Polarized pairing in neutron star crusts

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Early work done in collaboration with Joe Carlson (LANL)

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Neutron star inner crust

Low-density neutron matter in ion lattice

- Pairing significant: singlet in matter, possibly triplet in nuclei
- Nuclear toy problem of physical relevance
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Properties
- Scattering length ($a$)
- Effective range ($r_e$)
- Density ($\rho$) or ($k_F$)

“Observables”
- Energy ($E/E_{FG}$)
- Pairing gap ($\Delta/E_F$)
The meaning of it all

- Negligible contribution to specific heat consistent with cooling of transients:

- Young neutron star cooling curves depend on the magnitude of the gap:

- Superfluid-phonon heat conduction mechanism viable:

- Constraints to Skyrme-HFB calculations of neutron-rich nuclei:
$^1S_0$ neutron matter pairing gap

No experiment $\rightarrow$ no consensus
$^1S_0$ neutron matter pairing gap

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Cold atoms to the rescue

Theoretical many-body problem formulated by George Bertsch more than 10 years ago:

“What is the ground-state energy of a gas of spin-1/2 particles with infinite scattering length, zero range interaction?”

\[ E = \xi E_{FG} \quad E_{FG} = \frac{3}{5} N \frac{\hbar^2 k_F^2}{2m} \]
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Now within *direct* experimental reach!

Credit: Martin Zwierlein
Hamiltonian: unity in diversity

\[ \mathcal{H} = -\frac{\hbar^2}{2m} \sum_{k=1}^{N} \nabla_k^2 + \sum_{i<j'} v(r_{ij'}) \]

Neutron matter

$^1S_0$ channel of AV18 – later AV4

\[ a = -18.5 \text{ fm}, \quad r_e = 2.7 \text{ fm} \]

Cold atoms

modified Pöschl-Teller potential

\[ a = \text{tunable}, \quad r_e = \text{tunable/infinitesimal} \]
What do we know for sure?

**Weak Coupling**

Equation of state: \( \frac{E}{E_{FG}} = 1 + \frac{10}{9\pi} \xi_F a + \frac{4}{21\pi^2} (11 - 2 \ln 2)(\xi_F a)^2 \)

Pairing gap: \( \frac{\Delta}{E_F} = \frac{1}{(4e)^{1/3}} \Delta_{BCS} \)
What do we know for sure?

**Weak Coupling**

Equation of state: \[ \frac{E}{E_{FG}} = 1 + \frac{10}{9\pi} k_F a + \frac{4}{21\pi^2} (11 - 2 \ln 2) (k_F a)^2 \]

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**Strong Coupling**

Mean-field BCS is easy but unreliable:

\[ \Delta(k) = - \sum_{k'} \langle k | V | k' \rangle \frac{\Delta(k')}{2 \sqrt{\xi(k)^2 + \Delta(k)^2}} \]

*Ab initio GFMC* is difficult but accurate:

\[ \Psi_V = \prod_{i<j} f(r_{ij}) \mathcal{A} \left[ \prod \phi(r_{ij}) \right] \]
Equal Populations

Equation of state
Equations of state: results

- Results identical at low density
- Range important at high density
- Duke and ENS experiments at unitarity

Equations of state: comparison

- Lowest densities on the market; agreement with Lee-Yang trend
- At higher densities all calculations are in qualitative agreement

Odd-even staggering
Pairing gap
Pairing gaps: results

- Results identical at low density
- Range important at high density
- Two independent MIT experiments at unitarity

Pairing gaps: comparison

- Consistent suppression with respect to BCS; similar to Gorkov
- Disagreement with AFDMC studied extensively
- Emerging consensus?

Unequal Populations
Energy versus polarization

\[ \uparrow \uparrow \uparrow \uparrow + \downarrow \downarrow \]
Polarized neutron matter: energy

- Possibly relevant to magnetars and nuclear functionals
- Linear with respect to polarization:

\[ P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}} \]

Polarized neutron matter: momentum distribution

- Spread around $k_F$
- Known as “Fermi surface mismatch”

Conclusions

- Qualitative agreement on equation of state: benchmarking
- Emerging consensus (?) regarding the pairing gap
- Energy versus polarization is linear: benchmarking
The Present Future

- Extreme polarizations: the polaron problem
- Highly asymmetric nuclear matter
- Static response of neutron matter
Energy shell effects: backup

- No pairing term in the functional leads to large shell corrections
- All the while, we've only fit to QMC energies, not pairing gaps.

Neutron $r_e = \text{finite}$

Non-negligible finite-size effects: estimate the trend by comparing to the corresponding mean-field theory results.