VISCOSITY IN NUCLEI AND THE η / s RATIO

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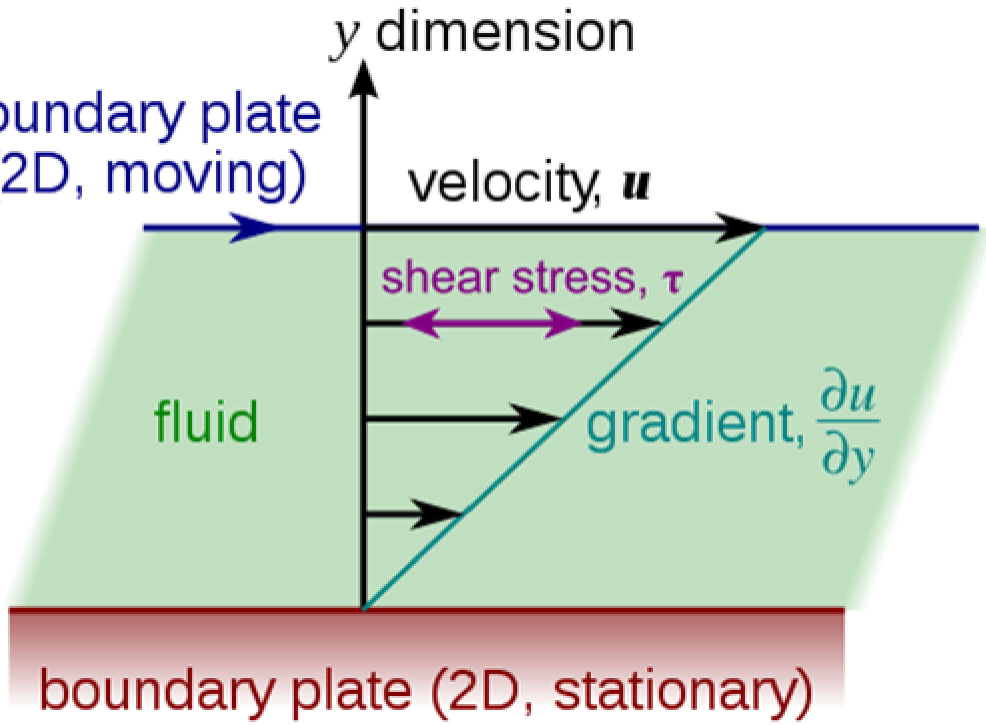
(N.Auerbach and S.Shlomo, Phys. Rev. Letters, **103**, 172501 (2009).)

Viscosity (from Wikipedia)

- **Viscosity** is a measure of the resistance of a fluid which is being deformed by either shear stress or tensile stress. In everyday terms (and for fluids only), viscosity is "thickness". Thus, water is "thin", having a lower viscosity, while honey is "thick", having a higher viscosity. Viscosity describes a fluid's internal resistance to flow and may be thought of as a measure of fluid friction .

viscosity

$$\tau = \mu \frac{\partial u}{\partial y}.$$



Viscosity

Viscosity is a reflection of the efficiency of momentum transfer. More efficient transfer means smaller viscosity and more fluidity.

Shorter is the mean free path smaller is the viscosity.

The η/s ratio.

[1].P.Kovtun, T.D.Son, and O.Starinets, Phys.Rev.Lett. 94, 111601 (2005). AdS/CFT

In certain supersymmetric gauge theories one finds [1] that the ratio of shear viscosity η to entropy density s is equal to:

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} \approx 6.05 \times 10^{-13} K\ s \qquad (1)$$

where k_B is the Boltzmann constant. It has been conjectured that this ratio is the lower limit for a large class of quantum field theories. The analysis of the ultrarelativistic heavy ion collisions data from RHIC seems to indicate that the state of matter produced behaves like a liquid with the above ratio being close to the lower limit [2]. Thus the matter produced behaves as a perfect fluid.

A simple derivation

- One can derive derive the above relation using a very simple consideration based on the uncertainty principle

$$\eta \sim \rho_m \mathrm{v} l \sim \rho_m \mathrm{v}^2 t \sim \rho m \mathrm{v}^2 t \sim \rho E t$$

where ρ_{m} is the mass density, ρ the particle density,

l mean free path,

E the energy and t - mean time between collisions.

$$\mathrm{s} \sim \rho \mathrm{k}_{\mathrm{B}}$$

$$\frac{\eta}{s} \sim \frac{Et}{k_B}, \text{ from the uncertainty principle } Et \geq \hbar$$

$$\frac{\eta}{\mathrm{s}} \sim \frac{Et}{k_B} \geq \frac{\hbar}{k_B}$$

or……

one can write :

$$\frac{\eta}{s} = \chi \frac{m\mathrm{v}l}{k_B} \quad \text{with } \chi \text{ being a constant}$$

$$\frac{\hbar}{m\mathrm{v}} = \lambda_d \quad \text{is the de Broglie wave length}$$

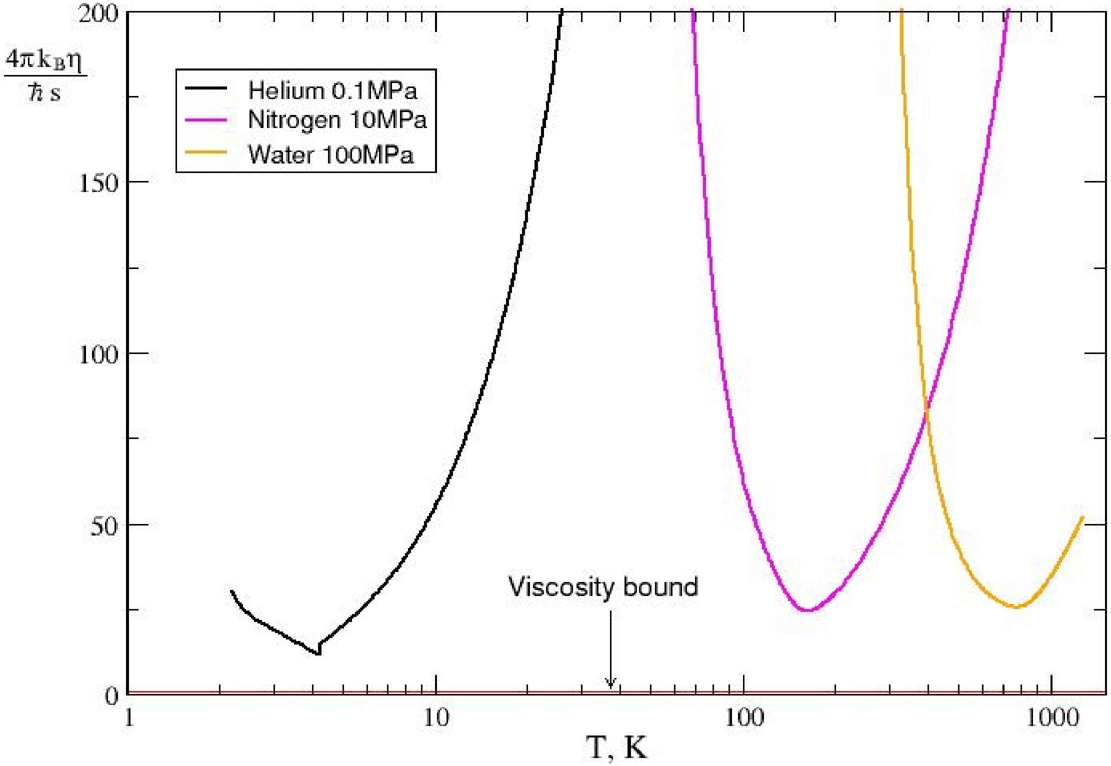
therefore

$$\frac{\eta}{\mathrm{s}} = \chi \frac{\hbar}{k_B} \frac{l}{\lambda_d}$$

but $\frac{l}{\lambda_d} \geq 1$ and therefore $\frac{\eta}{\mathrm{s}} \geq \chi \frac{\hbar}{k_B}$

A viscosity bound conjecture

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13}\, K \cdot s$$



Minimum of $\frac{\eta}{s}$ in units of $\frac{\hbar}{4\pi k_B}$

Xe	84
Kr	57
CO_2	32
H_2O	25
C_2H_5OH	22
Ne	17
He	8.8

P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231

The KSS postulate

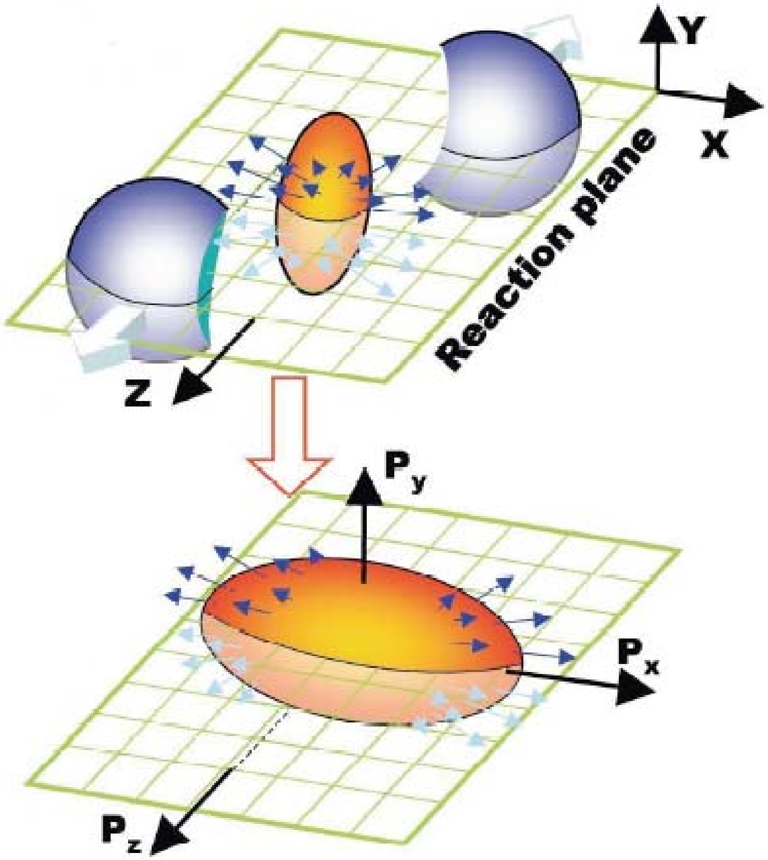
KSS postulate that the relation

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \text{ is universal}$$

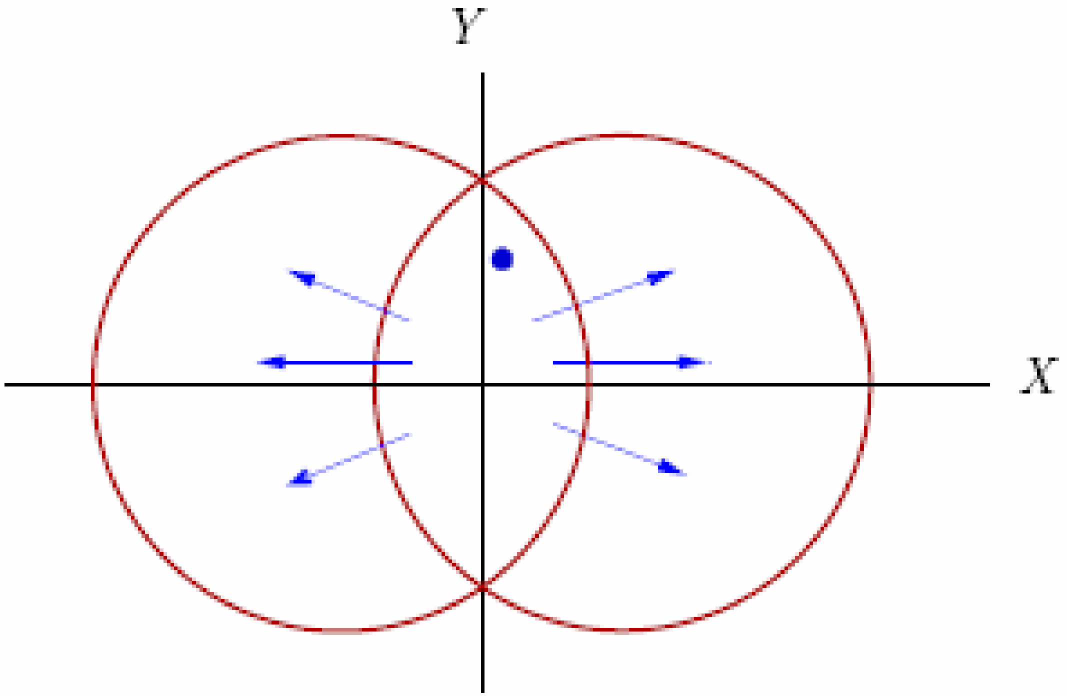
(See also P.Danielewicz and M.Gyulassy,

Phys. Rev D 31, 53 (1985))

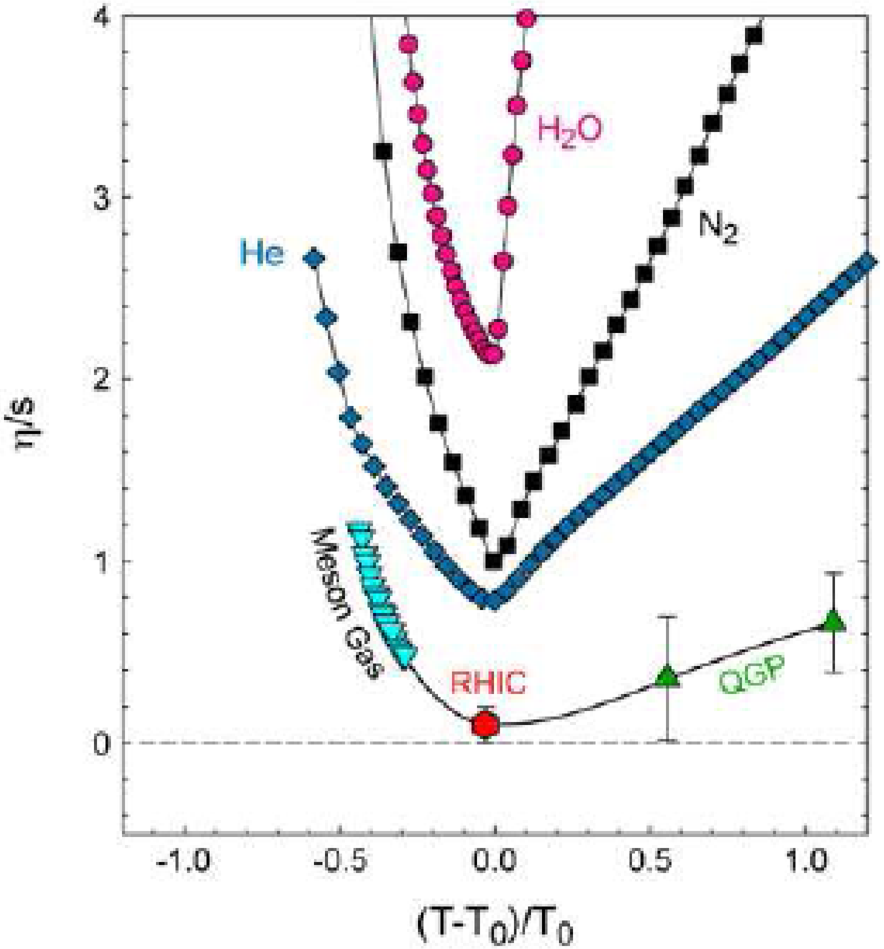
Relativistic Heavy Ions at RHIC



Elliptic Flow



Elliptic Flow



H2O
N2
He
Meson Gas
RHIC
QGP
η/s
(T-T0)/T0
4
3
2
1
0
-1.0
-0.5
0.0
0.5
1.0

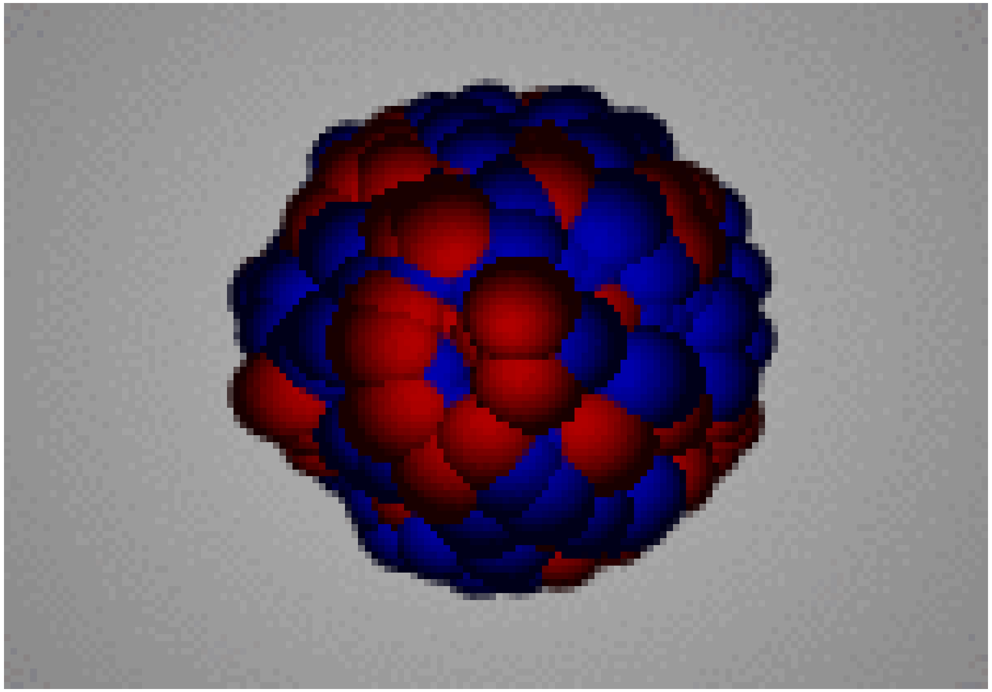
Finite Nucleus

It is however important to examine the values of the η/s in systems close to the ones obtained in the relativistic collisions at RHIC. One such system is a finite nucleus. It is quite clear that there is strong affinity of the matter formed in the RHIC experiments and "conventional" finite nuclei. The same forces are active in both systems and after all the state in RHIC experiments is due to the interaction of finite nuclei. In this work we first demonstrate that a consistent value for η is deduced from (i) analysis of the width of giant resonances within the hydrodynamic model, (ii) kinetic theory, and from (iii) the process of fission described using liquid drop models. We then provide a simple assessment of the entropy density.

Hydrodynamical Models in Nuclei

The use of hydrodynamical models in nuclei has a long history. The first successful calculations of nuclear masses were obtained from a model that has considered nuclei to behave as liquid drops. Hydrodynamical models were employed in the description of vibrational states in nuclei. In particular so called giant resonances were described as vibrations of proton and neutron fluids [3]. The isoscalar vibrations consist of proton and neutron fluids collectively vibrating in phase, while the isovector ones are described as vibrations of the proton liquid out of phase, with the neutron fluid. In most cases these models were successful in reproducing the experimental excitation energies and cross sections of reactions exciting such resonances.

Giant dipole



Giant resonances and their width

N.Auerbach and A.Yeverechyahu, Annals of Physics, **95**, 35 (1975).
("Nuclear Viscosity and Widths of Giant Resonances")

Most of the giant resonances, at excitation energies in the range of 10-40 MeV have a finite life time and carry a width. Following the success of the hydrodynamical models an attempt was made to link the widths of these resonances to the viscosity of the proton-neutron fluids [4]. In Ref. [4] a set of coupled hydrodynamical equations of the Navier- Stokes type was used to describe the flow of two viscous fluids, of protons and neutrons. The solution of these equations gave rise to giant resonances of both isoscalar (I=0) and isovector (I=1) type. For example the linearized Navier-Stokes equation for the isoscalar mode is:

Navier-Stokes equation

$$\frac{\partial \vec{v}_0}{\partial t} = -\frac{1}{\rho} u_0^2 \vec{\nabla} \rho + \nu \nabla^2 \vec{v}_0 + \frac{1}{3} \nu \vec{\nabla} \vec{\nabla} \cdot \vec{v}_0$$

where: $\rho = \rho_n + \rho_p$ (ρ_n and ρ_p being the neutron and proton densities, respectively), u_0 is the wave velocity, ν the kinematical viscosity and

$$\vec{v}_0 = \frac{\rho_n \vec{v}_n + \rho_p \vec{v}_p}{\rho_n + \rho_p}.$$

Solving this equation with appropriate boundary conditions and for various multipolarities one obtains eigenvalues containing a real and imaginary part.

The real part represents the energy of the excitation and the imaginary part depends on the kinematical viscosity parameter ν and represents the lifetime of the excitation.

The mass dependence A of the computed widths exhibits the experimental trends of the giant isoscalar resonances. As a result of such calculation one obtains a value for the kinematical viscosity [4]:

$$\nu = 0.6 \times 10^{22}\,\mathrm{fm}^2\,\mathrm{sec}^{-1} \tag{4}$$

The relation between the kinematical viscosity and the shear viscosity is:

$$\eta = \rho\nu \tag{5}$$

For the above value of ν the shear viscosity is:

$$\eta \approx 1 \times 10^{-23}\,\mathrm{MeV\ fm}^{-3}\ \mathrm{sec} \tag{6}$$

J.R.Nix and A.J.Arnold, Phys. Rev. C **21**, 396 (1980)

A later study $\eta \approx (1.9 \pm 0.6) \times 10^{-23} \mathrm{MeV\ fm}^{-3}\ \ \mathrm{sec}$

Fermi Liquid Drop Model

V.M.Kolomietz and S.Shlomo Phys.Rep. **390**, 133 (2004).

Recently, in Ref. [6], the authors described the dynamics of cold and hot nuclei within a generalized Fermi liquid drop model by employing a collision kinetic equation, which properly accounts for the dissipative propagation of sound waves in finite nuclei and nuclear matter. For a temperature $T < \varepsilon_F$ and excitation energy $\hbar\omega < \varepsilon_F$ of the sound wave, one finds for the collision viscosity

$$\eta = \frac{2}{5}\rho\varepsilon_F \frac{\tau_{coll}}{1+\left(\omega\tau_{coll}\right)^2} \tag{8}$$

where

$$\tau_{coll} = \frac{\tau_0}{1+\left(\hbar\omega/2\pi T\right)^2} \tag{9}$$

with

$$\tau_0 = \hbar\alpha / T^2 \tag{10}$$

- τ_{coll} is the Landau approximation for the collision relaxation time deducted from the collision integral.
- It was shown that the above form nicely describes both regimes of high and low frequencies ω, which correspond to zero sound (giant resonance) and first sound, respectively, as well as the intermediate regime.

At low temperature, $T \leq \varepsilon_F$ the relation between the thermal excitation energy *E** and *T* is given by

$$E^* = aT^2$$

where a is the level density parameter.

For $T \approx 1\,MeV$

$$\eta \approx 0.5\text{x}10^{-23}\,MeV\ fm^{-3}\ \text{sec}$$

The three values for η quoted here were obtained by considering the dissipation of collective motion as exhibited by the giant resonances. The values deduced in all three cases are consistent and the range of the values is:

$$\eta = (0.5 - 2.5)\text{x}10^{-23}\, MeV\ \ fm^{-3}\ \text{sec}$$

Fission

K.T.Davies, A.J.Sierk, and J.Nix, Phys. Rev. C 13, 2385 (1976).

Another type of collective motion encountered in the dynamics of nuclei is the process of fission. A number of works appeared in the literature which dealt with the dynamics of fission in heavy nuclei using viscous liquid drop models [9, 10, 11]. For example, in [9] the authors use a macroscopic approach and solve classical equations of motion for the fissioning nucleus. They apply this to spontaneous and induced fission. They find that the average value for the shear viscosity that reproduces best the data is:

$$\eta = (0.9 \pm 0.3)\mathrm{x}10^{-23}\, MeV \;\; fm^{-3}\ \mathrm{sec}$$

K.T.Davies, R.A.Managan, J.R.Nix, and A.J.Sierk, Phys. Rev. C 16, 1890 (1977).

R.Wieczorek, R.W.Haase, and G.Sussman, Proceedings of the Third Symposium of the Physics and Chemistry of Fission, Rochester, N.Y., 1973.

In a later study [10], applying a somewhat different model, the value deduced is twice as large:

$$\eta = (1.9 \pm 0.6)\mathrm{x}10^{-23}\, MeV \;\; fm^{-3}\ \sec$$

In another work that studied nuclear fission:

$$\eta \approx 1\mathrm{x}10^{-23}\, MeV \;\; fm^{-3}\ \sec$$

The values of η found in the studies of the fission process agree generally with the ones found from the work on giant resonances. We will therefore use for the shear viscosity the range of values obtained from the exploration of the widths of giant resonances.

Entropy

In order to evaluate the ratio in Eq. (1) one needs to evaluate the entropy. Our first, simplest determination of the nuclear entropy will be based on models of a free Fermi gas or of non interacting nucleons contained in an average nuclear potential, such as a finite Woods-Saxon well. For low temperature, $T < \varepsilon_F$ the entropy of such a system is given by the simple expression:

$$S = 2aT$$

and

$$s = \frac{\rho}{A} S$$

Values of eta/s

After taking into accounts the spread in the values of the shear viscosity and the values of parameters entering the entropy one arrives at the following limits

$$\boxed{\frac{\eta}{s} \approx (2.5 - 25)\text{x}\,\frac{\hbar}{4\pi k_B}}$$

eta/s ratio at RHIC

The experiments from RHIC provide data that can be used to determine the η/*s* ratio of the state created in these super-relativistic collisions of heavy nuclei. It is presently widely accepted that the form of matter created has a very high fluidity with:

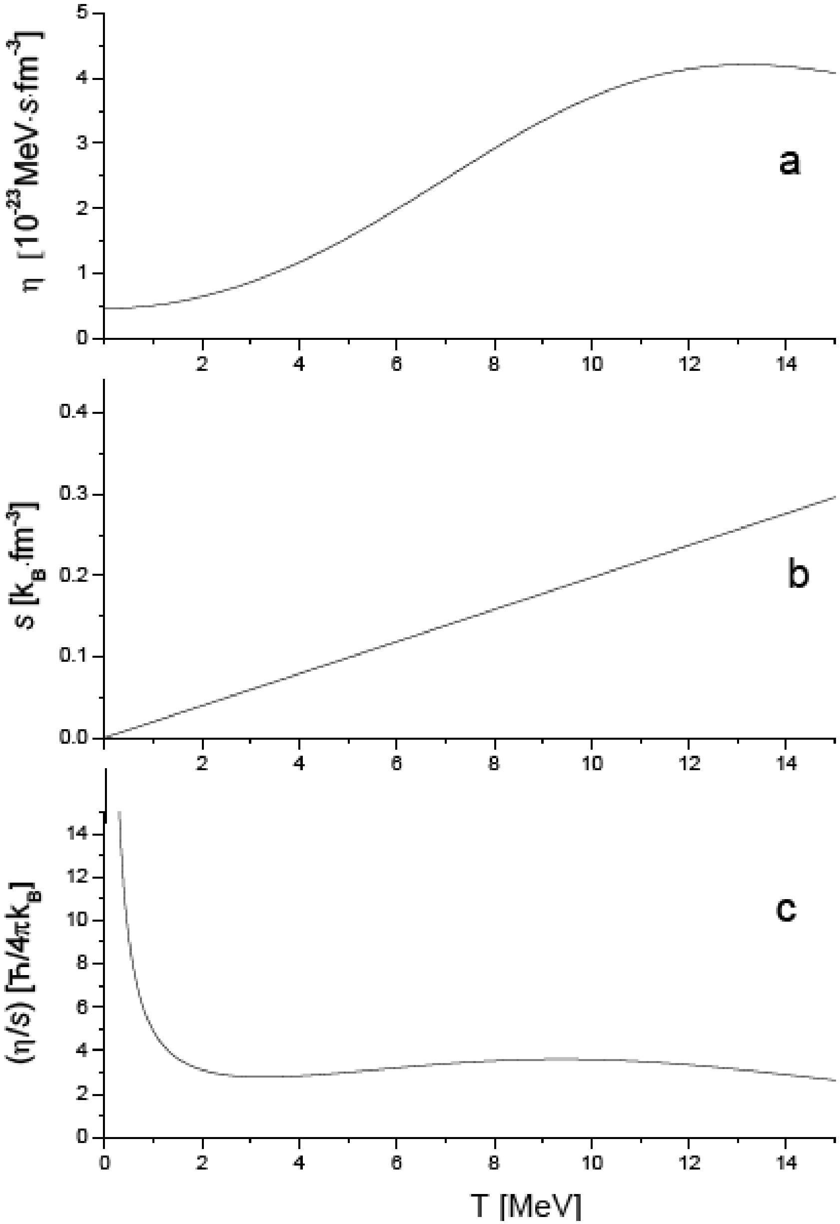
$$\frac{\eta}{s} \leq 5 \times \frac{\hbar}{4\pi k_B}$$

Comparing this value to our results for finite nuclei at low temperature we see that the deduced ratio (especially the lower limit) is not drastically different from the RHIC result. It is possible that the strong fluidity is a characteristic feature of the strong interaction of the many-body nuclear systems in general and not just of the state created in the relativistic collisions.

- Using the work Kolomietz and Shlomo one can write:

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}\frac{16}{5\pi}\varepsilon_F^2\frac{\alpha}{T}\frac{T^2 + (\hbar\omega/2\pi)^2}{(\hbar\omega\alpha)^2 + \left[T^2 + (\hbar\omega/2\pi)^2\right]^2}$$

In Fig.1c we show η/s as a function of temperature. The curve has the characteristic behavior found in other fluids [1], the minimum occurring at $T = 3$ MeV, with the value of $\eta/s \approx 3\times\hbar/4\pi k_B$.


η [10^-23 MeV·s·fm^-3]
a
s [k_B·fm^-3]
b
(η/s) [ħ/4πk_B]
c
T [MeV]

Consequences

The KSS postulate has a universal meaning.

Are there some consequences and useful applications from the KSS conjecture, for "regular" nuclei?

One can, for example, consider the question whether using the KSS inequality it will be possible to set an upper limit for the life time of a nuclear state (or a lower limit for its width), in cases when such state can be described by a hydrodynamical model

Lower limits for widths

- In previous work (N.Auerbach and A. Yeverechyahu) it was determined that the main contribution to the spreading width of a giant resonance can be written as:

$$\Gamma \approx \frac{\hbar \nu \xi_{J,I}^2}{R^2}$$

where ν is the kinematical viscosity, $\xi_{\mathrm{J,T}}$ is a constant characteristic for a giant resonance with total spin J and isospin I.

R is the nuclear radius.

Expressing η in terms of ν and using the above expression for the width and the KKS inequality one obtains :

$$\Gamma \geq \frac{0.5\xi_{\mathrm{J,I}}^2}{A^{2/3}} T$$

- For the isoscalar quadrupole

$$\xi_{2,0} = 3.1$$

and

$$\Gamma \geq \frac{4.8}{A^{2/3}} T.$$

For mass $\mathrm{A} = 125, \quad \Gamma \geq 0.2T \quad (\mathrm{T \text{ in } MeV})$.

For the isoscalar monopole $\xi_{0,0} = 4.5$ and

$$\Gamma \geq \frac{10}{A^{2/3}} T$$

For $\mathrm{A} = 125, \; \Gamma \geq 0.4T$

V.M.Kolomietz, V.A.Plujko and S.Shlomo. Phys. Rev. **C** 52,2480, (1995).

- In a model based on the Vlasov-Landau equation one derives the expression for the monopole:

$$\Gamma \geq \frac{7.2}{A^{2/3}} T$$

These considerations should be applied probably to a small range of low temperatures.

Perfect Fluid