Resolution of the proton radius puzzle via off shell form factors

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[Natural Resolution of the Proton Size Puzzle.](#)
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e-Print: **arXiv:1101.4073 to appear PRA RC**

Work stimulated by Pohl et al Nature 4466,213 (8 July 2010)

Experimental summary

Pohl et al Nature 466,213 (8 July 2010)
Pulsed laser spectroscopy

“measure a muonic Lamb shift of 49,881.88(76) GHz. On the basis of present calculations[11–15] of fine and hyperfine splittings and QED terms, we find $r_p = 0.84184(67)$ fm, which differs by 5.0 standard deviations from the CODATA value[3] of 0.8768(69) fm. Our result implies that either the Rydberg constant has to be shifted by $-110\,\mathrm{kHz}/c$ (4.9 standard deviations), or the calculations of the QED effects in atomic hydrogen or muonic hydrogen atoms are insufficient.”

- radius 4% smaller than previous
- Rydberg is known to 12 figures

$$R_\infty = \frac{m_e e^4}{8\varepsilon_0^2 h^3 c} = 1.097\,373\,156\,852\,5\,(73) \times 10^7\ \mathrm{m}^{-1},$$

Bohr

$$E = R_\infty / n^2$$
$$V \sim 1/r$$

- QED effects insufficient??
- Puzzle- why muon H different than e H?

Possible resolutions

"The 1S-2S transition in H has been measured to 34 Hz, that is, 1.4×10^{-14} relative accuracy. Only an error of about 1,700 times the quoted experimental uncertainty could account for our observed discrepancy."

- Experiment is wrong
  electron?
- muon interacts differently than electron
- QED bound states calculations slightly wrong

  Carroll, Thomas, Rafelski, Miller, Non-Perturbative Relativistic Calculation [arXiv:1104.2971 [physics.atom-ph]]. finds NO relevant effect

- our effect

Experimental analysis

Extract the proton radius from the transition energy,

compare measured ξ to the following sum of contributions:

ξ=206.2949(32) meV -One measured number

$\xi = \boxed{206.0573(45)} - 5.2262 r_p^2 + 0.0347 r_p^3$ meV

three computed numbers

To explain puzzle:

increase 206.0573 meV by 0.31 meV$=$ 3.1$\times 10^{-10}$ MeV

Pohl's Table of calculations

Lamb shift: vacuum polarization many, many terms

#	Contribution	Ref.	Our selection Value	Unc.	Pachucki[1–3] Value	Unc.	Borie[5] Value	Unc.
1	NR One loop electron VP	1,2			205.0074			
2	Relativistic correction (corrected)	1–3,5			0.0169			
3	Relativistic one loop VP	5	205.0282				205.0282	
4	NR two-loop electron VP	5,14	1.5081		1.5079		1.5081	
5	Polarization insertion in two Coulomb lines	1,2,5	0.1509		0.1509		0.1510	
6	NR three-loop electron VP	11	0.00529					
7	Polarisation insertion in two and three Coulomb lines (corrected)	11,12	0.00223					
8	Three-loop VP (total, uncorrected)				0.0076		0.00761	
9	Wichmann-Kroll	5,15,16	−0.00103				−0.00103	
10	Light by light electron loop contribution (Virtual Delbrück scattering)	6	0.00135	0.00135			0.00135	0.00015
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	1,2	−0.00500	0.0010	−0.006	0.001	−0.005	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17–19	−0.00150					
13	Mixed electron and muon loops	20	0.00007				0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21–23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	22,23	0.000047					
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	22,23	−0.000015					
17	Recoil contribution	24	0.05750		0.0575		0.0575	
18	Recoil finite size	5	0.01300	0.001			0.013	0.001
19	Recoil correction to VP	5	−0.00410				−0.0041	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	2,7	−0.66770		−0.6677		−0.66788	
21	Muon Lamb shift 4th order	5	−0.00169				−0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	2,5–7	−0.04497		−0.045		−0.04497	
23	Recoil of order α^6	2	0.00030		0.0003			
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	1,2,7	−0.00960		−0.0099		−0.0096	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	−0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

We want factor of 2

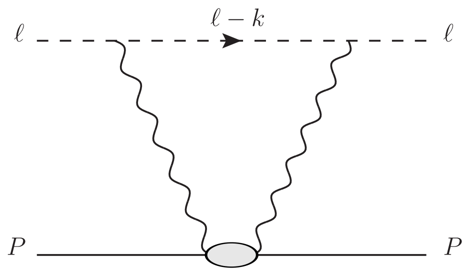
Table 1: All known radius-**independent** contributions to the Lamb shift in μp from different authors, and the one we selected. We follow the nomenclature of Eides *et al.*[7] Table 7.1. Item # 8 in Refs.[2,5] is the sum of items #6 and #7, without the recent correction from Ref.[12]. The error of #10 has been increased to 100% to account for a remark in Ref.[7]. Values are in meV and the uncertainties have been added in quadrature.

Why proton polarization?

proton structure effects $\sim \Phi(0)^2 \sim \alpha^3 \; m^2$

sufficient effect for μH too big for eH

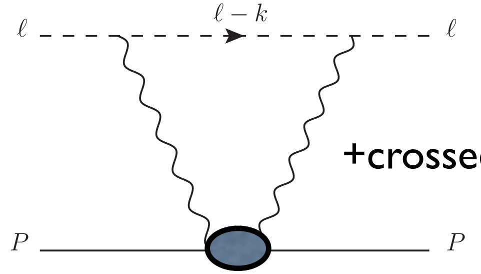
Polarization graph $\propto$ lepton mass m, effect $\sim \alpha^5 m^4$ OK



But Pachuchi: 0.015 meV, what's wrong ?

The Controversy- our effect is 20 times that of Pachucki, Martynenko... Carlson & Vanderhaeghan 1101.5965

Conventional approach $\sim$ Pachucki



+crossed photons

$$\Delta E \propto \alpha^5 m^3 \int \frac{d^4 q}{q^4} T^{\mu\nu} l_{\mu\nu}(m)$$

$T^{\mu\nu}$ is forward virtual-photon proton scattering amplitude,

$l_{\mu\nu}(m)$ is lepton-tensor

$$T^{\mu\nu}(q,P) = -i \int d^4x e^{iq\cdot x} \langle P|T(j^\mu(x) j^\nu(0)|P\rangle$$

$$T^{\mu\nu}(q,P) = -(g^{\mu\nu} - \cdots)T_1 + (P^\mu - \cdots)(P^\nu - \cdots)T_2$$

$Im(T_{1,2}) \propto W_{1,2}$ Measured structure functions

Cauchy plus data $\longrightarrow$ answers –rock solid (?)

Features

- need subtracted dispersion relation for T_1
- subtraction function ($q^0 = 0$, all q^2) totally unknown
- Insert complete sets of states in

$$-i \int d^4x e^{iq\cdot x}\langle P|T(j^\mu(x)j^\nu(0)|P\rangle$$

nucleon term treated as a Feynman diagram

needed because must remove iteration of potential that occurs by solving wave eqn

Totally ambiguous because nucleon is not an elementary Dirac particle

F_2 is not zero

Wrong to use Feyman propagator to represent intermediate states of composite fermions -Brodsky Primack 1969

Compton scattering

Fig. 10.10 Feynman diagrams for Compton scattering.

- You get different answers using the forms related by Gordon

$$\Gamma_1^\mu(P,q) = \gamma^\mu F_1(-q^2) + i\frac{\sigma^{\mu\alpha}q_\alpha}{2M}F_2(-q^2)$$

$$\Gamma_2^\mu(P,q) = \gamma^\mu(F_1(-q^2) + F_2(-q^2)) - \frac{(2P+q)^\mu}{2M}F_2(-q^2)$$

$$\Gamma_3^\mu(p,q) = \frac{(2P+q)^\mu}{2M}F_1(-q^2)+i\frac{\sigma^{\mu\alpha}q_\alpha}{2M}(F_1(-q^2)+F_2(-q^2))$$

Calculation of Born diagram is ambiguous, difference between using forms 1 and 2 gives **0.4 meV**- need another method

J.L. Powell first proton Compton calculation, PR75,32(1949)

"The assumption that the *proton* is correctly described by Pauli's equation is admittedly questionable. However, it is of interest to apply

Hill & Paz 1101.5965 similar worries

Our idea I-proton is not an elementary Dirac particle

- bound proton is off its mass shell
- form factor contains terms containing

$$p \cdot \gamma - M, \; p^2 - M^2$$ Inverse propagator

- ``virtuality'' terms important for EMC effect, Strikman Frankfurt, Kulagin, Petti
- Old idea-Zemach in 50's + many, use free proton form factors in atoms
- Bincer 1960 Naus & Koch PRC36, 2459(1987)- complete (half-on) vertex

function has 4 invariant functions

- Ball Chiu vertex QED why not?

Our idea II-specifics

$$\Gamma^\mu(p',p) = \gamma_N^\mu F_1(-q^2) + F_1(-q^2)F(-q^2)\mathcal{O}^\mu_{a,b,c} \qquad q = p' - p$$

$$\mathcal{O}^\mu_a = \frac{(p+p')^\mu}{2M}[\Lambda_+(p')\frac{(p\cdot\gamma_N - M)}{M} + \frac{(p'\cdot\gamma_N - M)}{M}\Lambda_+(p)]$$

$$\mathcal{O}^\mu_b = ((p^2 - M^2)/M^2 + (p'^2 - M^2)/M^2)\gamma_N^\mu$$

$$\mathcal{O}^\mu_c = \Lambda_+(p')\gamma_N^\mu\frac{(p\cdot\gamma_N - M)}{M} + \frac{(p'\cdot\gamma_N - M)}{M}\gamma_N^\mu\Lambda_+(p)$$

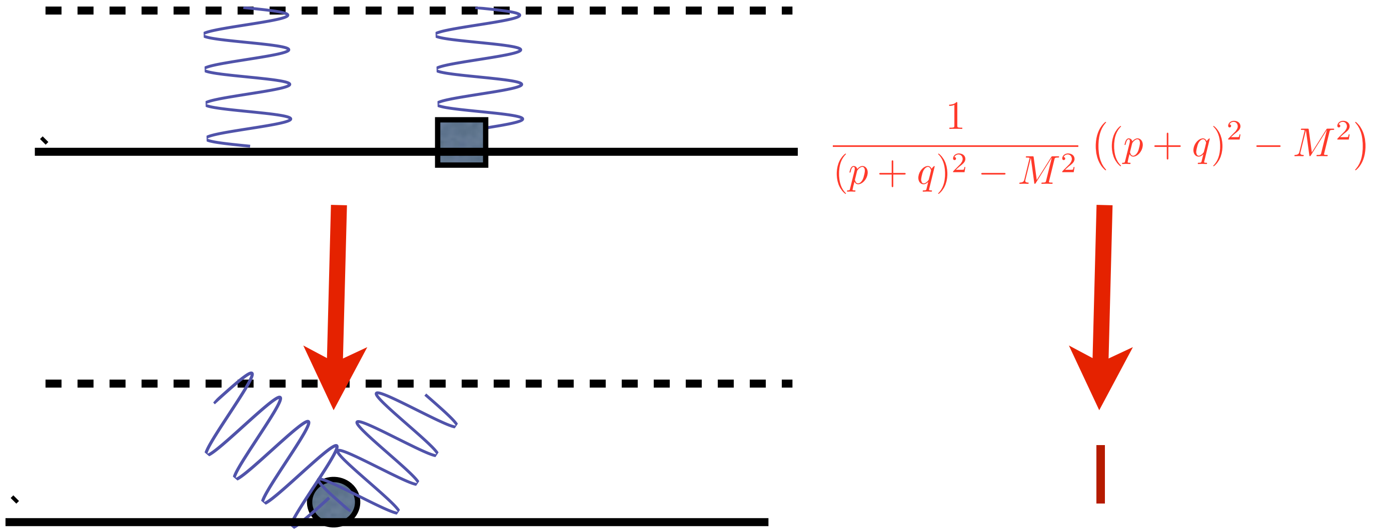
$$F(-q^2) = \frac{-\lambda q^2/b^2}{(1-q^2/\widetilde{\Lambda}^2)^{1+\xi}}.$$

F(0)=0,
off-shell proton charge = proton charge
gauge invariance

$$\text{parameters } \lambda/b^2,\ \tilde{\Lambda} = \Lambda\ ,\ \xi = 0$$

Basically one strength parameter

Cartoon of calculation



Results

$$\langle 2S|V|2S\rangle \;=\; \frac{-\alpha^5 m_r^3}{M^2}\frac{8}{\pi}\lambda\frac{mM}{b^2}F_L(m)\;,$$

α^3, m_r^3 from wave function at origin
$\alpha^2\ m$ from diagram evaluation

Effect $\propto m^4$, absent in hydrogen

$F_L(m)$ is almost independent of lepton mass

There is no hyperfine shift

$$\frac{\lambda}{b^2} = \frac{2}{(79\mathrm{MeV})^2}$$

This value gives entire discrepancy, could be 1/2 as much. Big or small?

Estimate ratio of correction to quasielastic scattering
Pretty big ~15%, right up against limits

- 2 photon exchange term in ep scattering
- gamma, Z exchange in PV ep scattering
- virtual Compton scattering
- .... idea is testable

Summary

- There is a puzzle with the proton radius
- Effects of off shell proton (contact interaction) can remove discrepancy without spoiling electron hydrogen
- previous calculations of proton polarizability missed our effect because proton is not an elementary Dirac particle- F_2 is not zero
- Effect is testable in other arenas

Spares follow

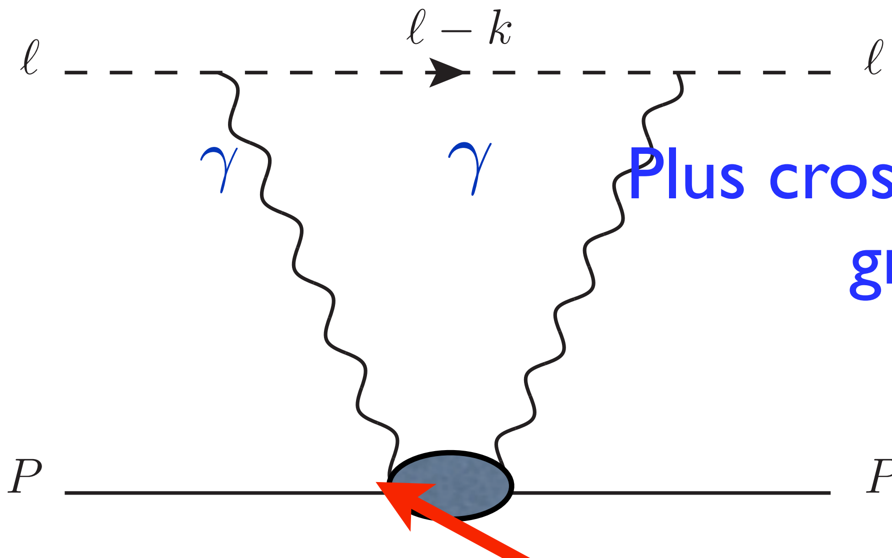
Support from Hill & Paz 1101.5965

- Feynman diagram for Born term is Sticking In Form Factor Model (SIFF)-``not derived from a well-defined local field theory"
    - more serious problem the subtraction function lacks theoretical control, goes as $1/Q^2$ not $1/Q^4$. Eqn used by Pachucki and CV not derived from first principles, uncertainly at least a factor of 10 higher than estimated

Our idea II-mechanism

lepton

proton



proton

Plus crossed photon graph

off-mass shell proton-
inverse Feynman propagator cancels Feynman propagator: makes contact interaction

electromagnetic vertex function parameters unknown

calculation is exploratory

needs off-shell form factor

$F(-q^2), F(0)=0$ charge and current conservation

unknown strength

Electron-proton interaction in atoms

Proton current

$$J^\mu = \bar{u}(p')\left(\gamma^\mu F_1(-q^2) + i\frac{\sigma^{\mu\nu}}{2M}q_\nu F_2(-q^2)\right)u(p),\ \ q \equiv p' - p$$

non-relativistic limit $J^0 \to G_E(\mathbf{q}^2) = F_1(\mathbf{q}^2) - \frac{\mathbf{q}^2}{4M^2}F_2(\mathbf{q}^2)$

Coon and Bawin

Phys. Rev. C 60, 025207 (1999

$$\Delta V_c(\mathbf{r}) = 4\pi\alpha \int \frac{d^3q e^{i\mathbf{q}\cdot\mathbf{r}}}{(2\pi)^3\mathbf{q}^2}(G_E(\mathbf{q}^2) - 1),\ \ G_E(\mathbf{q}^2) - 1 \approx 1 - \mathbf{q}^2 r_p^2/6$$

$r_p^2/6$: negative slope of G_E, not proton radius

$$\Delta V_C(\mathbf{r}) \approx -\frac{2\pi\alpha}{3}\delta(\mathbf{r})r_p^2,\ \ \Delta E = \langle\psi_S|\Delta V_C|\psi_S\rangle = \frac{2}{3}\pi\alpha|\psi_S(0)|^2 r_p^2$$

Karplus, Klein, Schwinger

r_p is **not** the proton radius

S-states only

Interpretation of Sachs - $G_E(Q^2)$ is Fourier transform of charge density **WRONG**

$$G_E(\vec{q}^{\,2}) = \int d^3r\rho(r)e^{i\vec{q}\cdot\vec{r}} \to \int d^3r\rho(r)(1 - \vec{q}^{\,2}r^2/6 + \cdots)$$

Correct non-relativistic:

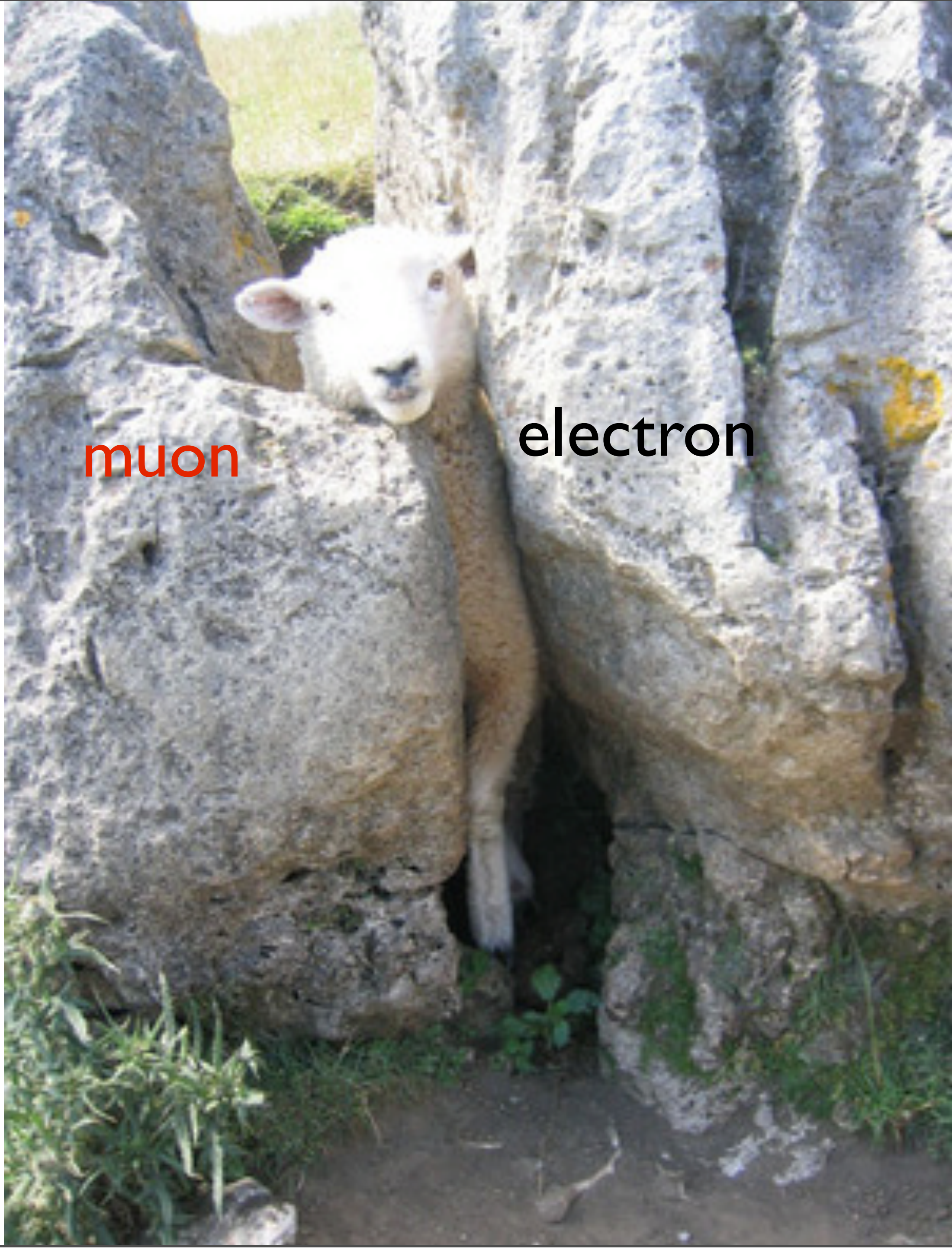
wave function invariant under Galilean transformation

Relativistic : wave function is frame dependent. Final wave function is boosted: initial and final states differ

there is no Ψ*Ψ

Interpretation of Sachs FF is wrong

Need relativistic treatment


muon
electron

Proton radius

Old non-covariant way $\rho_{3D}(r) = 4\pi \int \frac{q^2 dq}{(2\pi)^3} G_E(|\vec{q}|^2) \frac{\sin |\vec{q} r|}{|\vec{q}| r}$, 3 vector

$r_p^2 = \langle r_p^2 \rangle = \int d^3r \; r^2 \rho(r) = -\frac{1}{6} \frac{dG_E(|\vec{q}|^2)}{d|\vec{q}|^2}\Big|_{|\vec{q}|^2=0}$ **Not real radius**

Correct way $\rho(b) = \int \frac{Q\, dQ}{(2\pi)} F_1(Q^2) J_0(Qb)$, a true density, Q invariant

$\langle b^2 \rangle = \int d^2b \; b^2 \rho(b) = -\frac{1}{4} \frac{dF_1(Q^2)}{dQ^2}\Big|_{Q=0}$

b^2 is an honest moment

$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2) \rightarrow$

$$\langle b^2 \rangle = \frac{2}{3} r_p^2 - \frac{2}{3} \frac{1.79}{M^2}$$

Low energy muonic atom sees proton more closely