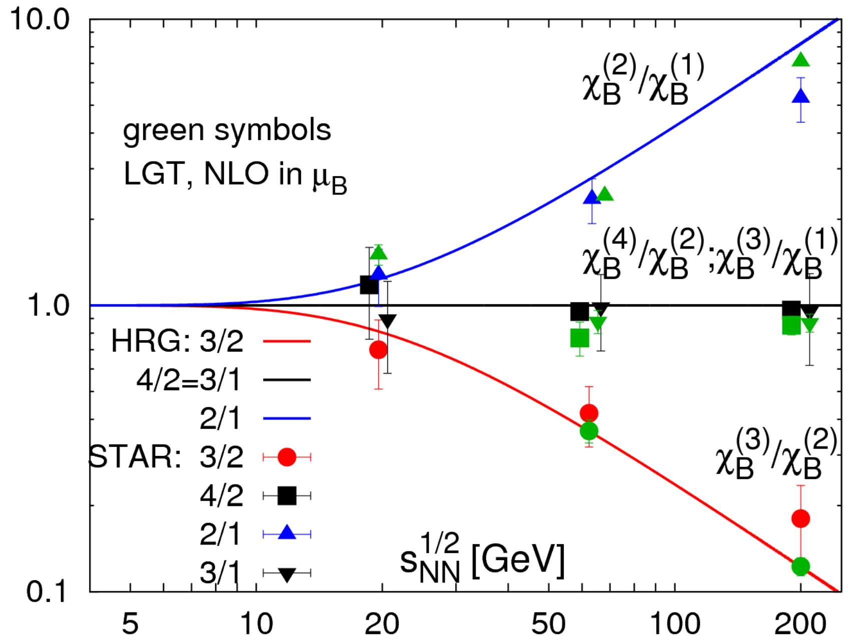
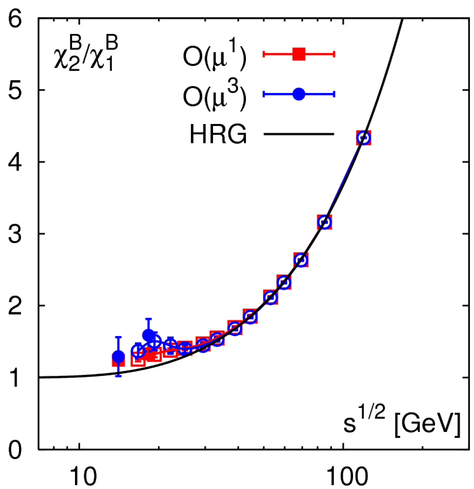
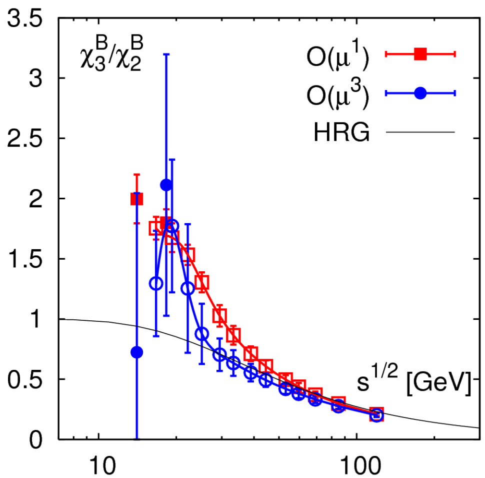
Moments of B-charge fluctuations







warning: at present this comparison does not use Tc(0) directly determined from LGT but assumes:

$$T_c(0) \gtrsim T_{freeze}(0)$$